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ARITHMETIC;

Ex

IN FOUR BOOKS.

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III. MERCANTILE ARITHMETIC,
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AUTHOR OF THE PRACTICAL ENGLISH GRAMMAR.

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1813.

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PREFACE.

THE Reader is here presented with a Treatise of *Practical Arithmetic for the Use of Schools*, being a complete Extract of the Practical Part from my *Treatise of Arithmetic in Theory and Practice.*

The first edition of which work being now mostly disposed of, and having received some friendly remonstrances as to the execution of it, with respect to paper and type, I resolved to publish a second edition in an elegant octavo volume by subscription; but as this proposed edition might be thought too expensive for a common school book, I (having by experience in my profession as well as by the growing demand, found its usefulness as such) have thus published the Practical Part distinct, at a low price, which I apprehend will be thought sufficient for the common use of school boys, while the work at large, will be highly useful to masters for a fund of instruction, as well as to students who are well accomplished in the practical part.

The method was partly suggested by a note on *Rollin's* thoughts concerning Education, p. 44, of the *Dublin* edition, which appearing to contain some useful hints, I transcribe at length, viz.

"Arithmetic might be taught in schools, in a much "more expeditious way than it generally is, by dividing "the scholars into forms, in the same manner as in "teaching languages, the whole may be divided into five, "six or more classes; ten of the pupils, for instance "may be in Multiplication, six in the Rule of Three, "thirteen in Practice, and so on. Whenever any class "is to proceed to a new rule, the master may explain to "them in chalk on two large boards, or some such thing, "the nature and genius of the rules into which they are "entering. A considerable time should be employed in "these explications, and the scholars might take places, "as in learning latin, &c. which could not fail of inspiring "them with great emulation. The several pupils in "a form should always be set the same sum or question, "but must be separated to prevent their copying one from "another. 'Twould also be proper to draw up for their "use, an epitome of Arithmetic, by way of question "and answer, containing the nature and explanation of "the several rules in that science, this they should copy, "and learn by heart perfectly; by which means they "would be able, not only to state the several questions "very expeditiously, but to give a reason for every "thing."

I apprehend the usual method of teaching Arithmetic is two-fold, either the master or assistant writes down the rules and questions for the boys, or causeth them to write them themselves from printed books or manuscripts: The former method is much more toilsome to the teacher, and besides takes up much time, which I presume might be better employed in instructing the pupils, and explaining to them the nature of their rules. And as most of the printed books are intended to explain the doctrine of Arithmetic in general to those who may want to improve themselves by reading, the questions are most or at least too many of them wrought, for which reason they are not suitable to be laid before the boys, as they would make no scruple of copying the work of their Author to ease themselves of the trouble of performing it; besides as the number of these in a school is generally few, it occasions perplexity and delay, while one waits for another to copy out his rule or question; the same objection lies against one general manuscript, although it consists of questions and their answers only.

To avoid this inconvenience, some teachers have disposed the several rules of Arithmetic, with proper questions and their answers, into little manuscripts of a sheet or two of paper orderly numbered; but these wearing out with continual use, it occasions very considerable labour to keep a school properly supplied therewith; from which considerations, the utility of introducing such a Book as this (properly drawn up) into our Schools, will be seen t the first view.

PREFACE.

Although regularity of method seemed to point out the order, or plan I have observed; yet I am not of opinion it will be necessary to teach all the parts in order as they lie; I think it will be best to begin with Whole Numbers, and proceed through Notation, Addition, Subtraction, Multiplication and Division: Addition and Subtraction of Numbers of divers Denominations, Reduction and the Rule of Three to Contractions, without meddling with the questions at the end of the rules: this may be a first course. *2dly*, Suppose they go over the same with the questions, and Multiplication and Division of Numbers of divers Denominations; through the Rule of Three entirely, and then proceed through Fractions, Vulgar and Decimal. *3dly*, Begin again at Multiplication of divers Denominations, and so proceed orderly quite through Mercantile Arithmetic, and in these reviews the learners may be detained a longer or shorter time in any rule, as they may appear more or less ready in the calculation thereof. *4thly*, And if again they went through the whole, with the reasons of the rules, I do not apprehend such a method absurd; or such a revisal of the same rules unnecessarily tedious: But I only mean just to hint my apprehensions of the use to be made of such a book, without pretending to prescribe to Teachers, many of whom, without doubt, are men of much more experience than myself, and of greater abilities likewise.

If any of these find any material imperfections in this work, have any amendments to propose, or any improvements of the plan, if they will be so kind as to impart them to the author, he hopes to receive them with candor, give them the due deference, and to make the necessary use of them (with a suitable acknowledgment) in a future edition of the work, if there should be hereafter occasion for it.

JOHN GOUGH.

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EXPLANATION *of certain* CHARACTERS *used in the following* WORK.

+ PLUS, or more, is the Mark of Addition; and denotes the Numbers it stands between are to be added together.

— Signifies Minus, or less, is the Sign of Subtraction; and when it stands between two Numbers, it denotes that the latter is to be taken from the former.

× Is the Mark of Multiplication; and denotes that the Numbers betwixt which it stands are to be multiplied together.

÷ Is the Mark of Division; and when two Numbers are placed in the same manner as the Points are here, it denotes, that the Number above the Line is to be divided by that below.

= Is the Mark of Equality; which being set between two Numerical Expressions, denotes they are equal between themselves.

: : : : Are marks of Proportionality; and denote that the Numbers betwixt which they are placed, are proportional Numbers.

Examples.

For $4+3=7$; read, 4 more 3 is equal to 7.

For $4-3=1$; read, when 3 is taken from 4, the remainder is equal to 1.

For $4\times 3=12$; read, 4 multiplied by 3 produces 12.

For $\frac{12}{3}=4$; read, 12 being divided by 3, the quotient is equal to 4.

For [illegible] : 4 : : 3 : 12; read, as 1 is to 4, so is 3 to 12.

A

TREATISE

OF

ARITHMETIC.

BOOK I.

ARITHMETIC is that Part of Mathematics which exhibits the Doctrine of *NUMBERS*, explains their Properties, and teaches the Art of calculating them.

CHAP. I.

NOTATION OF NUMBERS.

Notation is the writing of Numbers properly.

1. All Numbers are expressed by the different disposition of the following ten Characters, called Figures:

Cypher or Nought	One	Two	Three	Four	Five	Six	Seven	Eight	Nine
0	1	2	3		5	6	7	8	9

2. The value of these figures appears above at the first view; but we must observe, that besides their simple value, there expressed, they receive a new value from the place they possess in numbers expressed by several figures.

3. The value of the places encreases in a tenfold proportion infinite.

4. The first place is that next the right hand; the second the next on the left of it, and so on.

5. Every figure standing alone expresses its own value, as 7 expresses the number seven, 9 nine, &c.

6. If a number be expressed by more figures than one, (as every number above nine is) then the first figure signifies its own value or units, the second ten times its own value, or as many tens as singly it expresses units, *&c.* Thus in the following numbers:

17 Seventeen
71 Seventy-one
64 Sixty-four

The first figures 7, 1, 4, signify simply their own value, *viz.* 7 seven, 1 one, and 4 four; but the second figures 1, 7, 6, receive a new value from their place, and stand for ten times their simple value, *viz.* 1 signifies ten, 7 seventy, (seven tens) and 6 sixty, (or 6 tens.)

7. In like manner, a figure in the 3d place signifies ten times ten times, or a hundred times as much as it would in the first, as in the numbers:

317 Three Hundred and Seventeen.
571 Five Hundred and Seventy-one.
764 Seven Hundred and Sixty-four.

And so continually every figure to the left-hand has ten times the value it would have in the next place to the right, and signifies ten times its own value, so often repeated as its place is distant from the first or units place.

8. Whence it follows, and is easy to be conceived, that every ten of the value of a lower place is equal to 1 of the next higher.

9. But as the frequent repetition of tens would occasion obscurity and confusion, different terms are used to express the local value of figures, as by the following:

TABLE I.

	Place	Name
First Period.	First	Units
	Second	Tens——X
	Third	Hundreds—C
	Fourth	Thousands—M
	Fifth	X of Thousands
	Sixth	C of Thousands
Second Period.	Seventh	Millions
	Eighth	X of Millions
	Ninth	Hundreds of Millions
	Tenth	Thousands of Millions
	Eleventh	X of Thousands of Millions
	Twelfth	C of Thousands of Millions
Third Period.	Thirteenth	Millions of Millions or Billions
	Fourteenth	X of Billions
	Fifteenth	C of Billions
	Sixteenth	Thousands of Billions
	Seventeenth	X Thousands of Billions
	Eighteenth	C Thousands of Billions
	Nineteenth	Millions of Billions, or Trillions

10. Under every sixth place we draw a line of separation for this reason; because the next place (according to the preceding progression of the places) would be thousands of thousands; but again to prevent the obscurity that the frequent repetition of the same word would occasion, we use the term millions to stand for a thousand thousand, and then proceed to name the next 5 places successively, by prefixing to the word millions the same terms, in order as in the 5 places after units; and so on in a continual rotation; every round of six places is named a period; and the table may be yet continued further at pleasure, by learning names for the lowest place in each succeeding period, as in the following:

TABLE II.

Of the lowest Place of the	Period, the Name is
First	Units
Second	Millions
Third	Billions
Fourth	Trillions
Fifth	Quadrillions
Sixth	Quintillions
Seventh	Sextillions
Eighth	Septillions
Ninth	Octillions, *&c.*

11. Likewise each period may be sub-divided into two equal parts, which may be called members, consisting of three places each.

Of the Cypher.

12. The cypher by itself signifieth nothing, and put to the left-hand of another figure, altereth not its value, thus 7, 07, 007, all express the number seven, and no more, since the significant figure 7 is still in units place.

13. But a cypher, being put to the right-hand of a figure, encreases its value ten times, because it removes it to the second place, and two cyphers, in like manner put, encrease a figure's value an hundred times, by raising it to the third place, and so on, as,

7	Seven
70	Seventy
700	Seven Hundred
7000	Seven Thousand

14. Likewise a cypher to the left-hand of any number altereth not its value, since every significant figure continueth to possess the same place, as,

45
045
0045 &c.

15. But if a cypher be put to the right-hand of any

number, it encreaseth its value ten times, by removing every figure one place higher, two cyphers in like manner situate, cause that the significant figures express a number a hundred times as much as without the cyphers, since every figure is removed two places higher by such appositions of the cyphers, as,

45 Forty-five
450 Four hundred and fifty
4500, *&c.* Four thousand five hundred.

ROMAN NOTATION.

The characters used by the *Romans* to express numbers, were the following, *viz.*

I. One. V. Five. X. Ten. L. Fifty.

C. One hundred. D. Five hundred. M. One thousand

I.	One	X.	Ten	C.	One hundred
II.	Two	XX.	Twenty	CC.	Two hundred
III.	Three	XXX.	Thirty	CCC.	Three hundred
VI.	Six	LX.	Sixty	DC.	Six hundred
VII.	Seven	LXX.	Seventy	DCC.	Seven hundred
VIII.	Eight	LXXX.	Eighty	DCCC.	Eight hundred
IV.	Four	XL.	Forty	CD.	Four hundred
IX.	Nine	XC.	Ninety	CM.	Nine hundred

QUESTIONS
AND
PRACTICAL EXAMPLES
ON THE FIRST CHAPTER.

Ques. WHAT is Arithmetic?

Answer. The art of managing or calculating numbers.

Q. What is Notation?

A. The writing numbers properly.

Q. How are numbers written or expressed?

A. All numbers are expressed by the different disposition of the following characters:

Nought	One	Two	Three	Four	Five	Six	Seven	Eight	Nine
0	1	2	3	4	5	6	7	8	9

Q. How can all numbers be expressed by these characters?

A. By the APPLICATION of

NUMERATION TABLE.

C Millions, &c.	X Millions	Millions	C Thousands	X Thousands	Thousands	Hundreds	Tens	Units
9	8	7	6	5	4	3	2	1

Q. What have we observed of the cypher, or 0?

A. That by itself it signifies nothing; being put on the left-hand of any Number it altereth not its value; but being put to the right-hand, every cypher maketh the number ten times what it was before.

Examples *for Practice.*

Write down in figures.

1. Seventeen, *Answ.* 17.
2. Eleven — —
3. Fourteen — —
4. Thirty-six — —
5. Ninety-one — —
6. One hundred and fifty-four — — — 154
7. Two hundred and eighty-seven — — —
8. Six hundred and twelve — — — —
9. One hundred and fourteen — — — —
10. Five hundred and four — — — — 504
11. Nine hundred and nine — — — —
12. Seven hundred and sixty — — — —
13. Six hundred and ten — — — —

---

14. One thousand four hundred and twenty-five, *Answ.* — — — } 1425
15. Three thousand six hundred and forty-four —
16. Two thousand nine hundred and eleven —
17. Six thousand and eighty four — —
18. Six thousand eight hundred and four —
19. Six thousand eight hundred and forty —
20. Seven thousand and six — — —
21. Seven thousand and sixty — — —
22. Seven thousand six hundred — —
23. Four thousand — — — —
24. Eight thousand — — — —

---

25. Seventy-eight thousand six hundred and forty eight — — — } 78648
26. Ninety-one thousand three hundred and fifty-seven — — — }
27. Forty-thousand four hundred and fifty —
28. Eighty thousand and eighty — —
29. Ninety thousand and nine — —
30. Fifty thousand — — —

31. One hundred and twenty-seven thousand three hundred and ninety-four — 127394
32. Four hundred and fifty-six thousand seven hundred and eighty-nine —
33. Six hundred two thousand four hundred and nine — — —
34. Five hundred and forty thousand eight hundred and five — — —
35. Eight hundred thousand and eight — —
36. Three hundred thousand — — —
37. Nine hundred thousand five hundred —

38. Three millions one hundred and twenty-seven thousand three hundred and ninety-four 3127394
39. Four millions four hundred and fifty-six thousand seven hundred and eighty-nine
40. Eighty millions forty thousand and sixty —
41. Seven millions four hundred thousand —
42. Nine millions — — —

43. What is 7 in the third place, and how expressed?
44. What is 9 in the fifth place?
45. What is 5 in the second place?
46. What is 8 in the fourth place?
47. What is 6 in the sixth place?
48. What is 3 in the seventh place?

§ NUMERATION.

Is the right reading or reciting a number expressed by figures.

Rule I.

1. To read any number not exceeding three figures, consider the value each figure receives from its place, and read each with the name of its place adjoined thus, read three in the second place, thirty; in the third place, three hundred; and seven in the second, seventy; in the third, seven hundred, *&c.*

Hundreds

Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units
	1	7	8	6	4	0	2	5
	7	5	4	6	0	6	3	7
	4	6	4	0	6	9	0	0
	9	0	5	2	0	9	9	9
3	2	7	5	0	2	8	6	0

Rule II.

2. To read a number not exceeding six figures, or a * period.

Separate by a comma, the first members from the other figures, then read the second § member or a part of it, just as the first, or a part thereof; only call the second member thousands.

Example.

Thousand	Thousand	Thousand
5,423	74,370	753,753
2,735	93,058	4 7,047
3,025	60,108	370,000
2,400	30,000	507,208
9,003	90,008	530,003
		300,000

Rule III.

3 To read a number consisting of any number of figures whatsoever.

Divide it into Periods by prefixing a point to every sixth figure; then read every period alike, subjoining to each the name of the place of its lowest figure, which may be known from the second table preceding.

---

* *A Period is every 6 figures in any number taken from units.*

§ *A member is half a period, or 3 figures from units.*

Example.

	470	368432	078642
	98073	540087	248000
	476308	753246	538240
7532	400658	240865	429086
53784	279085	420006	278053
123456	789012	345668	901234
Trillions	Billions	Millions	Units

CHAP. II.

ADDITION.

20. ADDITION is the joining or collecting several numbers into one, or finding a number which shall be equal to any given numbers altogether.

GENERAL RULE.

Let the numbers marked A B C, be given to be added.

1. Place the numbers so that each figure may stand directly underneath (or in the same perpendicular row with) the figures of the same value, that is: units under units, tens under tens, hundreds under hundreds, &c.

54327	A
8062	B
5041	C
67430	

Then drawing a line under them; begin the Addition at the first place (or units) and add together all the figures in that place, and if their sum be under ten, set it down below the line underneath its own place; but if their sum be more than ten, set down only the overplus above the ten (or tens) and so many tens as the sum of these units amount to, carry to the place of tens, adding them and the figures which stand in the place of tens together; then proceed in the same manner to the third place, or hundreds, and so from place to place to the last, and set down the whole sum of the last place.

Application.

The marginal numbers being placed as before directed, I add together the first figures 1 and 2 (3) and 7 (10), and find their sum to be 10, I set down (0) the overplus above ten, and for the ten I add 1 with the figures of the next place, *viz.* 1 and 4 (5) and 6 (11) and 2 (13). Again, I set 3 the overplus above Ten, and add 1 for the (one) ten to the figures in the next place, *viz.* 1 and 3 is 4, which being under ten I set it down in the same place, and proceed in like manner to the last place, and the work is done.

54327
8062
5041
67430

TABLE *of* ADDITION.

To be committed to memory by the learner, previous to his entering into the Rule.

0	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

The manner of using the table is thus: take the greater of the two figures, whose sum is sought in the upper line, and the lesser in the left hand column in the same line with this, and underneath the other stands the sum. As suppose I wanted the sum of 9 and 7, then I look for 9 on the head of the table, and in the same line with 7 on the side, stands 16, the sum.

Rule II. *Example.*

When the numbers to be added are many, the following method may be practised: begin with the lowest figure of units place (as before) and joining it to the figures above it (as the last Rule) for every ten arising in the Addition, make a point over against the figure which added to the former maketh ten or more than ten, add the overplus above ten to the next figure above it, and so proceed to the top; then count the points and

7 4 3 6
2 1 7 9.
5 0 8 7.
6 8 5 3
2 4 6 4.
7 2 8 4
31 3 0 6

and how many they are, so many carry and add to the figures of the next place, and proceed in like manner thro' all the places, and the points of the last place collect, and set their number to the left-hand of the figure under the last place.

This rule doth not differ essentially from the last, being only a contrivance to help the memory.

Rule III.

To prove Addition.

Begin the Addition at the uppermost figure at the highest place, *viz.* next the left-hand, and add downward and place the lowest figure of the sum directly under the added figures, and the other figures of the sum on the left-hand of it: then begin with the uppermost figure of the next lower place, and add downwards in like manner, and still place the lowest figure of the sum under the added figures as before (29); so we shall get as many sums as the numbers (or greatest number) have places, and each one place nearer the right-hand: let those sums be added together (⅌ Rule 2,) and if their sum agree with the sum of the given numbers before found, the work may be presumed to be truly done.

```
 7436
 2179
 5087
 6853
 2464
 7287
------
29
 19
  37
   36
------
31306
```

Questions *and Practical* Examples.

Q. How are numbers to be placed, in order to be added?

A. Units under units, tens under tens, *&c.*

Q. How are numbers to be added?

A. Begin at units place, and add the lowest figure in that place to that above it; then add their sum to the next figure in the same place, and so on to the top: set down the overplus above ten or tens, and for every ten carry one to the next place, *&c.*

[1] 256	[2] 2486	[3] 3946	[4] 4675
643	3255	7932	7998
457	7667	8363	3997

[5] 245	[6] 6304	[7] 1524	[8] 2000
956	9120	7361	4798
201	9000	2000	2547
904	7660	5859	5000

[9]	[10]	[11]	[12]
9568	9658	4268	2823
5732	5973	2646	9208
9601	1579	3510	6524
2367	6137	6482	1282
8524	9795	8264	2627
5378	1793	6828	8344

[13]	[14]	[15]	[16]
576894	9768939	9864830	87698767
79865	8979695	572970	69789763
93784	9179642	3795	96827527
6798	9756879	43962	54328106
3297	3019431	869874	33708327
986	2137540	38237	90109461
4696	5967389	784	84176875
4796	4679849	77963	57807148
3210	4839518	6475	38210739
7826	8763649	328	93701283
5037	2198754	987	51068027
78989	9763475	6763974	72086015

QUESTIONS.

1. A merchant on settling his account, finds he owes A 60*l.* B 156*l.* C D 264*l.* and E 27*l.* I demand how much he owes in all? *Answ.* 507*l.*

2. A merchant sends his clerk with the following bills for payment, *viz.* one on I. J. for 60*l.* one on A. B. for 84*l.* one on C. D. for 167*l.* one on D. S. for 125*l.* and one on E. F. for 500*l.* I want to know the sum he received in all? *Answ.* 936*l.*

3. Bought 8 casks of indigo, N°. 1, 210℔. N°. 2, 196℔. N° 3, 4, 5, 205℔ each; N°. 6, 184℔. N°. 7, 225℔. and N°. 8, 174℔. How many pounds did the whole 8 casks contain? *Answ.* 1604℔.

4. A linen draper bought 10 bales of linen cloth, containing as follows, *viz.* N°. 1, 2, 367 yards each, N°. 3, 4, 5, each 407 yards; N°. 6, 7, 8, each 288 yards; N°. 9, 10, each 300 yards. I desire to know how many yards he bought in all? *Answ.* 3419.

5. A certain man being asked his age, answered, I have 7 sons, and between the birth of each was two years, at

the age of 21 I had my eldest son, which is now the age of my youngest; it is required to tell his age?

Answ. 54 years old.

6. A vintner buys 6 pipes of brandy, containing 120 gallons, 118, 125, 121, 127, and 119. How many gallons did he buy in all? *Answ.* 730.

7. How many days in a whole year? *

8. If from the creation to the flood was 1650 years; from the flood to the vocation of Abraham 427; from the vocation of Abraham to the founding of the temple 1010; from that to the foundation of Rome 266; thence to the birth of Christ 752; and since that 1809 years; how long is it since the creation? *Answ.* 5914 years.

9. How many strokes doth a regular clock strike in a week? *Answ.* 1092.

10. The lesser of two numbers is 276, the difference between them 96: how much is the greater?

Answ. 372.

11. What day of the year was the 27th of the 8th month, August 1765? *Answ.* 239th.

12. What day of the year was the 21st of the 6th month June 1764? *Answ.* 173rd.

13. If from Dublin to Naas be 14 miles; from Naas to Castledermot 19; from thence to Carlow 5; from Carlow to Kilkenny 18; thence to Clonmel 24; thence to Kilworth 23; and from Kilworth to Cork 18; how many miles then is it from Dublin to Cork? *Answ.* 121 miles.

14. Suppose the distance from Dublin to Holyhead is 60 miles; from Holyhead to Chester 87; from Chester to Whitchurch 20; from Whitchurch to Newport 20; from thence to Ivessey-bank 9; from thence to Meredon 34; from Meredon to Coventry 6; thence to Daventry 20; thence to Towcester 12; thence to Stoney-Stratford 8;

---

* *N. B. The number of days in the several months in the year by some authors, are comprized in the following lines:*

Thirty days are in September,
In April, June, and in November;
February hath twenty-eight alone,
And all the rest have thirty-one.

Now sum them up and let us hear
How many days in all the year. *Answ.* 365.

from Stoney-Stratford to Dunstable 18; thence to St. Albans 13; and thence to London 21: How many miles is it from Dublin to London? *Answ.* 328 miles.

15. Find how many years it was from the creation of Adam to the universal deluge; by the 5th chapter and 6th verse of the 7th chapter of Genesis. *Answ.* 1656 years.

CHAP. III.

SUBTRACTION.

SUBTRACTION is the taking a lesser number from a greater, and thereby finding the remainder or difference between them.

To prove Subtraction.

Rule.

Add the remainder to the lesser given number, and if the sum come the same with the greater given number, the work is right.

TABLE *of* SUBTRACTION.

0	1	2	3	4	5	6	7	8	9
1	0	1	2	3	4	5	6	7	8
2	-	0	1	2	3	4	5	6	7
3	-	-	0	1	2	3	4	5	6
4	-	-	-	0	1	2	3	4	5
5	-	-	-	-	0	1	2	3	4
6	-	-	-	-	-	0	1	2	3
7	-	-	-	-	-	-	0	1	2
8	-	-	-	-	-	-	-	0	1
9	-	-	-	-	-	-	-	-	0

The manner of using this table is the same with that of addition, only instead of adding the figures together, subtract them.

Questions *and Practical* Examples.

Q. What is Subtraction?
A. Subtraction from a greater take a less,
And thereby shews the difference or excess.
Q. How are the numbers to be placed?
A. The less beneath the greater we dispose,
Units and tens in correpondent rows.
Q. How is Subtraction performed?
A. Take ones from ones, and tens by tens decrease,
If still the figures to subtract are less;

But if the lower figure's greater, then
The lower figure we deduct from ten,
To what remains the upper figure add,
The just remainder of that place is had;
To the next lower figure carry one,
And thus from place to place the work is done.

From	[1] 8103	[2] 1969	[3] 1917	[4] 1549
Take	4193	1408	1718	1460

From	[5] 6119889	[6] 4090987	[7] 6278500
Take	3599907	1989849	1435045

From	[8] 60740167423	[9] 78000930402
Take	23420891075	53298495299

QUESTIONS.

Q. 1. A vintner bought 20 pipes of brandy, containing 2459 gallons, and sells 14 pipes containing 1680 gallons: How many pipes and gallons has he unsold?

Answ. 6 pipes, containing 779 gallons.

Q. 2. A merchant bought 564 tanned hides, weighing 16800lb. sells thereof 260 hides, weight 7809lb. How many hides has he unsold, and what do they weigh?

Answ. 304 hides, containing 8991lb.

Q. 3. Suppose Cork, Clonmel and Dublin, lie in a straight line, and the distance between Cork and Dublin is 121 miles, and from Cork to Clonmel is 41 miles. I demand the distance between Clonmel and Dublin? *Ans.* 80 miles.

Q. 4. How long is it since the happy revolution which happened in the year 1688, we are now in the year 1812?

Ans. 124 years.

Q. 5. What five different numbers make 100?

Q. 6. What number is that which being added to 977, the sum will be 2081? *Answ.* 1104.

Q. 7. What number must I subtract from 2081 that the remainder may be 1104? *Answ.* 977.

Q. 8. If a merchant owed 1000*l.* and hath paid in cash 280*l.* and hath likewise given an assignment on Jacob Payer for 156*l.* How much doth he still owe?

Answ. 564*l.*

Q. 9. Bought 2000 yards of linen for 466*l.* and sold 1476 yards for 369*l.* How many yards have I left, and what do I want to make up the first cost? *Answ.* I have 524 yards, and want 97*l.*

Q. 10. What difference is there between the age of A, born in the year 1693, and B that will be born 13 years hence: the question being put Anno 1773? *Answ.* 93 years.

Q. 11. If I was born in the year 1721, how many years old am I this year 1773? *Answ.* 52 years.

Q. 12. If I am 42 years old in 1773, what year was I born in? *Answ.* 1731.

Q. 13. If a lease bears date 1*st* of the 1*st* month 1722, how many years are unexpired thereof, suppose it granted for 99 years? *Answ.* 53 years in 1768.

Q. 14. In four bags were different species of coin to the value of 500*l.* in the first was 96*l.* in the second 120*l.* in the third 55*l.* I desire to know what sum the fourth contained? *Answ.* 229*l.*

Q. 15. A grant was made by the crown, Anno 1289 which was forfeited 137 years before the revolution in 1688: How long did it continue? *Answ.* 312 years.

Q. 16. Five notable discoveries were made in 215 years time, *viz.* The invention of the Compass. 2. Gunpowder. 3. Printing. 4. America. 5. the Reformation. The last was brought about Anno 1517: the third 77 years before: the second 42 years after the first, and the fourth 148 years after the second. The question is, in what year did each happen? *Answ.* Compass in 1302; Gunpowder 1344; Printing 1440; America 1492.

Q. 17. Thirty-three years before the Restoration in 1660, the crown granted Demesnes to certain uses for 210 years to come. The proprietor in 1715 procured a reversionary grant for 99 years, to commence after the expiration of the first. In what year will the second term end? *Answ.* Anno 1936.

Q. 18. A snail in getting up a pole, only 20 feet high, was observed to climb eight feet every day; but every night it came down again four feet. In what time by this method did he reach the top of the pole?

Answ. The night of the fourth day.

Q. 19. The semi-diameter of the earth's orbit, or annual path round the sun, is about 81,000,000 of miles, that of Venus 59,000,000; when they are both on the same side of the sun, they are in Perigœo; when on different sides, in Apogœo: What is the difference of their distances in both these positions? *Answ.* 118,000,000 miles.

Q. 20. If the mean distance between the earth and sun be 81 millions of miles, and between the earth and moon 240 thousand, how far are these two luminaries asunder in an eclipse of the sun, when the moon is lineally between the earth and sun? And in another of the moon, when the earth is in a line between her and him?

Answ. } In an eclipse of the sun, 80,760,000 miles.
In an eclipse of the moon, 81,240,000 miles.

CHAP. IV.

MULTIPLICATION.

1. A Number is said to multiply a number, when a number is produced which contains the multiplied number, as often as the multiplying number contains unity.

2. The multiplied number is called the *Multiplicand.*

3. The multiplying number, the *Multiplier.*

4. And the number produced, the *Product.*

THE TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

Rule.

Multiply the first figure of the multiplicand by the given multiplier, if the product be less than ten, set it down in the same place with the multiplied figure; but if the product be above ten (or tens) set down the overplus only, and reserve the ten or tens (in mind,) then by the same multiplier multiply the next figure of the multiplicand, and to the product add the ten or tens reserved, and proceed in the very same manner, until all the figures of the multiplicand are multiplied.

Examples.

[1] 256745
2
―――――
518490
―――――

[2] 785403
3
―――――

―――――

[3] 27540098
4
―――――

―――――

[4] 349621
5
―――――

―――――

[5] 909734
6
―――――

―――――

[6] 47109879
7
―――――

―――――

[7] 174675093
8
―――――

―――――

[8] 410097134
9
―――――

―――――

Case II.

When the multiplier consists of several figures.

Rule.

1. Multiply the multiplicand by the first or units figure of the multiplier, and subscribe the product (as in last;) in like manner multiply by the second figure of the multiplier, and so successively by every figure one by one; whereby there will be as many products as there are significant figures in the multiplier.

2. Place these products in order under one another, so that the first figure stand directly underneath, or in the same place with the multiplying figures; that is, the first figure of the second product must stand under the second figure of the first product, the first figure of the third, under the second figure of the second, and the third figure of the first, &c.

3. Add all these products together, and their sum is the product sought.

Examples.

[9] 3042
517

21294 First prod. by 7
3042. Second prod. by 10
15210.. Third prod. by 500

1572714 Sum of the 3 products.

[13] 7534628
25

[11] 37420768
349

[12] 5307652
154

[13] 7537269
387

[14] 42372432
7438

[15] 24372048
6323

[16] 753072468
57348

[17] 93724376072
471398

[18] 687654327
123456789

[19] 739425378395
47325612

Case III.

When the multiplier hath cyphers intermixed with the significant figures.

Rule.

1. Multiply first by the first significant figure, and by every other successively, (omitting the cyphers) so that there will be as many particular products as significant figures in the multiplier.

2. As before, let the first figure of every particular product be put in the same place with the multiplying figure.

3. Add these products together, *&c.* as in the last.

Examples.

[20] 570684
304

[21] 8504593
709

[22]

[22] 968347
506

[23] 9735088
7006

[24] 74086572
50402

[25] 63230875
58007

[26] 504030201
102030405

[27] 750037298
6003005

[28] 4750034076354
4000753009

Case IV.

When the multiplicand, or multiplier, or both have cyphers in the lowest places.

Multiply by the other figures, as before taught, neglecting the cyphers till the product be found; and then put all the cyphers, both at the end of the multiplicand and multiplier, to the right hand of the product.

[29] 75360
7200

[30] 5048600
307200

[31] 7325060
780200

[32] 7630400
7000

Case V.

When the multiplier is such a number that any two figures (in the table,) being multiplied together, will produce it.

Rule.

Multiply the given number by one of these figures, and that product by the other, which will give the desired product.

Examples.

[33] 346734 by 18 [34] 129746983 by 32
[35] 37,496 by 64 [36] 4070403204 by 96
[37] 413721 by 132 [38] 48149194 by 144

Multiplication was shewn to be a compendious method of adding a number to itself, a determinate number of times, and from this consideration the universal *Multiplication Table* was constructed: In like manner we may construct a table of any multiplicand whatever, *i. e.* find the product thereof by each of the single figures by *Addition* only, and this method is very useful in case of large multipliers.

Let the given multiplicand be the number 987654321.

Construction.

1	987654321
2	1975308642
3	2962962963
4	3950617284
5	4938271606
6	5925925926
7	6913580247
8	7901234568
9	8888888889
10	9876543210

Add the given multiplicand to itself, then we have the product thereof, multiplied by 2. Again, add the said product or sum, to the given multiplicand, and the sum is the product thereof by 3; and so adding every new product or sum to the given number, we may get the products to the said numbers multiplied by all the single figures.

2. Put each figure in the same (horizontal) row with the product it produceth, in a column to the left hand, as in the margin, and then the table is constructed.

3. A number is multiplied by 10 if a cypher be put before it: whence we get a most easy proof of the truth or error of the operation; for, if the product of the given number by 9 be added to the said given number, we get the product thereof by 10, which, if it comes out the said given number with a cypher annexed, proves every product right, otherwise some error is contracted.

THE USE.

The use is evident, for let the given multiplier be 123456789.

987654321

```
        987654321
        123456789
        ─────────
       8888888889
      7901234568
     6913580247
    5925925926
   4938271605
  3950617284
 2962962963
1975308642
987654321
──────────────────
121932631112635269
──────────────────
```

Look for first figure 9 in the column of single figures, and the number in the same line or horizontal row is the first product, and in the same manner we find the products by the other figures of the multiplicand successively, which we place and add as before taught.

To prove Multiplication.

Make the number which was the multiplicand, multiplier, then multiplying as usual, if the product be the same, the work is right.

Thus to prove the first Example in Case II.

```
Multiply  517
      by 3042
         ────
         1034
        2068
      15510
      ───────
      1572714  Product the same as
      ───────     there found.
```

QUESTIONS *in* MULTIPLICATION.

Q. What doth Multiplication teach?

A. From two numbers given to find a third, which shall contain one of the given numbers as often as the other contains unity.

Q. What are the numbers called?

A. 1. The number to be multiplied is called the Multiplicand; the number we multiply by, the Multiplier; and the number found, the Product.

Q. How is Multiplication performed when the multiplier is a single figure?

A. Of th' units o' th' multiplicand
  And given figure, th' product find;
Put down the product units, and
  The tens (if any) keep in mind:
The given tens next multiply:
  The product of these tens increase
By the tens reserved by:
  And so proceed from place to place.

Q. When the multiplier is any number, or consists of several figures.

A. 1. Multiply the multiplicand by the units figure of the multiplier; then by tens, and so successively by every other figure.

2. Let the units figure of each product stand in the same place with the multiplying figure, *&c.*

3. Add all the products together.

Q. When cyphers are intermixed with the significant figures of the multiplier?

A. Multiply the significant figures only, still observing to put the units figure of the product in the same place with the multiplying figure.

Q. How must we prove the work of Multiplication?

A. Make the multiplicand multiplier.

Q. Repeat the Multiplication Table?

A. Twice 2 is 4, *&c.*

CHAP. V.

DIVISION.

DIVISION teacheth to find how often one given number is contained in another.

The number which divides is called the Divisor.

The number which is to be divided is called the Dividend.

And the number found by dividing the greater by the less is called the Quotient.

Case I.

When the divisor is a single figure, and the dividend no more than two, and the divisor measures the dividend.

Rule.

Consider what number multiplying the divisor will produce the dividend, and that number is the quotient.

Examples.

Examples.

[1] [2] [3] [4]
3)27 3)21 4)16 5)35
[5] [6] [7] [8]
6)30 8)72 9)54 7)56

When the divisor doth not measure the dividend.

Rule.

Consider what number multiplying the divisor will make a product next less than the dividend, and that is the quotient figure sought.

2. Multiply the divisor by the quotient figure, and place the product under the dividend.

3. Subtract the product from the dividend, and if the remainder be less than the divisor, the quotient figure is truly taken.

Examples.

[9] [10] [11] [12]
2)15 3)25 3)29 4)37
[13] [14] [15] [16] [17]
5)28 6)34 7)48 8)57 9)39

When the divisor is a single figure and the dividend any number.

Rule.

Take the divisor in the last or highest figure of the dividend, if it be greater than the divisor, but if not, find how often the divisor is contained in the two last figures of the dividend, and having found the quotient figure, proceed exactly as in the last section, and find the remainder, as there taught.

2. To the remainder (if any) bring down the next lower figure, and let the number expressed by these two figures be esteemed a new dividend; or if there be no remainder, then esteem the next lower figures of the dividend a new dividend, which divide in like manner as in last section, and place the quotient figure to the right hand of that first found.

3. Proceed in the like manner from figure to figure, till the figures of the dividend be taken down successively one by one.

Case II.

When the divisor is any number whatever.

The proceſs is exactly the ſame as in laſt caſe, only the difficulty of determining the quotient figure is greater, to render which eaſy, obſerve the following rules:

1. By means of the laſt, or two laſt figures of the diviſor compared with as many, if greater, or, if not, one more of the dividend, diſcover the quotient figure as near as may be.

2. If the product of the quotient figure multiplied by the diviſor be greater than the aſſumed member of the dividend, the quotient figure is too great, wherefore make it leſs.

3. If the remainder, after the product is ſubtracted from the divided member, be greater than the diviſor, the quotient figure is too little and muſt be made greater.

4. If neither of theſe happen, the quotient figure is truly found.

QUESTIONS *and* PRACTICAL EXAMPLES.

Q. What is Diviſion?

A. The finding how often one given number is contained in the other.

Q. What are the given numbers called?

A. The leſſer is called the diviſor, (*i. e.* the divider,) and the greater the dividend, (*i. e.* the number to be divided.)

Q. What is the number found called?

A. The quotient, which ſhews how often the diviſor is contained in the dividend.

Q. How is the quotient found?

A. Seek how often the diviſor is contain'd
I' th' leading figures of the dividend;
So the firſt quotient's figures found; which by
The giv'n diviſor we muſt multiply;
The product take from what we did divide;
Next figure put by the remainder's ſide:
Repeat the proceſs, until one by one,
The figures of the dividend are gone.

Q. How ſhall I know when the quotient figure is truly found?

A. When the product of the ſaid figure and the diviſor is leſs than the divided member; and the remainder leſs than the diviſor.

Q. Suppose I bring down the next figure of the dividend to the remainder, and the number is still less than the divisor?

A. Put 0 for the quotient figure, and take down the next figure of the dividend.

Q. How must I prove Division?

A. Multiply the quotient by the divisor (adding in the remainder, if any) and if the product be equal to the dividend, the work is right.

Examples.

[18] 2) 3754638 ([19] 3) 42754372 (
[20] 4) 2608735 ([21] 5) 796047320 (
[22] 6) 47345243 ([23] 7) 187342358 (
[24] 9) 34227234 ([25] 11) 472430734 (
[26] 12) 348073620 (

It is usual in Practice to put down the quotient only, performing the rest of the operation by memory.

[27] 2) 43782 [28] 3) 784375 (
21891

[29] 4) 2724865 ([30] 5) 74340845 (
[32] 7) 3425168 ([31] 6) 75324374 (
[34] 9) 352463) [33] 8) 54372582(
[35] 11) 34267345(

Case II.

Examples.

[36] 27) 31865372 ([37] 83) 54768724 (
[38] 47) 3986294823 ([39] 53) 79852375 (
[40] 325) 3784270981 ([41] 4376) 2428907324 (

[42] 53428) 824675402378 (
[43] 753364) 5864732164372 (
[44] 6398607) 2835729034758 (
[45] 137864272) 5430726894136 (

Case III.

When the divisor hath cyphers in its lowest place, cut off the cyphers (with a dash of the pen) and as many of the lowest figures of the dividend, and then divide the other figures of the dividend by the significant figures of the divisor, as before taught; and the figures cut off from the dividend must be brought down to the remainder, if not cyphers.

Application.

56|00(75489|32 (1347
56
―――
194
168
―――
268
224
―――
446
392
―――
5432
―――

Examples.

[46] 2400) 72579482 ([47] 3600) 7529176586 (
[48] 3000) 427587761([49] 9000) 6876752871 (
[50] 9000) 4786560000([51] 4720) 6843248 (
[52] 20) 3724865([53] 37482000) 478652725314(

When

Case IV.

When the divisor is such a number, that any two figures, (in the multiplication table) being multiplied together, will produce the said divisor.

Rule.

Divide the given number by one of these figures, and that quotient again by the other, which will give the quotient required,

Note, If there be a remainder in the last division, it will be so many times the first divisor, which added to the first remainder (if any) will give the true one.

Examples.

[54] Divide 1206817 by 16. [55] 42768 by 48.
[56] 14652 by 64. [57] 74682 by 72. [58] 417681 by 81
[59] 34672 by 108. [60] 217904 by 120. [61] 14276 by 144

Case V.

Division may be performed with much ease and certainty by constructing a table of the products of the divisor multiplied by the single figures, in the same manner as the table of the multiplicand was constructed (Page 30.)

Application.

1	987654321
2	1975308642
3	2962962963
4	3950617284
5	4938271605
6	5925925926
7	6913580247
8	7901234568
9	8888888889
10	9876543210

987654321) 121932631112635269 (123456789
987654321........
———————
2316719901
1975308642
———————
3414112592
2962962963
———————
4511496296
3950617284
———————
5601720123
4938271605
———————
6705185185
5925925926
———————
7792592592

7792592592
6913580247

8791023456
7201234568

8888888889
8888888889

The Use.

The use of this table is very easily apprehended: For we find the first member of the dividend as before, *viz.* the same number of figures as the divisor hath, if the highest figures of the dividend be greater than the highest of the divisor, or 1 more, if less. Then look in the table for that product which is immediately next less than the first member of the dividend, and place it under the said member; and the figure in the column to the left-hand is the quotient figure, which is thus known, without any doubtful trials, as before. The rest of the work is the same, as in the common method before taught. This method may be of good use to a learner, and likewise in making large divisions.

PROBLEMS,

Resulting from the Comparison *of the preceding Rules.*

Problem I.

Having the sum of two numbers and one of them given to find the other.

Subtract the given number from the given sum, and the remainder will be the number required.

Example.

Let 144 be the sum of two numbers; one of which is 96, the other is required.

From 144 the Sum
Take 96 the given number

Remains 48 the other.

Proof 144

Problem

Problem II.

Having the greater of two numbers, and the difference between it and the lesser given, to find the lesser.

Subtract the one from the other.

Example.

From 144 the greater
Take 96 the difference

Remains 48 the lesser.

Problem III.

Having the lesser of two numbers given, and the difference between it and a greater, to find the greater.

Add them together.

Given { 96 the lesser number
48 the difference

Sum 144 the greater number required.

Problem IV.

Having the product of two numbers, and one of them given to find the other.

Divide the product by the given number, and the quotient will be the number required.

Let the product of two numbers be 144, and one of them 3; I demand the other?

3) 144

Answ. 48

Problem V.

Having the dividend and quotient to find the divisor.

Divide the dividend by the quotient.

Cor. Hence we get another way of proving Division.

Problem VI.

Having the divisor and quotient given, to find the dividend.

Multiply them together.

Now by a due consideration and application of these problems only, the following questions may be resolved in a short and elegant manner, altho' some of them are generally supposed to belong to higher rules.

QUESTIONS.

1. What number is that, which being added to 9709 makes 10901? *Answ.* 1192.

2. The lesser of two numbers is 9709, the difference between them is 1192; what is the greater?
Answ. 10901.

3. What number must I multiply by 7 that the product may be 623? *Answ.* 89.

4. The product of two numbers is 31383450, and one of them 4050: the other factor is required?
Answ. 7749.

5. What is the difference, and what the sum of six dozen dozen, and half a dozen dozen? *Answ.* diff. 792, sum 936.

6. The sum of two numbers is 300; the less 114: What is their difference and product? *Answ.* 132 and 28044.

7. The remainder of a Division sum is 423; the quotient 423; the divisor is the sum of both and 19 more: What then was the number to be divided?
Answ. 366318.

8. There is a certain number, which being divided by 7, the quotient resulting multiplied by 3, that product divided by 5, from the quotient subtract 20, to the remainder add 30, and half the sum shall make 35?
Answ. 700.

$$\frac{35\times2-30+20\times5\times7}{3}$$

9. What number added to the forty-third part of 4429 will make the sum 240? *Answ.* 137.

10. What number deducted from the 26th part of 2262 will leave the 87th part of the same? *Answ.* 61.

11. A sheepfold was robbed three nights successively; the first night half the sheep were stolen, and half a sheep more; the second, half the remainder were lost, and half a sheep more; the last night they took half what were left, and half a sheep more; by which time they were reduced to 20: How many were there at first? *Answ.* 167.

12. What number multiplied by 72084 will produce 5190048? *Answ.* 72.

13. There are two numbers; the greater of them is 73 times 109, and their difference 17 times 28. I demand their sum and product? *Answ.* 15438 and 59526317.

Of Numbers *of divers* Denominations.

NUMBERS of divers denominations are those whereby we express the sundry denominations or divisions of money, weights and measures.

1. *Of MONEY.*

In England and Ireland accounts are kept in pounds, shillings and pence, which are compounded as follow:

4 Farthings make 1 Penny
12 Pence —— 1 Shilling
20 Shillings —— 1 Pound.

Abbreviations,

d. Pence. *s.* Shillings. *£.* or *l.* Pounds.

Farthings are expressed as parts of a penny, thus:

$\frac{1}{4}$ expresses One farthing,
$\frac{1}{2}$ ———— One halfpenny,
$\frac{3}{4}$ ———— Three farthings.

But payments are made in the following coins.

GOLD COINS	weighing		Current in Eng. at			Current in Ireland at		
	dwt.	*gr.*	*£.*	*s.*	*d.*	*£.*	*s.*	*d.*
Guinea	5	8	1	1	0	1	2	9
Half guinea	2	16	0	10	6	0	11	4½
SILVER COINS				*s.*	*d.*		*s.*	*d.*
Crown piece	Current in England at			5	0	Current in Ireland at	5	5
Half ditto				2	6		2	8½
English shilling				1	0		1	1
Half ditto				0	6		0	6½

In the United States gold and silver coins are passed at the following rates.

Weights of Coin.			*N. Ham Massa R. Isl. Connect. Virgin.*			*N. York* and *N. Car.*			*N. Jer. Pennsyl. Delaw.*			*S. Car.* and *Georgia.*		
	dwt.	*gr.*	*£.*	*s.*	*d*	*£.*	*s.*	*d.*	*£.*	*s.*	*d*	*£.*	*s.*	*d.*
French Guinea	5	5	1	7	6	1	16	0	1	14	6	1	1	9
English ditto	5	6	1	8	0	1	17	4	1	15	0	1	1	5
Crown	19	0	0	6	8	0	8	9	0	8	4	0	5	0
Half Joe	9	0	2	8	0	3	4	0	3	0	0	2	17	0
Pistole	4	5	1	2	0	1	8	0	1	7	0	0	17	0
Dobloon	17	6	4	8	0	5	16	0	5	12	0	3	10	0
Moidore	6	18	1	16	0	2	8	0	2	5	0	1	8	0
Dollar	17	6	0	6	0	0	8	0	0	7	6	0	4	8
English Shilling			0	1	4	0	1	9	0	1	8	0	1	0

Farth.	Pence		
4	1	Shilling	
48	12	1	Pound
960	240	20	1

2. *TROY WEIGHT.*

By *Troy weight* are weighed gold, silver, jewels and liquors.

The denominations are

24 Grains make 1 Penny weight
20 Penny-weights make 1 Ounce
12 Ounces make 1 Pound

Abbreviations.

Abbreviations.

℔. Pounds,	*dwt.* Penny-weights,
oz. Ounces,	*grs.* Grains.

Grains	P. Weights	Ounces	Pound.
24	1		
480	20	1	
5760	240	12	1

3. *AVOIRDUPOISE WEIGHT.*

Is the weight generally used for weighing goods of all kinds, (the above excepted) and its divisions are as follow:

16 Drams	make	1 Ounce,
16 Ounces	——	1 Pound,
14 Pounds	——	1 Stone,
28 Pounds	——	1 Quarter of an hundred,
4 Quarters or	112 Pounds	1 Hundred weight,
20 Hundreds	——	1 Ton.

Abbreviations.

drs. Drams,	*qrs.* Quarters,
oz. Ounces,	*Cwt.* Hundred weight.
℔. Pounds,	

NOTE. 16 Pound is a stone of wool in Ireland. Therefore 7 stone of wool is 1 *Cwt.*

Drams	Ounces	Pounds	Stones	Qrs.	Cwt.	Ton.
16	1					
256	16	1				
3584	224	14	1			
7168	448	28	2	1		
28672	1792	112	8	4	1	
573440	35840	2240	160	80	20	1

4. *APOTHECARIES' WEIGHT.*

By the following divisions Apothecaries compound their medicines;

medicine; but buy and sell by Avoirdupoise weight.

20 Grains make 1 Scruple,
3 Scruples —— 1 Dram,
8 Drams —— 1 Ounce,
12 Ounces —— 1 Pound.

Abbreviations.

grs. Grains,
℈ Scruples,
ʒ Drams,
℥ Ounces,
℔. Pounds.

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5. *CLOTH MEASURE.*

4 Nails make 1 Quarter of a yard,
4 Quarters —— 1 Yard,
3 Quarters —— 1 Ell *Flemish*,
5 Quarters —— 1 Ell *English*,
6 Quarters —— 1 *French* Ell.

Abbreviations.

Na. Nails,
qr. Quarter,
Yd. Yard,
E. *F.* Ells *Flemish*,
E. *E.* Ells *English*,
F. E. *French* Ells.

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6. *LONG MEASURE.*

This is used to measure the distances of places one from another, and every thing where length only is considered; likewise to take the dimensions of the length, breadth and thickness of all bodies. Its divisions are as follow:

3 Barley corns make 1 Inch,
12 Inches —— 1 Foot,
3 Feet —— 1 Yard, 2 yards 1 fathom,
$5\frac{1}{2}$ Yards —— 1 *English* Perch
7 Yards —— 1 *Irish* Perch
40 Perches or Poles —— 1 Furlong,
8 Furlongs —— 1 Mile,
3 Miles —— 1 League.
$69\frac{1}{2}$ *English* Miles }
$54\frac{17}{8}$ *Irish* Miles } —— 1 Degree of a great circle,
360 Degrees —— A great circle of the earth.

Or in measuring distances.

7$\frac{92}{100}$ Inches make 1 Link *English*
10$\frac{8}{100}$ Inches —— 1 Link *Irish*,
25 Links —— 1 Perch,
100 Links —— 1 Chain,
10 Chains —— 1 Furlong,
8 Furlongs —— 1 Mile.

Inches	Feet				
12	1	Yards			
36	3	1	Perches		
252	21	7	1	Furlongs	
10080	840	280	40	1	Mile.
80640	6720	2240	320	8	1

English	Inches	Links					
7$\frac{92}{100}$	10$\frac{8}{100}$	1	Poles or Per.				
198	225	25	1	Chains.			
792	1008	100	4	1	Furl.		
7920	10080	1000	40	10	1	Mile.	
63360	80640	8000	320	80	8	1	

Note. The last column to the left hand marked *English*, shews the inches of the *English* pole, chain, *&c.* and the next marked inches, shews the inches in the *Irish.*

7. *SQUARE MEASURE.*

Is used in finding the contents of surfaces, when we measure both the length and breadth.

144 Square inches make 1 Square foot,
9 Square feet —— 1 Square yard,
49 Square yards —— 1 Square perch,
40 Square perches —— 1 Rood,
4 Roods —— 1 Acre.

ENGLISH SQUARE MEASURE.

30$\frac{1}{4}$ Square yards 1 Square perch.
39$\frac{1}{16}$ Square yards, 1 Square perch, Cunningham measure.

Otherwise thus:

7$\frac{92}{100}$	Inches	make	1 Link *English,*
10$\frac{8}{100}$	Inches	——	1 Link *Irish,*
625	Links square	——	1 Square perch,
100,000	Square links	——	1 Acre.

---

8. *LIQUID MEASURE.*

4 Naggins	make	1 Pint,
2 Pints	——	1 Quart,
2 Quarts	——	1 Pottle,
2 Pottles	——	1 Gallon,*
42 Gallons	——	1 Tierce,
63 Gallons	——	1 Hogshead,
84 Gallons	——	1 Puncheon,
2 Hogsheads	——	1 Pipe or Butt,
2 Pipes	——	1 Tun.

Pints	Quarts	Pottles	Gal.	Tierces	Hogsh.	Punch.	Pipe or Butt	Tun.
2	1							
4	2	1						
8	4	2	1					
336	168	84	42	1				
504	252	120	63	1$\frac{1}{2}$	1			
672	336	168	84	2	1$\frac{1}{2}$	1		
1008	504	152	126	3	2	1$\frac{1}{2}$	1	
2016	1008	504	252	6	4	3	2	1

9. *DRY MEASURE.*

Is used to measure corn and other dry goods; but as they are generally sold by weight in Ireland, this measure is little used; however the denominations are as follow, *viz.*

2 Pints

* The gallon appointed to be used for measuring all kinds of liquids in Ireland is 217$\frac{6}{10}$ cubic inches, and 40 gallons are a barrel of ale, tho' the vessel usually holds 42.

In England the wine gallon is 231; and the beer or ale gallon 282 cubic inches.

2 Pints	make	1 Quart,
2 Quarts	——	1 Pottle,
2 Pottles	——	1 Gallon,
2 Gallons	——	1 Peck,
4 Pecks	——	1 Bushel,
4 Bushels	——	1 Barrel,
2 Barrels	——	1 Quarter.

10. *TIME.*

60 Seconds	make	1 Minute,
60 Minutes	——	1 Hour,
24 Hours	——	1 Day,
7 Days	——	1 Year.
365 Days	——	1 Year.

ADDITION *of Numbers of Divers Denominations.*

Rule.

PLACE the numbers so, that those which are of the same denomination stand exactly underneath each other. Then beginning with the figures of the lowest or least denomination, add them together into one sum; divide this sum by that number, which 1 of the next higher contains of the denomination added, the remainder set down underneath the added figures, and carry the quotient figure to be added with the figures of the next superior denomination, and so proceed from one denomination to another until all be finished.

PENCE TABLE.

		s.	d.
20	Pence is	1	08.
30	——	2	06
40	——	3	04
50	——	4	02
60	——	5	00
70	——	5	10
80	——	6	08
90	——	7	06
100	——	8	04
110	Pence is	09	02
120	——	10	00
130	——	10	10
140	——	11	08
150	——	12	06
160	——	13	04
170	——	14	02
180	——	15	00

Examples *of* Money.

[1] *l.*	*s.*	*d.*
710	11	06
616	14	03
419	15	07

[2] *l.*	*s.*	*d.*
516	14	08
319	18	06
724	17	09

[3] *l.*	*s.*	*d.*
965	19	11
876	17	09
742	10	01
967	19	10
435	14	11

[4] *l.*	*s.*	*d.*
998	18	02
873	01	09
175	12	03
789	14	06
966	10	06

[5] *l.*	*s.*	*d.*
306	02	08
1090	13	01
984	12	03
442	13	00
653	11	09

[6] *l.*	*s.*	*d.*
1342	03	01
827	11	11
371	14	03
1208	17	09
799	17	06

[7] *l.*	*s.*	*d.*
105	17	07
193	10	11
1200	13	00
319	19	07
1004	04	11
96	16	06
111	09	09
976	13	10
449	12	06
7	14	11

[8] *l.*	*s.*	*d.*
940	10	07
28	11	06
308	18	04
1003	11	04
153	12	09
380	13	07
1003	14	06
796	17	06
874	16	07
9	03	05

[9] *l.*	*s.*	*d.*
999	18	11
808	17	10
777	18	01
666	16	09
505	15	08
677	13	07
333	12	06
735	11	05
313	10	01
331	19	03
113	17	01
419	14	04

[10] *l.*	*s.*	*d.*
177	18	11
878	19	10
909	06	11
1851	17	10
765	17	02
983	10	03
1796	11	05
641	13	04
1351	15	07
235	16	06
7	07	07
1065	11	03

[11] *l.*	*s.*	*d.*
875	18	09½
1898	17	08
798	19	09¾
896	18	11
1768	05	09¼
585	16	08
1079	14	10½
898	17	07
1199	16	09¾
426	13	02¼
1519	12	09

[12] *l.*	*s.*	*d.*
1007	17	07½
489	19	09
1998	17	10¾
996	16	11
1548	18	05½
798	15	05
1654	11	04¾
607	14	03
1596	14	07½
849	13	03
1700	00	00¾

[13] *l.*	*s.*	*d.*
1695	18	10
4304	17	09
3975	15	03
8162	10	11
7594	11	10
9150	13	10
5329	19	11
1904	10	07
4985	04	02
6403	16	10
9108	19	10
2399	18	11
6624	12	09

[14] *l.*	*s.*	*d.*
9790	17	10½
7549	18	07
6854	15	09½
8162	18	04
9971	12	06½
8986	11	11
8796	14	10¾
9875	13	11
6914	15	10¼
7896	17	04
9713	18	06½
9162	15	11
6397	19	10¼

[15] *l.*	*s.*	*d.*
1103682	15	11¾
19020	17	04
1032	19	08¾
50233	16	04½
1006	13	02
103021	17	04½
2071069	18	10
104	12	06¼
503019	08	00¼
50017	15	04
1188	10	03
106	18	06½
9986	17	11
93009	03	08¼
170066	18	02
31117	17	06½
5006	08	08¼
4117	05	04
2118	03	11¾
12144	08	08
3331	12	07½
113217	16	06
172101	17	03
1716	16	10
14	13	05¼
179	19	07
20016	18	07
8003	14	06¼
3	07	06½

[16] *l.*	*s.*	*d.*
18101935	15	04
16173260	13	03½
17121960	14	06
1218130	18	06¼
154188	13	07¾
176099	09	11
136033	17	10½
13093	02	07½
121966	14	05
13666	12	08½
27333	13	04½
17613	19	08
27333	18	06½
3217	12	08
13666	12	04½
1832999	17	02½
466	06	05½
1783	13	07
18926	19	11¼
17412	18	06
176849	13	07½
332191	13	07
182	06	06½
1827	09	07
32115	08	11¾
36419	16	08¾
177722	16	02
1133219	17	03¼
12991	19	08¼
1	12	03¾

Note. When the columns of the money to be added are long (as frequently happens in business) it is troublesome and liable to error to collect them into one continued sum: to remedy which, *practice* hath invented sundry contrivances for ease and dispatch; some point for every 12 in pence, and every 20 in shillings; this is esteemed inelegant and unclerk like; but instead thereof we may very conveniently point in pence at 120 or 10*s.* In shillings and pounds at 100, and then we must carry 10 for every point.

By

2. *Troy Weight.*

[17] ℔.	*oz.*	*dwt.*	*gr.*
886	11	15	23
239	10	10	20
147	05	13	19
455	10	09	18
559	09	18	05
254	10	05	01

[18] ℔.	*oz.*	*dwt.*	*gr.*
809	06	09	07
908	10	15	23
089	01	04	20
876	07	19	20
789	07	08	14
570	06	04	10

3. *Avoirdupois Weight.*

[19] *C.*	*qrs.*	℔.
943	2	11
386	3	25
479	2	27
421	3	19
784	1	26
234	2	09
432	3	03
867	1	06

[20] *C.*	*qrs.*	℔.
1153	1	09
151	2	22
1519	3	21
195	0	19
1567	1	23
567	2	01
1267	3	09
2643	1	11

By applying this to the last example (page 50) my meaning may be more clearly comprehended.

First then adding the farthings, I find them 35, *i. e.* 8¾*d.* carry 8; then say 8 I carry and 8 is 16; *&c.* up to the sum *17613 19 8 where the pence amount to 124: I point for 120 or 10*s.* and then proceed saying 4 and 4 is 8, *&c.* to the top, where the sum amounts to 75*d.* *i. e.* 6*s.* 3*d.* put down 3 and carry 6+10 for the point, *i. e.* 16 to the shillings; then I begin with the units of shillings and say 16 I carry and 9 is 25 and 7 is 32, *&c.* up to the line *1832999 17 2½, where the sum amounts to 106: I point for 100 and carry 6; and so adding to the top, when the sum is 82, I set down 2 and carry 8+10 for the point, *i. e.* 18 to the tens of shillings, which I count up, saying 18, 19, 20, *&c.* up to the top, where they amount to 41. Then since every 2 tens make 20 shillings or 1*l.* I say half of 41 is 20 and 1 over, which odd 1 remaining is to be put to the 2 before set down to make it 12, *&c.* Then carry the 20 and proceed to the pounds, and make a point for every 100 in like manner.

Note. Some chuse to point at 60 or 5*s.* in the pence

[21]	*Ton*	*C.*	*qrs.*	℔.
	194	19	3	21
	890	13	2	15
	908	16	1	24
	577	18	0	23
	789	15	2	27
	865	14	3	26
	468	14	2	23
	779	16	3	16

[22] ℔.	*oz.*	*dr.*
27	10	10
17	08	04
25	13	03
29	12	11
16	10	15
26	14	08
17	15	13
24	10	05

4. *Liquid Measure.*

[23]	*Tun*	*Hhd.*	*Gal.*
	31	3	59
	29	2	41
	54	3	62
	25	1	39
	31	1	62

[24]	*Tun*	*Hhd.*	*Gal.*
	85	1	61
	84	2	59
	97	3	40
	69	0	59
	53	3	29

[25]	*Hhd.*	*Gal.*	*Pints.*
	27	53	1
	17	29	3
	25	17	5
	37	45	4
	13	19	6
	25	28	2

[26]	*Tun.*	*Hhd.*	*Gal.*	*Pints.*
	4	3	59	2
	7	1	60	3
	6	2	15	4
	2	0	24	7
	3	3	33	1
	5	1	27	0

SUBTRACTION *of Divers Denominations.*

Rule.

PLACE each denomination under that of the same kind. Then begin with the lowest denomination and subtract it from the number of the same denomination above it, (if it be greater) but if the lower denomination be greater, then subtract it from the number which one of the next highest contains of the lower denomination, and to the remainder add the upper number of the lower denomination, the sum is the true remainder required: Then subtract the next higher denomination of the lower line from 1 less than the number of the same denomination in the upper; or else add 1 to the lower and subtract the sum from the upper; and thus proceed from one denomination to another until all be subtracted.

Examples *of* Coin.

	[1] *l.*	*s.*	*d.*	[2] *l.*	*s.*	*d.*
Received	1098	19	10	979	15	11
Paid	134	15	06	200	19	11

	[3] *l.*	*s.*	*d.*	[4] *l.*	*s.*	*d.*
Received	9275	02	03	8500	19	07
Paid	7869	19	11	985	18	09

	[5] *l.*	*s.*	*d.*	[6] *l.*	*s.*	*d.*
Received	1000	19	09¼	4351	17	10½
Paid	999	09	11½	2359	19	11¾

	[7] *l.*	*s.*	*d.*	[8] *l.*	*s.*	*d.*
Received	14931	16	10	3687	07	08¾
Paid	14919	17	09½	1457	19	06¼

Troy Weight.

	[9] ℔.	*oz.*	*dwt.*	*gr.*	[10] ℔.	*oz.*	*dwt.*	*gr.*
Bought	554	9	19	23	946	00	00	01
Sold	97	0	17	15	125	11	17	23

[11]	℔.	*oz.*	*dwt.*	*gr.*
Bought	304	11	15	13
Sold	196	10	19	21

[12]	℔	*oz*	*dwt.*	*gr.*
	917	0	14	09
	798	9	18	17

Avoirdupois Weight.

[13]	*C.*	*qrs.*	℔.
Bought	200	2	26
Sold	99	3	15

[14]	*C.*	*qrs.*	℔.
	275	2	15
	27	2	07

[15]	*C.*	*qrs.*	℔.
Bought	6040	1	21
Sold	1908	3	27

[16]	*C.*	*qrs.*	℔.
	1937	0	00
	889	3	27

[17]	*C.*	*qrs.*	℔.
From	8340	2	03
Take	1090	3	16

[18]	*C.*	*qrs.*	℔.
	9074	3	25
	7849	1	16

[19]	*Tons.*	*C.*	*qrs.*	℔.
Bought	6904	16	3	11
Sold	987	14	0	27

[20]	*Tons.*	*C*	*qrs.*	℔.
	7650	10	0	13
	4876	10	3	25

Liquid Measure.

[21]	*Tun*	*Hhd.*	*Gal.*
Bought	31	3	15
Sold	29	2	26

[22]	*Tun*	*Hhd.*	*Gal.*
	54	0	27
	21	3	42

[23]	*Tun*	*Hhd.*	*Gal.*
Bought	304	0	54
Sold	109	3	54

[24]	*Tun*	*Hhd.*	*Gal.*
	721	1	33
	247	2	11

QUESTIONS

Shewing the use of Addition *and* Subtraction *of Money.*

Dublin, 5th of July, 1809.

1. Bought of George Grocer,

12*C*, 2*qrs.* of sugar, at 52*s.* ⅌ *Cwt.*	£.	32	16	03
28℔ of rice, at 3*d.* ⅌ ℔ —		0	07	00
10 loaves of sugar, *wt.* 35℔ at 1*s.* 1*d.* —		1	17	11
3*C* 2*qrs.* 12℔. of raisins, at 36*s.* ⅌ *Cwt.*		6	18	10
20℔ of tea, at 5*s.* 6*d.* ⅌ ℔. —		5	10	00

2. Laid out at market, *viz.*

For meat, seven groats and two pence
Butter, fourteen pence — —
Onions, five farthings — —
Eggs, three halfpence — —
Linen, two and twenty pence —
Potatoes, a groat — —

£. 0 6 0¾

3. How much is the sum of

Seven and thirty shillings and six pence
Nine and thirty shillings and three halfp.
Four and forty shillings and nine pence
Twenty-nine shillings and three pence
Fifty shillings — — —

£. 10 0 7½

4. A nobleman going to the country, orders his tradesmen's bills for payment, which are as follow, *viz.* the brewer's 41*l.* 10*s.* the butcher's 212*l.* 6*d.* the baker's 24*l.* the tallow chandler's 13*l.* 8*s.* the taylor's 137*l.* 9*s.* 9*d.* the

draper's 74*l*. 13*s*. 6*d*. the coach-maker's 214*l* 16*s*. 6*d*. the wine merchant's 68*l*. 12*s*. the confectioner's 16*l*. 2*s*. his rent 50*l*. his servant's wages came to 46*l*. 5*s*. and he would carry with him 50*l*. to defray his expences: For what sum must he draw on his banker to answer all these?
Answ. 948*l*. 17*s*. 3*d*.

5. Received of Peter Paywell in payment of his bill of 10*l*. 1 Portugal piece of 3*l*. 17*s*. 8*d*. 2 moidores, 2 guineas and an half, a crown and 1 half crown: I desire to know whether I must receive or return change, and how much? *Answ*. I must return 1*s*. 2*d*.

6. Bought 6 bags of hops, weighing, *viz*, N°. 1, 2 *C*. 2*qrs*. 10℔. N°. 2, 2*C*. 1*qr*. 16℔. and No. 3, 2*C*. 0*qrs*. 24℔. N°. 4, 2*C*. 2*qrs*. N°. 5, 2*C*. 1*qr*. 12℔. and N° 6, 2*C*. 1*qr*. 16℔. How many *Cwt*. have I bought? *Answ*. 14*C*. 2*qrs*. 22℔.

7. In a gentleman's service of plate there are 14 dishes weighing 16℔ 10*oz* 13*dwt*. 36 plates weighing 85℔ 10*oz*. 11*dwt*. 6 salts weighing 2℔. 8*oz*. knives and forks 6℔. 11*oz*. 2*dwt*. 4 salvers 9℔. 5*oz*. 4*dwt*. cups, tankards, *&c*. 22℔. 0*oz*. 18*dwt*. a silver tea kettle and lamp 10℔ 6*oz*. 9*dwt*. What quantity of plate had the butler under his care? *Answ*. 102℔. 11*oz*. 4*dwt*.

8. A merchant's clerk receives of sundry persons for his master, *viz*. of A 13*l*. and half a crown, of B 2*l*. 13*s*. of C 2 pistoles and half a guinea, of D 1*l*. 9*s*. 8½*d*. of E 11*l*. and 6¼*d*. of F a moydore and 13 English shillings, of G a bank-note of 20*l*. of H 15*l* and 3 crown pieces over. I desire to know what sum he had in charge?
Answ. 68*l*. 13*s*. 4¾*d*.

9. A merchant bought 600 salt ox hides, weighing 561 *C*. and 2℔. of which he sells 250, weighing 239*C*. 3*qrs*. 25℔. I demand how many hides he hath left, and what they weigh? *Answ*. 350 hides, weighing 321*C*. 0*qrs*. 5℔.

10. A shop-keeper bought a piece of cloth containing 42 yards for 22*l*. 10*s*. of which he sells 27 yards for 15*l*. 15*s*. I demand how many yards he hath left, and what they stand in? *Answ*. 15 yards, which stand in 6*l*. 15*s*.

11. A merchant who had 209 casks of butter, weighing 400*C.* 2*qrs.* 14℔. shipped off 173 casks thereof, weighing 413*C.* 2*qrs.* 27℔. I demand how many casks he had left and their weight? *Answ.* 36 casks, containing 136*C.* 3*qrs.* 15℔.

12. What five numbers of pounds, shillings and pence, all different, will make just 100*l.*

13. Paid A B, in full for E F's bill on me for 75*l.* *viz.* I gave him Richard Drawer's note for 7*l.* 12*s.* 6*d.* Peter Johnson's ditto for 5*l.* an assignment on Robert Dealer for 17*l.* 13*s.* 9½*d.* In bank notes 40*l.* the rest I make up in cash. I want to know what sum will make up the deficiency? *Answ.* 4*l.* 13*s.* 8½*d.*

14. A trader failing, was indebted to A 71*l.* 12*s.* 6*d.* to B 34*l.* 9*s.* 9*d.* to C 16*l.* 18*s.* 8*d.* to D 41*l.* to E 66*l.* 7*s.* 6*d.* to F 11*l.* 2*s.* 3*d.* to G 19*l.* 19*s.* to H 20*l.* At the time of this disaster he had by him in cash 3*l.* 13*s.* 6*d.* in commodities 23*l.* 10*s* in household furniture 13*l.* 8*s.* 6*d.* in plate 7*l.* 18*s.* 5*d.* in a tenement 56*l.* 15*s.* in recoverable book debts 87*l.* 13*s.* 10*d.* Supposing these things faithfully surrendered to his creditors, what will they lose by him? *Answ.* 91*l.* 10*s.* 5*d.*

15. Bought 5 hogsheads of sugar, weight as follows:

		C.	*qrs.*	℔.		*C.*	*qrs.*	℔.
Nº. 10	Gross	7	1	18	Tare		3	12
11	——	6	4	24	——		3	04
12	——	6	2	14	——		2	25
13	——	7	0	05	——		3	08
14	——	7	3	26	——		3	19

I demand the nett weight of the sugar when the tare is taken away? *Answ.* 31*C.* 3*qrs.* 19℔.

MULTI-

MULTIPLICATION *of Divers Denominations.*

Case I.

When the multiplier is less than 12, multiply first the lowest denomination thereby, and manage the product directly in the same manner as the sum in *Addition of divers Denominations*, and so proceed from one denomination to another till they all be multiplied.

Remark.

The learner will with a little practice, be able to perform all the work of this multiplication by the memory. But as the shillings may frequently be more than 12, in that case, multiply the units by the multiplier, adding to the product the shillings reserved in the product of the pence, and for every ten reserve 1 to be carried, and set down the remainder (or overplus) above the tens, for the units of the shillings in the product: Then multiply the tens, adding thereto the tens reserved from the product of the units, and divide the sum by 2, the quotient is the pounds to be carried, and if 1 remains put it down in the second place of the shillings in the product.

Application.

Suppose as before the sum to be multiplied is 5*l.* 13*s.* 6½*d.* where 2 is reserved from the product of the pence——I multiply the shillings thus, 4 times 3 is 12, and 2 I carry is 14, put down 4 and carry 1; then I say 4 times 1 is 4 and 1 I carry is 5, the half of 5 is 2, or 2 in 5 goes twice and 1 remains, so I put the 1 remaining in the second (or tens) place of the shillings, whereby the shillings in the product become 14, and reserve the quotient 2 to be carried to the pounds.

l.	*s.*	*d.*
5	13	6½
		4
22	14	2
	2) 5	
	2	1

Examples to Case I.

	l.	*s.*	*d.*			*l.*	*s.*	*d.*	
[1] Multiply	1	7	6	by 2	[2]	4	13	2	by 3
	l.	*s.*	*d.*			*Yds.*	*qr.*	*na.*	
[3] Multiply	0	17	8½	by 4	[4]	13	2	2	by 5

C. *qrs.* ℔. — *l.* *s.* *d.*
[5] Multiply 4 2 12 by 6 — [6] 12 18 7½ by 7
l. *s.* *d.* — *l.* *s.* *d.*
[7] Multiply 0 13 6½ by 8 — [8] 5 17 10½ by 9
[9] Multiply 1 8 4 by 10 — [10] 13 16 8 by 11
C. *qrs.* ℔.
[11] Multiply 6 2 24 by 12.

Case II.

When the multiplier is greater than 12, but the product or composit of two numbers both less than 12.

Rule.

Multiply first by one component part of the given multiplier, and the product so found multiply by the other; this last product will be the answer.

Application.

Let 4*l.* 17*s.* 10*d.* be given to be multiplied by 21.

Multiply first	4	17	10	
by			3	one component part } 21
And the product	14	13	6	
by			7	the other comp. part } 21
Answ.	102	14	6	

Examples.

	l.	*s.*	*d.*			*l.*	*s.*	*d.*
[12] Multiply	1	8	4	by 21	*Answ.*	29	15	0
[13] Multiply	0	17	4¼	by 32	——	27	16	0
[14] Multiply	1	10	8	by 44	——	67	9	4

		l.	s.	d.				l.	s.	d.
[15]	Multiply	4	16	1½	by	56	*Answ.*	269	12	4
[16]	Multiply	3	17	8	by	64	——	248	10	8
[17]	Multiply	1	13	6	by	121	——	202	13	6
[18]	Multiply	0	18	3	by	72	——	65	14	0
[19]	Multiply	0	4	9	by	144	——	34	4	0

Case III.

When the multiplier is not composite of two numbers less than 12, but exceeds some such composite number by a number not greater than 12.

Rule.

For the composite number next lest than the given multiplier, find the product (☞ last rule) then multiply the given multiplicand by the overplus, (or number by which the assumed composite number is less than the given multiplier) and add the product together; the sum is the product required.

Application.

Let 1*l.* 17*s.* 4½*d.* be multiplied by 23.

	l.	s.	d.	
Multiply	1	17	4½	
			3	(2
	5	12	1½	
			7	
	39	4	10½	Product by 21 ☞ last.
	3	14	9	—by 2 the overplus of 23 above 21.
Answ.	42	19	7½	—by 23.

Or thus:

1	17	4½	22
		2	
3	14	9	
		11	
41	02	3	Product by 22.
1	17	4½	——— by 1, the N°. wanting to make 23.
42	19	7½	——— by 23.

Examples.

		l.	*s.*	*d.*				*l.*	*s.*	*d.*
[20]	Multiply	3	13	4	by	31	*Answ.*	113	13	4
21]	Multiply	1	11	6	by	23	——	36	4	6
2]	Multiply	0	16	6½	by	47	——	38	17	5½
[23]	Multiply	1	18	10	by	68	——	132	0	8
[24]	Multiply	1	2	9	by	75	——	85	6	3
[25]	Multiply	0	16	8	by	112	——	93	6	8
[26]	Multiply	0	5	5	by	139	——	37	12	11
[27]	Multiply	1	15	4½	by	155	——	274	3	1½

QUESTIONS

Shewing the Use of this *MULTIPLICATION*.

1. *In casting up the price of Goods.*

1. Bought a piece of broad cloth containing 24 yards, at 15*s.* 3*d.* ℔ yard; what comes it to? *Answ.* 18*l.* 6*s.*

2 What cost a chest of tea; weighing 98℔. at 5*s.* 6*d.* ℔ ℔.? *Answ.* 26*l.* 19*s.*

3. Bought 39℔. of butter, at 1*l.* 5*s.* 6*d.* ℔ ℔. what comes it to? *Answ.* 49*l.* 14*s.* 6*d.*

4. What cost 1 hundred *wt.* of sugar at 7½*d.* ℔ ℔.? *Answ.* 3*l.* 10*s.*

5. Bought a hogshead of wine at 5*s.* 4*d.* ꝑ gallon; what did it cost? *Answ.* 16*l.* 16*s.*

6. If I hold 120 acres of land at 14*s* 6*d.* ꝑ acre; what is my yearly rent? *Answ.* 87*l.*

2. *In casting up Coins.*

A merchant sends his clerk to the bank to receive cash for a note of 75*l.* which he receives in the following species, *viz.*

		l.	*s.*	*d.*				*l.*	*s.*	*d.*
27	at	1	2	9	—	—	—			
23	at	1	9	1	—	—	—			
40	at	0	5	5	—	—	—			
							Change	00	00	2
								75		

Paid Daniel Raymond in full for John Denham's bill on me, 100*l.* *viz.*

		l.	*s.*	*d.*				*l.*	*s.*	*d.*
60	at	1	2	9	—	—	—			
21	at	1	9	3	—	—	—			
1	at	0	19	6	—	—	—			
							Change	00	01	3
								100		

Received of Abraham Accepter in full for Richard Drawer's bill on him, 150*l.* *viz.*

		l.	*s.*	*d.*				*l.*	*s.*	*d.*
36	at	1	18	10	—	—	—			
11	at	1	9	1	—	—	—			
158	at	0	5	5	—	—	—			
20	at	1	2	9	—	—	—			
								150	1	8
							Change returned		1	8
								150		

Received of D B, in full for tobacco sold him the 27*th* of September, 143*l.* 12*s.* 9*d.* *viz.*

		l.	*s.*	*d.*				*l.*	*s.*	*d.*
29	at	1	9	3	—	—	—			
13	at	3	17	8	—	—	—			
28	at	0	18	3	—	—	—			
16	at	0	18	1	—	—	—			
115½	at	0	1	1	—	—	—			
16¼	at	0	5	5	—	—	—			
								143	12	9

Paid John Hammond 250*l.* for 250 barrels of beef, *viz.*

		l.	*s.*	*d.*				*l.*	*s.*	*d.*
10	at	3	17	8	—	—	—			
25	at	3	17	6	—	—	—			
14	at	1	18	10	—	—	—			
20	at	1	9	3	—	—	—			
33	at	0	5	5	—	—	—			
8	at	0	1	1	—	—	—			
								250	0	1
							Received back			1
								250		

Received of James Thomas 327*l.* for my bill of 300*l.* English, which I have drawn on C B of London at 9 ℔ *Cent.*

		l.	*s.*	*d.*				*s.*	*d.*		*l.*	*s.*	*d.*
13	at	3	17	8	More	3	at	9	10				
25	at	3	17	6	——	2	at	9	8				
14	at	1	18	10	——	1	at	9	2				
29	at	0	19	6	——	1	at	9	0				
27	at	0	19	4	——	2	at	4	11				
21	at	1	2	9	——	1	at	4	9				
7	at	1	9	3									
9	at	1	9	1							327	1	4
16	at	1	8	11						Returned		1	4
26	at	0	18	3							327		

A passed a bond for 144*l.* 10*s.* the interest came to 19*l.* He then paid off 40 guineas, and gave a fresh bond for what was behind. By the time there was 13*l.* 4*s.* 8*d.* due on the second for interest, he paid off 24 moydores, 2 guineas and 6*s.* 8*d* more, took up the old bond, and signed a new one still for the residue. The principal again ran on till there was 9*l.* 11*s.* 3*d.* more due, and then he determined to take it up. How much had his creditor to receive. *Answ.* 73*l.* 1*s.* 9*d.*

DIVISION *of Divers Denominations.*

Case I.

To divide a number of divers denominations by a number less than 12.

Rule.

Divide the highest denomination by the divisor: then multiply the remainder (if any) by that number which unity or one of the same denomination contains of the next lower, and to the product add the number of the lower denomination in the given dividend; which sum divide by the given divisor, and the quotent in the number of the lower denomination; in like manner proceed from denomination to denomination till the lowest be come to.

		l.	*s.*	*d.*	*Application.*			*l.*	*s.*	*d.*	
[1]	Divide	2	13	6	by 2	[8]	Divide	32	14	0	by 9
[2]	———	1	17	9	by 3			*C.*	*qrs.*	℔.	
[3]	———	3	18	0	by 4	[9]	———	10	3	16	by 10
[4]	———	54	17	6	by 5			*Yds.*	*qrs.*	*na.*	
[5]	———	236	10	0	by 6	[01]	———	17	0	3	by 11
[6]	———	1	19	$2\frac{3}{4}$	by 7			*l.*	*s.*	*d.*	
[7]	———	27	18	6	by 8	[11]	———	182	16	0	by 12

Case II.

To divide by a composite number greater than 12.

Rule.

Divide first by one component part, and the quotient by the other, the last quotient is the answer sought.

Application.

Let 102*l.* 6*d.* be given to be divided by 21.

		l.	*s.*	*d.*	
21	7)	102	14	6	divided by 7.
	3)	14	13	6	the first quotient to be divided by 3.
	Answ.	4	17	10	

The reason is evident.

		l.	*s.*	*d.*			*l.*	*s.*	*d.*
[12]	Divide	29	15	0	by 21	*Answ.*	1	8	4
[13]	———	27	16	0	by 32	———	0	17	$4\frac{1}{4}$
[14]	———	67	9	4	by 44	———	1	10	8
[15]	———	269	12	4	by 56	———	4	16	$2\frac{1}{2}$
[16]	———	248	10	8	by 64	———	3	17	8
[17]	———	202	13	6	by 121	———	1	13	6
[18]	———	65	14	0	by 72	———	0	18	3
[19]	———	34	04	0	by 144	———	0	4	9

Case III.

If the divisor falls under neither of the foregoing cases, the quotient may be found by *long division,* as follows, *viz.*

Divisor 23)42 19 $7\frac{1}{2}$(1 17 $4\frac{1}{2}$
23
———
Remainder 19*l.*
Multiply by 20 shillings in 1*l.* and add in 19*s.*
———
23)399(17 shillings.
23
———
169
161
———
Remain 8 shillings,
Multiply by 12 the pence in a shilling, and add 7*d.*
———
23)103(4 pence,
92
———
Remain 11
Multiply by 4 farthings in 1 penny, and add $\frac{1}{2}$*d.*
———
23) 46 (2 farthings.
46
———

		l.	s.	d.				l.	s.	d.
[20]	Divide	113	13	4	by	31	*Answ.*	3	13	4
[21]	———	38	17	$5\frac{1}{2}$	by	47	———	0	16	$6\frac{1}{2}$
[22]	———	132	0	8	by	68	———	1	18	10
[23]	———	85	6	3	by	75	———	1	2	9
[24]	———	740	16	8	by	100	———	7	8	2

Case IV.

If the given quantity or divisor consists of $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$.

Rule.

Multiply the given quantity by 4, adding to the product 1 for $\frac{1}{4}$, 2 for $\frac{1}{2}$, 3 for $\frac{3}{4}$; and it will give the divisor, which divide with as before, and the quotient multiply by 4, will give the answer.

Examples.

Suppose I give for $6\frac{1}{4}$ yards of cambrick 4*l.* 10*s.* $7\frac{1}{2}$*d.* at what rate did I buy it ⅌ yard?

```
 6¼        l.   s.   d.
 4    {5)  4   10   7½
 ──       ────────────
 25   {5)  0   18   1½
          ────────────
           0    3   7½
                    4
          ────────────
       £.  0   14   6  Answ.
```

25. Suppose a person in trade to clear 1035*l.* 9*s.* $0\frac{3}{4}$*d.* in $10\frac{1}{2}$ years equally, what was his yearly increase of fortune? *Answ.* 98*l.* 12*s.* $3\frac{1}{2}$*d.*

26. Suppose another to clear 518*l.* 8*s.* $0\frac{3}{4}$*d.* equally in $8\frac{3}{4}$ years, what was his yearly profit? *Answ.* 59*l.* 4*s.* 11*d.*

QUESTIONS *shewing the Use of this* DIVISION.

Quest. 1. A piece of broad cloth containing 24 yards, cost 18*l.* 6*s.* what did it cost ⅌ yard? *Answ.* 15*s.* 3*d.*

2. Bought 39℔ of butter, for 49*l.* 14*s.* 6*d.* what did it cost ⅌ *Cwt.*? *Answ.* 1*l.* 5*s.* 6*d.*

3. If a hogshead of wine cost 16*l.* 16*s.* what was it ⅌ gallon? *Answ.* 5*s.* 4*d.*

4. If 27 *C.* of sugar cost 47*l.* 13*s.* 9*d.* what cost 1 *Cwt.*?
Aw. 1*l.* 15*s.* $3\frac{3}{4}$, $\frac{5}{9}$*d.*

5. If a *Cwt.* of sugar cost 3*l.* 10*s.* what is it ꝑ ℔.
Answ. $7\frac{1}{2}$*d.*

6. If I held 120 acres of land, and my yearly rent is 87*l.* what do I pay an acre? *Answ.* 14*s.* 6*d.*

7. If a reckoning of 6*l.* 14*s.* 2*d.* be to be paid by 35 persons, what must they pay a piece? *Answ.* 3*s.* 10*d.*

8. A man dying left his estate worth 1000*l.* betwixt his wife and 3 sons, *viz.* to his wife $\frac{1}{3}$; to the eldest son $\frac{1}{4}$, and the remainder to the second and third, share and share alike: what is the share of each? *Answ.* The wife 333*l.* 6*s.* 8*d.* the first son 250*l.* the second and third sons 208*l.* 6*s.* 8*d.*

9. Three merchants, A, B and C, have a ship in company: A hath $\frac{5}{8}$, B $\frac{1}{4}$, and C $\frac{1}{8}$; and they receive freight 228*l.* 16*s.* 8*d.* It is required to divide it among the owners according to their respective shares? *Answ.* A's share 143*l.* 0*s.* 5*d.* B's 57*l.* 4*s.* 2*d.* and C's 28*l.* 12*s.* 1*d.*

10. Four merchants have a ship in company, *viz.* A $\frac{11}{32}$, B $\frac{1}{4}$, C $\frac{7}{32}$, and D $\frac{6}{32}$, and they let it out a voyage to the West Indies on freight, at 22*l.* 10*s.* 8*d.* ꝑ month; she performs the voyage in 18 months; what is each man's share of the freight? *Answ.* A's 139*l.* 8*s.* 6*d.* B's 101*l.* 8*s.* C's 88*l.* 14*s.* 6*d.* and D's 76*l.* 1*s.*

11. A father left among 5 sons an estate consisting of 500*l.* in cash, with 5 bills, each 48*l.* 10*s.* 6*d* he ordered 20*l.* to be laid out on his funeral, and his debts to be paid, amounting to 164*l.* the rest he leaves among his sons, thus, to the eldest $\frac{1}{3}$, and to the other four equal shares; what is the share of each son? *Answ.* 186*l.* 4*s.* 2*d.* the eldest, and the others 93*l.* 12*s.* 1*d.* a piece.

12. A privateer having taken a prize worth 1025*l.* it is divided into 100 shares, of which the captain is to have 11; 2 lieutenants, each 5; 12 midshipmen, each 2; and the remainder is to be divided equally among the sailors who are 120 in number? *Answ.* Captain's share 112*l.* 15*s.*

15*s.* lieutenant's 51*l.* 5*s.* a midshipman's 20*l.* 10*s.* and a sailor's 4*l.* 10*s.* 11½*d.*

13. Ten pounds a quarter is allowed to five auditors of a fire office. They attend about seven times in the quarter, and the absentees' money is always divided equally among such as do attend. A and B on these occasions never miss; C and D are twice in a quarter absent, and E only twice: At the payment what had each man to receive?

CHAP. VI.

REDUCTION.

REDUCTION is the changing a number of a higher or greater denomination to a lesser; likewise a number of divers denominations to the lowest of them, and the contrary, *viz.* to change a number of a lesser denomination to a greater.

I.

To bring a number of a higher or greater denomination to a lower or less.

Rule.

Multiply the given number of the higher by that number which one of the greater contains of the less.

So 27*l.* will be reduced into 540 shillings; for if 27 be multiplied by 20 (the shillings in a pound) the product is 540: in like manner 540 shillings may be reduced into 6480 pence: for if 540 be multiplied by 12 (the pence in 1 shilling) the product is 6480.

l.
27
20 shillings = 1*l.*
——
540 shillings.
12 pence = 1 shilling.
——
6480
——

II.

II.

To bring a number of divers denominations to the lowest mentioned.

Rule.

To the product found (by last) still add the odd number of the same denomination.

Application.

So 364*l.* 15*s.* 5*d.* may be reduced into 87545 pence, thus: 364*l.* multiplied by 20 (because 20 shillings make one pound,) makes 7280, to which, adding 15*s.* the sum is 7295 shillings; which shillings, being multiplied by 12 (the pence contained in a shilling) produce 87540 pence, to which add the 5 odd pence, the sum is 87545 pence.

	l. *s.* *d.*	
	364 15 5	
Mult. by	20	shil. in 1*l.* and add 15*s.*
	7295	shill. in 364*l.* 15*s.*
Mult. by	12	pence in 1*s.* and add 5*d.*
	87545	pence in 364*l.* 15*s.* 5*d.*

Note. The practical method of reducing the pounds is thus: say 0 is nothing, but 5 is 5, (the units of the shillings,) then say twice 4 is 8, and 1 (the ten shillings in 15) is 9, *&c.*

III.

To bring a number of a lesser denomination to a greater.

Rule.

Divide the given number by that which one of the greater contains of the less; the quotient is the answer.

Application.

s.
2|0)54|0
Answ. 27*l.*

So 540 shillings may be reduced to 27*l* For if I divide 540 by 20, (the shillings in 1 pound) the quotient will be 27.

Note. If after division any thing remains, the remainder is of the same name or denomination with the dividend. For it is the remaining part of the dividend after the divisor is taken away as often as possible.

Examples.

In 87545 pence how many pounds?

First, I divide 87545 pence by 12, (because 12 pence make a shilling) and thereby get the quotient 7295 shillings and there remain 5 odd pence over: Again dividing 7295 shillings by 20, the quotient is 364*l.* and 15 remain, *viz.* 15 odd shillings: So 87545 pence are reduced to 364*l.* 15*s.* 5*d.*

```
12) 87545
    ─────────
2|0) 729|5  5d. rem.
─────────────────
  364l. 15s. 5d.
─────────────────
```

Note. The bringing a greater name to a lesser is called *Reduction descending*, and bringing a less to a greater *Reduction ascending*.

Reduction descending and ascending mutually prove each other, as may appear by comparing the preceding operations.

Questions concerning Reduction.

Q. What is Reduction?

A. The bringing a greater denomination to a lesser, called Reduction descending; or a less to a greater, called Reduction ascending.

Q. How is Reduction performed?

A. The greater to the less is brought, when I have multiplied,
But to the greater to reduce the less, I must divide.

Q. If there is a remainder after I divide?

A. The remainder is of the same name with the dividend.

Q. How is Reduction proved?

A. Reduction descending, and Reduction ascending, prove each other.

1. *Money.*

1. *Money.*

1. In 36*l.* how many shillings?

2. In 720 shillings, how many pounds?

3. In 357*l.* how many pence?

4. In 85680 pence what pounds?

5. In 476*l.* how many farthings?

6. In 456960 farthings how many pounds?

7. In 49*l.* 18*s.* how many shillings?

8. In 998 shillings how many pounds?

9. In 36*l.* 7*s.* 9*d.* how many pence?

10. In 8733 pence what pounds?

11. In 375*l.* 17*s.* 10¾*d.* how many farthings?

12. In 360859 farthings how many pounds?

13. In 48*l.* 17*s.* 11½*d.* how many halfpence?

14. In 23471 halfpence how many pounds?

---

2. *Troy Weight.*

15. In 354℔ how many grains?

16. In 2039040 grains of silver how many ℔.?

17. In 7℔. 2*oz.* 15*dwt.* how many penny-weight?

18. 1735 *dwts.* how many ℔.?

19. In 428℔. 7*oz.* 13*dwt.* how many grains?

20. In 2779992 grains how many ℔.?

Frequent mention being made in this Treatise of Foreign Coins, now disused in Ireland, the following Table is here inserted.

	English.			*Irish.*		
	l.	*s.*	*d.*	*l.*	*s.*	*d.*
Portugal Piece - - - - - -	3	12	0	3	17	8
A Moydore - - - - - - - -	1	7	0	1	9	3
A *Louis d'or* or *French* pistole				0	18	3

Half and quarter the above in proportion.

3. *Of Avoirdupois Weight.*

21. In 27*C.* 2*qrs.* 12℔. how many pounds?

22. In 3092 pounds how many hundred weight?

23. In 24*Ton.* 14C. 3*qrs.* 15℔. how many pounds?

24. In 55427 pounds how many tons?

25. In 18℔. 10*oz.* 8*drs.* how many drams?

26. In 4776 drams how many pounds?

27. In 24 *Ton* 17C. 3*qrs.* 17℔. 5*oz.* 14*drs.* how many drams?

28. In 14275934 drams how many tons?

Note. Since 112 pounds make 1C. weight, hundred-weights may be brought into pounds by multiplying them by 112; and contrary wise, pounds may be reduced into hundred-weights by dividing them by 112.

Examples.

C.
Reduce 42 into pounds.
112℔. 1*Cwt.*

84
42
42

4704℔.

Note. If there be odd quarters and pounds, for 1 *qr.* put 28℔. for 2 *qrs.* 56℔. and 3 *qrs.* 84℔. which with the odd pounds add together with the products, placing them units under units, &c.

C. qr. ℔.
Bring 27 1 24 into
pounds. 112

54
27
27|28 for 1*qr.*
24 odd pounds.

3076 pounds.

Bring 4704 pounds into hundred weights.

℔.
112)4704(42*C.* *Answ.*
448

224
224

Note. If a remainder results after division, the remainder is pounds (131) divide the remainder by 28, the quotient will be quarters, and the last remainder (if any) pounds.

Bring 3076 pounds into hundred weights.
℔. C. qr. ℔.
112)3076(27 1 34
224

836
784

28)52(1
28

24

Now it is shewn that any number being added to itself, the sum will be equal to the same number multiplied by 2, and the other figures of 112 being 1, a number is multiplied by 112 when put down four times, *viz.* under itself units under units; again, units in the second place; and again, units in the third place, and then these numbers collected into one sum.

Hence it may be easily conceived how any number of hundreds, quarters and pounds may readily be reduced into pounds by addition only, as follows:

Let it be required to reduce 7*C.* 3*qrs.* 12℔. into pounds.

I put 7 under itself, which when added will be equal to the product of 7 multiplied by 2, then 7 placed twice more as in the margin, is equal to 7 multiplied by 10 and 100 respectively.

C.
7 3 12
7
7
784
12
——
880℔.
——

7 } = 7 × 2
7 }
7 . mult. by 10
7 . . mult. by 100
—— ——
784 mult. by 112
84℔. = to 3*qrs.*
12 odd pounds.
—— *C. qrs.* ℔.
880℔. in 7 3 12

Examples.

29. In 124*C.* 1*qr.* 13℔. how many pounds? *Answ.* 13929.

30. In 275*C.* 3*qrs.* 27℔. how many pounds? *Answ.* 30911.

31. In 75*C.* 2*qrs.* 27℔. how many pounds? *Answ.* 8483.

32. In 72*C.* 1*qr.* 8℔. how many pounds? *Answ.* 8100.

---

4. *Cloth Measure.*

33. In 75 yds. 1 qr. 3 na. how many nails?

34. In 1207 nails, how many yards?

35. In 720 yards, how many ells English?

36. In 576 ells, how many yards?

---

5. *Liquid Measure.*

37. In 65 tun, 2*hhds.* 7 *gal.* how many gallons?

38. In 16513 gallons, how many tuns?

39. In 27 tun, 3hhds. 53 gallons, 5 pints, how many pints?

40. In 56373 pints, many tuns?

QUESTIONS TO EXERCISE REDUCTION.

Quest. 1. In 3400*l.* how many crowns, half crowns, shillings, groats and three pences, and of each an equal number? *Answ.* 7004, and 68 half-pence over.

2. In 12 bags of wheat, each 2½*C.* how many stone, and allowing 20 stone to the barrel, how many barrels?
Answ. 240 stone, and 12 barrels.

3. How many strokes doth a regular clock strike in a year? *Answ.* 56940.

4. How many minutes since the commencement of the Christian Æra, allowing it to be 1809 years?

5. If from Dublin to Cork be 101 miles, I demand how many barley corns will reach between the two places, allowing 3 barley corns to make 1 inch?
Answ. 24433920.

6. How often will a chariot wheel 13 feet 4 inches in circumference, turn round in running from Dublin to Drogheda, supposing the distance 22 miles?
Answ. 8064 times.

7. Admit a ship's cargo from Bourdeaux to be 250 pipes, 130 hogsheads, and 150 quarter casks, (½ hogsheads) how many gallons in all: and allowing every pint to be a pound, what burden was the ship of?
Answ. 44415 gallons, and the ship's burden was 158 tuns, 12*C.* 2*qrs.*

8. Suppose a merchant hath orders to ship 892*C.* 3*qrs.* 12℔. of beef in barrels, each to contain 200℔. how many barrels will he want? *Answ.* 500.

s of wool, *viz.*

	C.	*qrs.*	℔.
o. 1	5	1	17
2	5	3	14
3	6	2	04
4	4	3	25
5	6	1	12

I want to know how many stones are therein?
Answ. 204 stone.

10. How many boxes, each to hold 12℔. may be filled out of a hogshead of tobacco containing 7½*C.*?
Answ. 70.

11. Received from Jamaica 56 hogsheads of sugar, each 12*C.* 1*qr.* 10℔. (100℔, being their *Cwt.*) how many *Cwt.* here of 112℔? *Answ.* 617*C.* 2*qrs.*

12. Imported from Rotterdam 46 bales of cloth, each containing 24 pieces, and each piece 42 ells Flemish; how many yards were therein? *Answ.* 34776 yards.

13. In 150000 crusadoes, each 400 reas, 1000 reas to a milre of 5*s.* 6*d.* how many pounds sterling?
Answ. 16500*l.*

14. In 506*l.* 12*s.* 6*d.* how many Portugal reas at 20 for 3*d.* *Answ.* 810600.

CHAP VII.

THE RULE OF THREE DIRECT.

THE method of finding a fourth proportional, from three numbers given, is called by some the *Rule of Proportion;* by others the *Rule of Three*, because 3 numbers are given to find a fourth; and again, the *Golden Rule* for its extraordinary use.

In practical questions where three (applicate) numbers are given to find a fourth proportional; the greatest difficulty will be in stating the question, or abstracting the numbers out of the words in the question, and placing them down in their proper order.

Now two of the three given terms are homogeneal, or of the same kind, one of which * asks or moves the question, and is to be put in the third place, the other number of the same kind in the first; and the remaining number in the second, being of the same name or kind with the fourth required. For instance,

If 3 yards cost 9 shillings, what will 6 yards cost at the same rate or proportion?

Here the first clause (if 3 yards cost 9 shillings) assigns or supposes the rate; then follows the question, what will 6 yards cost? Consequently the numbers must be ranged thus; by the rule, *viz.*

yds. *shil.* *yds.*
3——9——6

6 *yds.* which moves the question -	3*d.*
3 yards the same kind - - -	1*st.*
9 shillings - - - -	2*d.*

The numbers being thus stated, the rule for the operation is as follows:

Rule for the Operation.

Multiply the second and third terms, and divide their product by the first: The qoutient is the fourth proportional sought of the same name and kind with the second.

yds. *s.* *yds.*
3——9——6
6
——
3)54 prod. 2 and 3.
——
18 shillings.

If the first number be 1, the answer or fourth is found by multiplying the second and third: And if the second or third be 1, the fourth will be found by dividing the other number by the first.

---

* *Asks or moves the question)* The term which moves the question hath generally some words like these before it, *viz.* What will? What cost? How many? How far? How long? or How much?

To prove the work. Multiply the first and fourth terms together; likewise the second and third: and if the products be equal the work is right.

	1st.	Proof.	
4th : :	18	9	: : 2d.
1st : :	3	6	: : 3d.
	—	—	
	54	54	

Otherwise by varying the stating, as follows:

1. Directly (ꝑ *qu.*)	or 2. Inversely.	3. Alternately.
3 : 9 : : 6 : 18	18 : 6 : : 9 : 3	3 : 6 : : 9 : 18
	6	6
2. Inversely.	—	—
9 : 3 : : 18 : 6	18)54(3	3)54
	54	—
	—	18

That is when the fourth number is found, if it be made the first of another stating, the third given number the second; the second the third; and we work by the general rule, if the answer be the first given number, the work is right other ways.

These things being premised, I intend to lead the learner gradually through the varieties of this excellent rule, in the sundry cases following.

Case I. *Rule.*

The fourth number is always found in the same name which the second is given in, or reduced to; which if it be not the highest denomination of its kind, reduce it to the highest when it can be done.

Application and Reason.

Let the following question be proposed, *viz.* If 1℔. of sugar cost 7*d* what will 112℔. come to?

Multiplying the third 112 by 7 the second, I find the product 784, which is the fourth number sought, since the first number is one, I say it is 784 pence, the same name with the second number, as is self-evident (I think) from the least consideration of the nature of the question:

for if 1℔. is equivalent to 7*d.* 112℔. of the same must be worth 112 times as much, *viz.* 112 times 7*d.* *viz.* 784 pence. But then since accompts are kept in pounds, shillings and pence, and payments made in coins which are calculated in shillings and pence, or pounds, shillings and pence, it is proper that these pence be brought into pounds by Reduction, and then the answer will be 3*l.* 5*s.* 4*d.* Q. E. I.

```
℔.   d.     ℔.
1 ——7—— 112
           7
         ———
     12)784
         ———
    2|0)6|5  4
    ——————————
 Answ. 3l. 5s. 4d.
    ——————————
```

Examples.

Quest. 1. What cost 327 yards of canvas, at 8*d.* ℘ yard? *Answ.* 10*l.* 18*s.*

2. What will 17 tons of tallow come to at 25*l.* the ton? *Answ.* 425*l.*

3. At 7*s.* ℘ ℔. what cost 128℔. of tea? *Answ.* 44*l* 16*s.*

4. What cost 139 barrels of barley, at 6*s.* ℘ barrel? *Answ.* 41*l.* 14*s.*

5. How much is the price of 178℔. of merchandize, at 17*d.* ℘ ℔.? *Answ.* 12*l.* 12*s.* 2*d.*

6. What cost 1727℔. of rice, at 9 farthings ℘ ℔. *Answ.* 16*l.* 3*s.* 9$\frac{3}{4}$*d.*

Case II.

When the second number is of divers denominations, bring it to the lowest mentioned, and the fourth will be found in the same name, to which the second is reduced, which reduce back to the highest possible.

Examples.

Let it be required to find what 178 yards of linen will cost, at 4*s.* 8*d.* ℘ yard?

```
1 ———— 4  8
```

```
        s.  d.
1 ———— 4   8 ———— 178
       12          56
       ——         ————
       56         1068
                  890
                  ————
               12)9968
               ———————
              2|0)83|0  8
              ———————————
              £. 41 10 8
              ———————————
```

7. What is the amount of 324 pieces, at 2*s.* 8½*d.* ℘ piece? *Answ.* 43*l.* 17*s.* 6*d.*

8. How much will 120*C.* come to, at 18*s.* 6*d.* ℘ *Cwt.*? *Answ.* 111*l.*

9. What will 24 yards of broad cloth cost, at 15*s.* 3*d.* ℘ yard? *Answ.* 18*l.* 6*s.*

10. Bought 39C. of butter, at 1*l.* 5*s.* 6*d.* ℘ *Cwt.* what comes it to? *Answ.* 49*l.* 14*s.* 6*d.*

11. If I hold 120 acres of land, at 14*s.* 6*d.* ℘ acre, what is my yearly rent? *Answ.* 87*l.*

12. If I expend one week with another 2*l.* 13*s.* 5½*d.* ℘ week, what is my expence in 52 weeks? *Answ.* 138*l.* 19*s.* 10*d.*

Case III.

If the first and third be of different names, or one or both of divers denominations, reduce them both to one denomination, that is, to the lowest mentioned in either.

13. What quantity of brandy can I get for 18*l.* 18*s.* at *s.* ℘ gallon? *Answ.* 1 hogshead.

14. At 5*s.* ℘ ounce, what silver will 6*l.* pay for?
Answ. 2℔.

15. If a yard of broad cloth cost 12*s.* how many yards can I have for 15*l.*? *Answ.* 25 yards.

16. If one ounce of spice cost 4*d.* what is the price of 112℔.? *Answ.* 29*l.* 17*s.* 4*d.*

17. At 3*s.* 4*d.* ℘ pair, what cost 17 dozen and 4 pair of stockings? *Answ.* 34*l.* 13*s.* 4*d.*

18. If 1 ton of cheese cost 16*l.* 16*s.* what cost 20 ton, 17*C.* 2*qrs.*? *Answ.* 350*l.* 14*s.*

19. If 1℔. of silk be worth 1*l.* 9*s.* 6*d.* what cost 25℔. 8*oz.*? *Answ.* 37*l.* 12*s.* 3*d.*

20. If 1*C.* of sugar cost 1*l.* 6*s.* 8*d.* what cost 17*C.* 2*qrs.* 14℔.? *Answ.* 23*l.* 10*s.*

21. At 16*l.* 16*s.* ℘ hogshead, what come 27*hhds.* 9 *gal.* to? *Answ.* 456*l.*

22. If 6*l.* 4*s.* 11¼*d.* be paid for the carriage of 17*C.* 3*qrs.* 11℔. what was paid for the carriage of 1℔.?
Answ. ¾*d.*

23. If 2*C.* 3*qrs.* 21℔. of sugar cost 6*l.* 1*s.* 8*d.* what cost 35¼*C.*? *Answ.* 73*l.*

24. Bought a butt of wine for 62*l.* 8*s.* at 5*s.* 4*d.* ℘ *gal.* how many gallons did it contain? *Answ.* 234 *gal.*

Case IV.

When the product of the second and third is divided by the first; if there happen a remainder after the division is ended, and the quotient is not the least denomination of its kind; then multiply the remainder by that number, which one of the same denomination with the quotient contains of the next lesser, and divide this product again by the first number; and proceed in the same manner till the least denomination be found, or till nothing remain.

Let it be required, If 1 ton of cheese cost 16*l.* 16*s.* how much can I buy for 350*l.* 14*s.*?

```
l.   s.         Ton          l.   s.
16   16 ———————— 1 ————————— 350  14
20                            20
——                           ———
336                      336)7014(20
                             672
                             ———
                             294
                              20
                             ———
                         336)5880(17
                             336
                             ————
                             2520
                             2352
                             ————
                              168
                                4
                             ————
                          336)672(2
                              672
                              ———
```

Examples.

25. What quantity of wine can I buy for 456*l.* at 16*l.* 16*s.* ℘ hogshead? *Answ.* 27 *hhds.* 9 *gal.*

26. A goldsmith sold a tankard for 10*l.* 12*s.* at 5*s.* 4*d.* ℘ *oz.* I demand the weight thereof?
Answ 39 *oz.* 15 *dwts.*

27. Bought a hogshead of tobacco for 17*l.* 9*s.* which weighed 5*C.* 2*qrs.* 16℔. I demand the price of 4 hogsheads containing 23*C.* 1*qr.* 3℔.? *Answ.* 71*l.* 19*s.* 7½*d.*

28. If a butt of wine, containing 234 gallons, cost 62*l.* 8*s.* what was it ℘ gallon? *Answ.* 5*s.* 4*d.*

29. If an ingot of silver weigh 36*oz.* 10*dwts.* what is it worth at 5*s.* ⅌ ounce? *Answ.* 9*l.* 2*s.* 6*d.*

30. What will the carriage of 17*C.* 3*qrs.* 11℔. come to at the rate of 7*s.* ⅌ *Cwt.*? *Answ.* 6*l.* 4*s.* 11¼*d.*

Note. 1. If the divisor have a cypher or cyphers in its lowest place or places, and if the dividend hath the same number of cyphers in the same places, cut off the cyphers from both and reject them entirely, and the significant figures of the divisor only, will remain the divisor, from denomination to denomination, as for

Example.

31. How much sugar can I buy for 23*l.* 10*s.* at 1*l.* 6*s.* 8*d.* ⅌ *C.*?

Here 320 is the divisor, and 5640 the dividend; from both which I reject the cyphers in units place, and divide the significant figures of the one by those of the other, whereby I procure the true quotient 17.

l. *s.* *d.* *C.* *l.* *s.*
1 6 8——1——23 10
20 20
—— ——
26 470
12 12
—— ——
320 32|0)564|0(17
32
——
244 *C.* *qrs.* ℔.
224 (17 2 14
——
20
4
——
32)80(2
64
——
16 *&c.*

But if the figures cut off from the dividend be not all (or any of them) cyphers, as many as are not must be restored to the remainder, and the same number of cyphers retained in the divisor; as for

Example.

32. I want to know the price of 5 chests of tea, containing 4*C.* 2*qrs.* 14℔. when 1 chest containing 3*qrs.* 6℔. cost 18*l.*

```
qrs. ℔.      l.         C. qrs. ℔.
 3   6-------18---------4   2   14
28                    444
--                     56
90                     14
--                    ---
                      518
                       18
                      ---
                     4144
                      518
                     ----
                9|0)932|4
                    103l. 54 rem.
                          20
                         ---
                   9|0)108|0
                       ---
                       12s.
                       ---
```

Here the divisor is 90; the dividend 9324 wherein the figure cut off is 4, which I restore to the remainder, which becomes (not 5 but) 54, which being multiplied by 20, the product 1080 I divide again by 90, retaining as many cyphers as the figures restored or brought down. The *Answ.* 103*l.* 12*s.*

33. Bought for 125*l.* 8*s.* 4*d.* 9 pieces of broad cloth, at 16*s.* 8*d.* ⅌ yard; how many yards were in them?
Answ. 150 *yds.* 2*qrs.*

34. If for 9*l.* I buy a hogshead of brandy, how much can I buy for 144*l.* 5*s.*? *Answ.* 16 *hhds.* 1 *gal.* 6 pints.

35. Sold 21*C.* 1*qr.* 20℔. of beef for 8*l.* which being liked, a merchant wants 650*C.* 2*qrs.* 19℔. of the same; quere the amount? *Answ* 242*l.* 18*s.* 4*d.*

36. What is the interest of 575*l.* for 1 year, at 5 ⅌ *cent.*? *Answ.* 28*l.* 15*s.*

The work of this question is only the Application of Preposition 20, and this rule or case: And in this method we calculate all premiums, allowances or rates ⅌ *cent.* or 100, *viz.* the first number being 100.

```
 l.      l.      l.
100------5------575
                  5
               ----
             l 28|75
                 20
               ----
             s. 15|00
```

I demand

37. I demand the price of 2160 skains of worsted, at 23s. ⅌ 100 skains? *Answ.* 24*l.* 16*s.* 9½, $\frac{40}{100}$*d.*

38. What is the commission on 1024*l.* at 2 ⅌ *cent?* *Answ.* 20*l.* 9*s.* 7⅕*d.*

39. What is the exchange between London and Dublin, on 585*l.* at 7 ⅌ *cent?* *Answ.* 40*l.* 19*s.*

Case V.

If the first number be greater than the product of the second multiplied by the third; then bring the second to a lower denomination.

40. If 17 ton 12*C.* of iron cost 165*l.* what is that for 2*C?* *Answ.* 18*s* 9*d.*

```
Ton C.        l.        C.
 17  19 ——————165——————2
 20           20
 ———          ———
 352         3300
                2
             ——— s.  d.
          352)6600(18  9
              352
              ————
              3080
              2816
              ————
               264
                12
              ————
              3168
              3168
              ————
```

41. If 27 hogsheads of Brandy cost 456*l.* what is that ⅌ gallon? *Answ.* 5*l.* 4¾, $\frac{603}{1701}$

42 If 16*l.* 16*s.* 6*d.* pay for 18*C.* 0*qrs.* 3℔ of cheese, how much can I buy for 1*s.*? *Answ.* 6lb.

43. If 16 gallons cost 2*l.* 8*s.* what cost 1 quart? *Answ.* 9*d.*

44. If 153*C*. 0*qrs*. 16lb. cost 536*l*, what cost 14lb?
Answ. 8*s*. 9*d*.

45. If a man's yearly income be 300*l*. what is that ꝑ day? *Answ*. 16*s*. 5¼, $\frac{15}{365}$*d*.

Case VI.

When any number of barrels, bales or other packages or pieces are given, each containing an equal quantity, let the content of one reduced to the lowest name, be multiplied by the given number of packages, or pieces.

Application and Reason.

46. Bought 4 pieces of cloth, each 12 ells, for 7*l*. 10*s*, what cost 1 ell?

```
4 pieces, each 12 ells.
                4
               ——
               48
               ——
   Ells     l.  s.    Ell.
 If 48 cost 7   10 what 1
           20
          ——— s.  d.
      48)150(3   1½
          ———
```

47. If an ounce of Silver be worth 5*s*. what is the price of 14 ingots, each ingot 7lb. 5*oz*.? *Answ*. 311*l*. 10*s*.

48. Bought 6 hogsheads of sugar, each 6*C*. 3*qrs*. at 56*s*. ꝑ *C*. what come they to? *Answ*. 113*l*. 8*s*.

49. Bought 14 bags of hops, each 4*C*. 3*qrs*. 14lb. for 237 guineas; what do they stand me in ꝑ *Cwt*.?
Answ. 3*l*. 19*s*.

50. If 10 pieces of cloth, every piece 42 yards, cost 126*l*. what will one yard cost? *Answ*. 6*s*.

Case

Case VIII.

If the given pieces, barrels, bales, &c. be of unequal contents, (as it most generally happens) put the separate contents of each orderly under each other, and add them into one sum, whereby we obtain the whole quantity.

Application.

51. Bought 3 hogsheads of Brandy, containing 61, 62, 62½ gallons at 6*s.* 8*d.* ꝑ gallon. I demand how much they amount to?

Gal.
61
62
62½
Gal. *s.* *d.*
If 1 cost 6 8 what 185½
2 12 2
2 80 371
80
2)29680
12)14840
2|0)123|6 8
Answ. £61 16 8

52. Bought 3 pipes of wine containing 120½, 124, 126¾ gallons, at 5*s.* 6*d.* ꝑ gallon: what come they to?
Answ. 102*l.* 1*s.* 10½*d.*

53. What is the price of 4 pieces of cloth, containing 23, 24, 25, and 27 yards, at 5*s.* 5*d.* ꝑ yard?
Answ. 26*l.* 16*s.* 3*d.*

54. Bought 4 parcels of Butter, the first weight 10*C.* 3*qrs.* 27lb. the second 13*C.* 0*qrs.* 1lb. the third 23*C.* 2*qrs.* 0lb. the fourth 19*C.* 3*qrs.* 14lb. at 16*s.* 8*d.* the *Cwt.* what do they amount to? *Answ.* 56*l.* 2*s.* 11*d.*

55. Sold 4 hogsheads of tobacco, No. 1 weight 5*C.* 2*qrs.* No. 2, 5*C.* 1*qr* 14lb. No. 3, 5*C.* 0*qr.* 7lb. No. 4. 5*C.* 1*qr.* 21lb. at 10½*d.* ⅌ lb. I demand the amount?
Answ. 104*l.* 14*s.* 9*d.*

56. Bought 4 bags of wool containing; No. 1 4*C.* 3*qrs.* 15lb. No. 2. 5*C.* 2*qrs.* 12lb. No. 3. 7*C.* 2*qrs.* 5lb. No. 4. 6*C.* 1*qr.* 10lb. at 10*s.* 8*d.* ⅌ stone. How many guineas will pay for them? *Answ.* 80 guineas.

When the foregoing cases are well understood, the learner will be able to solve (it is hoped) the most common question in this Rule; I will now proceed to shew how to shorten the work in some particular cases.

CONTRACTIONS *in the* RULE *of* THREE.

Case I.

When the first and third numbers are such as fall under the cases of Multiplication and Division of Divers Denominations, and the second is of divers denominations; then if we multiply and divide as is there taught, it will shorten the work by saving the trouble of Reduction in form. As for

Example.

57. If 14 ton of tallow cost 338*l.* 6*s.* 8*d.* what cost 17 ton?

```
Ton        l.    s.   d.       Ton
14 ——————  338   6    8 ——————  17
                      4 }
          ——————————————}
          1353   6    8 { 16 }
                      4 }    {  ⅌ Mult. of divers deno-
          ——————————————          minations.
          5413   6    8      {        Case 3.
           338   6    8   1  }
          ——————————
⅌ Div. 14 { 2)5751  13  4 — 17
Case 2    {  ——————————————
          {  7)2875  16  8
             ——————————————
   Answ.  £410  16  8
          ——————————————
```

58. If the freight of a ship amount to 124*l.* 17*s.* 6*d.* what must I receive for my $\frac{5}{32}$ parts? *Answ.* 19*l.* 10*s* 2¾*d.*

59. If a Captain's pay for 3 weeks be 6*l.* 17*s.* 6*d.* what is his yearly pay? *Answ.* 119*l.* 3*s.* 4*d.*

60. at 18*s.* 8*d.* ꝑ *Cwt.* what cost a parcel containing 34℔? *Answ.* 5*s.* 8*d.*

61. If a hogshead of Brandy cost 16*l.* 16*s.* what must I give for five gallons? *Answ.* 1*l.* 6*s.* 8*d.*

Case II.

When the first term is an aliquot part of the second or third; divide by the first that of which it is an aliquot part, and multiply the other by the quotient.

If 27℔ of chocolate cost 6*l.* 15*s.* what is that ꝑ *Cwt.*?

```
 ℔.        l.  s.       ℔.
 27 ——————— 6  15 ——————— 112
           20               5
          ————            ————
       27)135(5        2|0)56|0
          135             ————
          ————        Answ. 28l.
```

62. What is the price of 12 ton 4C. of Iron, when 5C. cost 3*l.* 10*s.*? *Answ.* 170*l.* 16*s.*

63. At 36*l.* 10*s.* ꝑ annum, what is that ꝑ week? *Answ.* 14*s.* $0\frac{1}{2}$, $\frac{4}{5}\frac{4}{2}$*d.*

64. If 12 tun of brandy cost 1296*l.* what cost 19 tun? *Answ.* 2052*l.*

65. If 9 gallons of rum cost 2*l.* 4*s.* 10*d.* what is that ꝑ hogshead? *Answ.* 15*l.* 13*s.* 10*d.*

66. If 14℔ of sugar cost 13*s.* at what rate is that ꝑ *Cwt.*? *Anw.* 5*l.* 4*s.*

67. If 25℔ cost 2*l.* 6*s.* 8*d.* what is that for 100℔? *Answ.* 9*l.* 6*s.* 8*d.*

68. If 13*l.* gain 1*l.* 15*s.* what will 104*l.* gain at that rate? *Answ.* 14*l.*

69. If 2 ounces of silver be worth 11*s.* 6*d.* what is 1lb. worth? *Answ.* 3*l.* 9*s.*

70. At 9*s.* 9*d.* ⅌ stone, what cost 1 *Cwt.* of wool? *Answ.* 3*l.* 8*s.* 3*d.*

Case III.

On the contrary, when the second or third is an even part of the first, divide the first by that which is a part of it, and by the quotient divide the other.

71. If 3*l.* 10*s.* buy 5 *Cwt.* what will 64*l.* 8*s.* buy? *Answ.* 92 *Cwt.*

72. If 1296*l.* pay for 12 tun of brandy, how much then can I get for 2052*l.* *Answ.* 19 tun.

73. If the price of a hogshead of rum be 15*l.* 13*s.* 10*d.* what must I give for a cask containing 9 gallons? *Answ.* 2*l.* 4*s.* 10*d.*

74. At 5*l.* 4*s.* ⅌ *Cwt.* what is that per stone? *Answ.* 13*s.*

75. Bought 4 bags of wool, weighing in all 28*C.* 3*qrs.* 14lb. I want to know the trett of said wool, being 8lb for every 3 *Cwt?* *Answ.* 2*qrs.* 21lb.

76. If 3 hogsheads of brandy cost 68*l.* 17*s.* what will 7 gallons be worth? *Answ.* 2*l.* 11*s.*

Case IV.

When the first and second, or the first and third have a common measure; (*i. e.* when some number will divide both) divide them by their greatest common measure, and work with the quotients instead of the given numbers.

Application

Application.

77. If 63 gallons of wine cost 16*l*. 16*s*. what is the price of 84 gallons?

Gal.	*l.*	*s.*	*Gal.*
21)63	16	16	21)84
—		4	—
3			4
	3)67	4	
Answ.	£22	8	

78. How much beef can I buy for 100*l*. If 802C. 1*qr*. 17lb. cost 650*l* ? *Answ.* 123C. 1*qr*. 22lb

79. If 1*l*. English money be worth 1*l*. 13*s*. 4*d*. Flemish, how many pounds Flemish are in 279*l*, 8*s*. English?
Answ. 465*l*. 13*s*. 4*d*.

80. I a chest containing 48 pounds of tea, cost 24*l* 10*s*. 6*d*. what is it ⅌ *Cwt.* ? 58*l*. 3*s*. 9*d*.

81. Sold 3 hogsheads of port wine containing 240 gallons for 58*l*. and am to receive ready money for two of them, containing 126 Gallons! what must I receive?
Answ. 30*l*. 9*s*.

QUESTIONS *for* EXERCISE.

82. What must I pay for 1635*l*. deducting 1 ⅌ cent. for prompt payment? *Answ.* 1618l. 13*s*.

83. Bought 6 pipes each containing 121 gallons of wine, at 4*s*. 9*d*. ⅌ gallon, by auction; and for prompt payment am allowed 1*s*. in the pound; what must I pay for said wine, and what am I abated ⅌ *cent.*?
Answ. I must pay 163*l*. 16*s*. 1*d*. and am abated 5 ⅌ *cent.*

84. If I have owing to me 1000*l*. and compound with my debtor at 12*s*. 6*d*. ⅌ pound; how much must I receive?
Answ. 625*l*.

85. A sets

85. A sets out from a certain place and goes 12 miles a day, 5 days after B sets out from the same place, the same way, and goes 16 miles a day; in how many days will he overtake A? *Answ.* 15 days.

86. If I buy tallow at 35*l.* ⅌ ton, how must I sell a ton to gain by 10 ton as much as 1 ton cost? *Answ.* 38*l.* 10*s.*

87. A goldsmith bought of a merchant a wedge of gold, which weighed 14℔ 3*oz.* 8*dwt.* for 514*l.* 4*s.* what did he pay ⅌ ounce? *Answ.* 3*l.*

88. A grocer bought 3C. 1*qr.* 14℔ of cloves at 2*s.* 4*d.* ⅌ pound, and sold them for 52*l.* 14*s.* whether did he gain or lose by the bargain? *Answ.* he gained 8*l.* 12*s.*

89. A draper bought of a merchant 8 packs of cloth, each pack had 4 parcels in it, and each parcel contained 10 pieces, each piece 26 yards; he gave after the rate of 4*l.* 16*s.* for 6 yards; what came the 8 packs to, and what is it worth ⅌ yard? *Answ.* 6656*l.* at 16*s.* ⅌ yard.

90. How many dozen of stockings at 11 groats a pair can I buy for 190*l.* 9*s.* 8*d.* *Answ.* 86 dozen 7 pair.

91. If I buy 100 yards of Ribbon at 2 yards for a shilling, and 100 yards *ditto*, at 3 yards for a shilling, and sell them a gain for 2 shillings the 5 yards, whether do I gain or lose, and how much? *Answ.* I lose 3*s.* 4*d.*

92. Bought 45 barrels of beef at 21 shillings ⅌ barrel, among which are 16 barrels, which, being damaged, I take them upon being allowed 4 instead of 3; I demand what I must pay for them? *Answ* 43*l.* 1*s.*

93. A merchant bought 5 tun of wine for 285*l.* by the misfortune of a pipe staving he lost 120 gallons, but is willing to sell it so as to sustain no loss; I desire to know how he must sell it ⅌ gallon? *Answ.* 5 shillings.

94. A gentleman who hath an estate of 265*l.* 19*s.* 2*d.*

yearly rent, would regulate his expence in such a manner as to lay up 60 guineas a year; I desire to know how much he must spend a day? *Answ.* 16s. 10*d.*

95. Imported from Holland 84 pieces of linen, which cost 537*l.* 12*s.* at 4*s.* ⅌ ell Flemish; how many yards were there in all; in one piece; and what cost it per yard?
Answ. 2016 yards in all; 24 yards in one piece; at 5*s.* 4*d.* ⅌ yard.

96. Two men depart from one place, the one goes north 7 miles a day, the other south 11 miles a day; how far are they distant the 12th day after their departure?
Answ. 216 miles.

97. A merchant would lay out in spices 560*l.* *viz.* cloves at 4*s.* ⅌ ℔ mace 7*s.* cinnamon 3*s.* and nutmegs at 2*s.* and he would have an equal quantity of each sort, I demand the quantity? *Answ.* 700℔ of each.

98. Shipped for Jamaica 550 pair of silk stockings, at 11*s* 6*d* ⅌ pair, and 460 yards of Stuff at 14*d.* ⅌ yard; in return for which I had 46C. 3*qrs.* of sugar at 24*s.* 6*d.* ⅌ *Cwt.* and 1570℔ of indigo at 2*s.* 4*d.* ⅌ ℔. what remains due to me of my adventure?
Answ. 102*l.* 12*s.* 11½*d.*

99. A factor bought a certain quantity of broad cloth and drugget, which together cost 81*l.* the quantity of broad cloth was 50 yards at 18*s.* ⅌ yard; and for every 5 yards of broad cloth he had 9 yards of drugget: I demand how many yards of drugget he had, and what it cost him ⅌ yard? *Answ.* 90 yards, at 8s. ⅌ yard.

100. A merchant shipped for Spain 400 cloths, which produced neat 12*l.* *sterl.* ⅌ cloth, and is willing to have return, one half in wine at 30*l.* ⅌ tun, and the other half in rice at 28*s.* ⅌ *Cwt.* I demand how much of each sort must be returned for the cloths? *Answ.* 80 tun of wine, and 1714C. 1*qr.* 4℔ of rice.

101. A debtor who owes several persons 1400*l.* 5*s.* 10*d.* compounds and pays them as far as his effects will go, which amount to no more than 931*l.* 8*s.* 7¾*d.* how much do the creditors receive ⅌ pound? *Answ.* 12*s.* 6*d.*

102. If 30 pence and 40 groats buy 50 pints of wine, What is the cost of 60 quarts in current sterling coin?
Answ. 1*l.* 18*s.*

103. Whereas a moydore and a crown, just 15 yards did buy;
How many ells of that same cloth for 50*l.* had I?
Answ. $346\frac{64}{216}$.

104. If from a rule of 3 foot long the shadow 5 is made,
What is the steeple's height in yards that's 90 feet in shade? *Answ.* 18 yards.

105. If 2℔ of pepper cost 25*d.* what will 60℔ of cloves come to if 3℔ of cloves be worth 16℔. of pepper?
Answ. 16*l.* 13*s.* 4*d.*

106. How many dozen of gloves at 8*d.* the pair, will pay for 36 dozen and 8 pair of stockings at 3*s.* 6*d.* per pair? *Answ.* $192\frac{1}{2}$ dozen.

OF SOUND.

Sound not interrupted, is by experiments found uniformly to move about 1150 feet in one second of time.

107. How long after firing a cannon off Carrickfergus may the report be heard at Belfast, taking the distance at 9 miles? *Answ.* 52 seconds, $35\frac{12}{23}$ thirds.

108. If I see the flash of a piece of ordnance, fired by a vessel in distress at sea, which happens we will suppose nearly at the instant of its going off, and hear the report a minute and three seconds afterwards, how far is she off, reckoning for the passage of sound as before?
Answ. 10 miles, $6\frac{1}{4}$ furlongs.

OF THE LEVERS.

There being three orders of Levers, or three varieties, wherein the weights, prop, or moving powers may be differently applied to the vectis, or inflexible bar, in order to effect mechanical operations in a convenient manner.

For the First order, see Page 100.

Of the Second and Third Order of LEVERS.

In mechanics, a Lever of the second order, is where the power acts at one end, the prop fixed directly at the other, and the weight somewhere between them.

In this order of Levers, their force is a contra proportion to their length.

In a Lever of the third order, the prop is planted at one end of the bar, the weight at the other end, and the moving force somewhere between.

109. If a lever be 100 inches long, what weight, lying 7½ inches from the end, resting on a pavement, may be moved with the force of 168lb lifting at the other end of the lever? *Answ.* 2072lb

110. A water wheel turns a crank, working 3 pump-rods, fixed just six feet from the joint or pin; by which their several levers, each 9 feet in length, are fastened, for the sake of the intended motion; at one end of the suckers of the pumps being wrought by the other, shews them to be levers of the third order: now I would know what the length of the stroke in each of the barrels will be, if the crank be made to play just nine inches round its centre? *Answ.* 27 inches.

111. With what force ought that water wheel to be driven, which, circumstanced as in the last question, raises 3 cubic feet of water at every revolution of the wheel, each experimentally weighing 62½lb. Avoirdupois, the friction of the machine rejected? *Answ.* 281⅙lb.

MOTION *of* BODIES *with their* VELOCITIES.

1. If the quantities of matter in any two or more bodies put in motion be equal, the forces wherewith they are moved will be in proportion to their velocities.

2 If the velocity of these bodies be equal, their forces will be directly as the quantities of matter contained in them.

3. If both the quantities of matter and the velocities be unequal, the forces with which the bodies are moved,

will be in proportion compounded on the quantities of matter they contain, and of the velocities wherewith they move.

112. There are two bodies the one contains 25 times the matter of the other (or twenty-five times heavier) but the lesser moves with 1000 times the swiftness of the greater: in what proportion are the forces by which they are moved? *Answ.* lesser 40 to 1.

113. There are two bodies, one of which weighs 100lb. the other 60lb. but the lesser body is impelled by a force 8 times greater than the other, the proportion of the velocities wherewith these bodies move is required?

Answ. as $13\frac{1}{3}$ to 1

1. In comparing the motions of bodies, if their velocities be equal, the spaces described by them are in direct proportion of the times in which they are described.

2. If the times be equal, then the spaces described will be as their velocities.

3. If the times and velocities be unequal, the spaces will be in a proportion compounded of the times and velocities.

114. There are two bodies one of which moves forty times swifter than the other, but the swifter body has moved but one minute, whereas the other has been in motion two hours: the ratio of the spaces described by these two bodies is required?

Answ. the swifter to the slower as 1 to 3

115. Suppose one body to move thirty times swifter than another; as also the swifter to move 12 minutes, the other only 1, what difference will there be between the spaces by them described, supposing the last has moved 60 inches? *Answ.* 1795 feet.

116. There are two bodies, one whereof has described 50 miles, the other only 5, but the first hath moved with 5 times the velocity of the second: what is the ratio then of the times they have been describing those spaces?

Answ. the first body hath been in motion double the time of the latter.

QUESTIONS

QUESTIONS.

Q. What is the Rule of Three?

A. That which teaches from three numbers given to find a fourth proportional.

Q. How are the given numbers to be managed?

A. The first and third must be reduced to the same name. *viz.* the lowest mentioned in either, and the second likewise to its lowest name.

Q. How must the numbers be stated?

A. So that the first and third may be of one name or kind.

Q. How is the operation performed?

A. Multiply the second and third together, and divide their product by the first, the quotient is the fourth number sought, in the same name with the second.

CHAP. VIII.

THE RULE OF THREE INVERSE.

HITHERTO the questions proposed were such that the first number of the stating was always to the third as the second was to the fourth required, which is called direct proportion, or the *Rule of Three Direct*: and this is the most useful and general property of proportionals, as will appear by the sequel.

But (the question being stated according to the Rule before laid down) the nature and conditions of several questions are such, that, as the first is to the third, so reciprocally must the fourth be to the second: that is, the greater the third is in proportion to the first, the less must the fourth be in respect of the second: or the less the third is in proportion to the first, the greater the fourth must be in proportion to the second. This is called reciprocal or inverted, or indirect proportion, or the *Rule of Three Inverse.*

The principal difficulty that will embarrass the learner, will be, to distinguish when the proportion is direct, and when indirect. This is done from an attentive consideration of the sense and tenor of the question proposed, for if thereby it appears that when the third term of the stating is less than the first, the answer must be less than the second, or when the third is greater than the first, the answer must be greater than the second; then the proportion is direct.

But if the third is less than the first, and yet the sense of the question requires the fourth to be greater than the second, or if the third being greater than the first, the answer must be less than the second, the proportion is indirect.

Application.

First, this question being proposed. If 12 men make 4 perch of ditching in one day, how many perch will 24 men make in the same time? The stating will stand thus:

Men	Perch	Men
12———	4———	24

where it is very manifest that 24 men will do more in the same time, than 12 at the same proportion of working, *viz.* in proportion as 24 to 12 *i. e.* twice as much, *viz.* 8 perches: so that 24 men being more than 12, and requiring the answer more than 4 perches (the second number) this question is direct.

But if the question proposed were this, *viz.* if 12 men make 16 perch of ditching in 4 days, in what time will 24 men perform the same at the same rate of working?

Here 16 is a superfluous term, having no corresponding term, which being rejected, state the other terms of the question.

Men	Days	Men
12———	4———	24

In which stating it is very evident that 24 men will perform 16 perch in less time than 12 men, and that therefore the fourth required must be less than the second, in the same proportion as the third is greater than the first, therefore the proportion is inverse or indirect.

Rule for the Operation.

The question being stated as already directed in the Rule of Three Direct, multiply the first and second numbers together, and divide the product by the third, the quotient is the answer required, in the same name with the second.

Men Days Men
12——4——24
12
—
24)48(2 days *Answ.*
48
—

QUESTIONS.

Q. What is the *Rule of Three Inverse?*

A. When three numbers are given to find a fourth, which shall have such proportion to the second as the first to the third.

Q. How is a question distinguished whether it belongs to the Rule of Three Inverse or Direct?

A. If more do more or less do less respect,
It is a question in the Rule Direct;
But less requiring more, and greater less,
A question of the Inverse Rule express.

Examples.

Q. 1. There was a certain building raised in 8 months by 120 workmen; but the same being demolished, it is required to be rebuilt in 2 months: how many men must be employed about it? *Answ.* 480 men.

2. If 28*s.* will pay for the carriage of an hundred weight 150 miles; how far may 6*Cwt*, be carried for the same money? *Answ.* 25 miles.

3. If for 5*l* 5*s.* I have 14*Cwt.* carried 136 miles; how many miles may I have 24 *Cwt.* carried for the same money? *Answ.* $79\frac{1}{3}$ miles.

4. If a footman perform a journey in 3 days, when the days are 16 hours long, how many days will he require of 12 hours long to go the same journey in?
Answ. 4 days.

5. How many yards of plush are sufficient to make a cloak of equal magnitude with one which had in it 4 yards of 7 quarters wide, when the plush is but 3 quarters wide?
Answ. $9\frac{1}{3}$ yards of plush.

6. How many yards of canvas that is ell wide, will be sufficient to line 20 yards of sey, that is 3 quarters wide?
Answ. 12 yards.

7. If a man performs a journey in 6 days, when the day is 8 hours long; in what time will he do it, when the day is 12 hours long. *Answ.* 4 days.

8. If I lend my friend 100*l.* for 6 months, (allowing the month to be 30 days) how long ought he to lend me 1000*l.* to requite my kindness? *Answ.* 18 days.

9. If 6 mowers can mow a field in 12 days, in what time will 24 mowers do it? *Answ.* 3 days.

10. Suppose 800 Soldiers were placed in a garrison, and their provisions computed sufficient for 2 months; how many soldiers must depart that the provisions may serve them 5 months? *Answ.* 480 men.

11. Admit that I lent to a friend on his occasion 100*l.* for 6 months, and he promised me the like kindness when I desired it; but when I came to request it, he could lend me only 75*l.* The question is, how long I may keep his money to recompense my courtesy to him?
Answ. 3 months.

A LEVER OF THE FIRST ORDER.

A Lever of the first order hath the power at one of its ends, the weight to be raised is put at the other, and the fulcrum or prop somewhere between them.

In this order, the power applied at one end will be reciprocally proportional to the distances of those ends from the fulcrum or point supported: or in the steel yards as the distance of the weight from the point of suspension.

Examples.

12. What weight will a man be able to raise, who presses with the force of an hundred weight and an half on

the end of an equipoised handspike 100 inches long, which is to meet with a convenient prop exactly 7⅓ inches above the other end of the machine? *Answ.* 18¼ *Cwt.*

13. What weight hung at 70 inches distance from the fulcrum of a steel yard, will equipoise a hogshead of tobacco, weighing 9½ *Cwt* freely suspended at two inches distance on the contrary side?

Answ. 30℔ 6*oz.* 6½*drs.* nearly.

MOTIONS *of* BODIES *with their* VELOCITIES.

In comparing the motion of bodies, the ratio or proportion between their velocities will be compounded of the direct ratio of the forces wherewith they are moved, and the reciprocal of the quantities of matter they contain.

Examples.

14. The battering ram of Vespasian weighed; suppose 100,000℔ and was moved, let us admit, with such a velocity, by strength of hands, as to pass through 20 feet in one second of time, and this was found sufficient to demolish the walls of Jerusalem: with what force must a bullet that weighs about 30℔. be moved in order to do the same execution? *Answ.* 66,666 feet, 8 inches ⅌ second.

15. A body weighing 200lb. is impelled by such force as to send it 100 feet in a second; with what velocity would a body of 8lb move, if it were impelled by the same force? *Answ.* 2500 feet ⅌ second.

CHAP. IX.

THE DOUBE RULE OF THREE.

THIS probably is called the Double Rule of Three, because questions therein may be solved by two statings of the Single Rule of Three. It is likewise by some called the Rule of Five, because generally, five numbers are given to find a sixth, of which five given numbers, 3 are conjoined in form of supposition; and upon that supposition a question is raised of the other two, which with the number sought, are respectively like the former three.

I. To solve questions in this rule by two statings of the Single Rule of Three, this is the Rule:

1. Let either of the two numbers of which the question is raised, be put in the third place, and the correspondent number of the same name or kind in the first, the second will be that which hath no correspondent number given.

2. Three of the first given numbers being thus stated, find a fourth proportional.

3. Put this fourth number resulting from the work of the first stating, for the second number of a second stating, the remaining number of which the question is raised the third, and its corresponding number of the same name the first, and the fourth number resulting will be the answer.

Application.

Let this question be proposed, *viz.*

If the carriage of 25 stone weight for 16 miles cost 15*l.* 10*s.* what will 40 stone cost for 9 miles?

Here of the five given numbers 25 stone, 16 miles, and 15*l.* 10*s.* are conjoined in form of a supposition, and thereupon a question is raised concerning 40 stone for nine miles: wherefore let either of the two numbers, 40 stone or 9 miles be put for the third number of the first stating and its corresponding term, 25 stone or 16 miles.

Sto. *l.* *s.* *Sto.* *l.* *s.*
1...25—15 10—40 *Answ.* 24 16

Miles *l.* *s.* Miles *l.* *d.*
2...16—24 16—9 *Answ.* 13 19

Or thus:

Miles *l.* *s.* Miles *l.* *s.* *d.*
1...16—15 10—9 *Answ.* 8 14 4½

Sto. *l.* *s.* *d.* *Sto.* *l.* *s.*
2...25—8 14 10½—40 *Answ:* 13 9

Such questions as (being stated) are found to have both the statings in direct proportion, may be solved more readily by one compound stating and operation, thus: Place the two terms of which the question is raised under one another in the third place; their corresponding terms under each other in the first; and the remaining term in the middle:

dle; then multiply both these first terms and third terms into each other, and so the double stating is reduced to a simple stating of the Rule of Three Direct, *viz.* the product of the two first terms is the first of a simple stating; the second term is the second; and the product of the third term is the third number to find a fourth proportional.

So the first example will stand thus:

l. *s.*
Sto. 25———15 19———40 *Sto.*
Mi. 16 20 9 *Mi.*

150 310 360
25 360

400 186
93

4|00)1116|00 2|0)27|9
279 *Answ.* 13*l.* 19*s.*

Contraction.

Questions in this Rule may be contracted as in the Rule of Three, particularly if any of the first and last terms have a common measure we may divide them, and use the quotients instead of the numbers themselves, by which means the last question will stand thus, *viz.*

5 $\cancel{25}$———15 10———$\cancel{40}$ $\cancel{8}$
2 $\cancel{16}$ 9 9

10 1|0)13|9 10

Answ. £. 13 19

Questions belonging to this Rule are such as follow:

1. If 4 students spend 19*l.* in 3 months; how much will serve 8 students 9 months? *Answ.* 114*l.*

2. If the carriage of 8 *Cwt.* 128 miles cost 48 shillings, for how much may I have 4 *Cwt.* carried 32 miles after the same rate? *Answ.* 6*s.*

3. If 240*l.* in 16 months gain 64*l.* how much will 60*l.* gain in 6 months? *Answ.* 6*l.*

4. A merchant agrees with a carrier to carry 15 *Cwt.* of goods 40 miles for 10 crowns, each crown 65 pence; how much must one pay in proportion to have 3 *Cwt.* carried 32 miles? *Answ.* 17*s.* 4*d.*

5. If 20 *Cwt.* is to be carried 50 miles for 5*l.* how much will 40 *Cwt.* cost if it was to be carried 100 miles?
Answ. 20*l.*

6. With how many pounds sterling could I gain 5*l.* ⅌ annum, if with 450*l.* I gain in 16 months 30*l.*?
Answ. 100*l.*

7. If 8*l.* is gained in 12 months with 100*l.* with how much money can I gain 8*l.* 12*s.* in 5 months?
Answ. with 258*l.*

8. If 60*l.* in 6 months gain 6*l.* what will 240*l.* gain in 16 months? *Answ.* 64*l.*

9. If 1 pound of thread makes 3 yards of linen 5 quarters broad; how many pounds of thread would be wanted to make a piece of linen 45 yards long, and 1 yard broad?
Answ. 12 pounds.

10. If 200℔. of merchandize is carried 40 miles for 3 shillings, how many pounds may be carried 60 miles for 22*l.* 14*s.* 6*d.*? *Answ.* 20200℔.

11. If for 3 shillings 200℔. of goods are carried 40 miles; how many miles might 20200℔. be carried for 22*l.* 14*s.* 6*d.*? *Answ.* 60 miles.

12. If 200℔. are carried 40 miles for 3 shillings; how much must be paid for carrying 20200℔. 60 miles?
Answ. 22*l.* 14*s.* 6*d.*

13. If 3℔. of worsted make 10 yards of stuff of 1 *yd.* 2*qrs.* broad, how many pounds will be wanted to make a piece 100*yds.* long and 3 *qrs.* broad? *Answ.* 15℔.

14. If a footman travel 240 miles in 12 days when the day is 12 hours long; in how many days may he travel 720 miles when the day is 16 hours long?
Answ. 27 days.

15. If 12 men in 8 days gain 8*l.* 8*s.* what will 21 men gain in 15 days? *Answ.* 27*l.* 11*s.* 3*d.*

16. What is the interest of 200*l.* for three years and 9 months, at 5 ⅌ *cent.* ⅌ annum? *Answ.* 37*l.* 10*s.*

17. If 80,000 *Cwt.* of ammunition was to be removed from a place in 9 days, and that in 6 days time I find 4500*Cwt.* is carried away by 18 horses, how many horses would be wanted to carry away the remainder in 3 days?
Answ. 604 horses.

18. A and B are on opposite sides of a wood, 134 fathoms about, they begin to go round it both the same way at the same instant of time; A goes 11 fathoms in two minutes, and B 17 in 3: the question is, how many times will they surround this wood, before the nimbler overtakes the slower?

Answ. { 17 times by A.
17½—— by B.

19. A weight of 1½℔. laid on the shoulders of a man is no greater burthen to him than its absolute weight; what difference will he feel between the said *wt.* applied near his elbow, at 12 inches from the shoulder, and in the palm of his hand 28 inches therefrom; and how much more must his muscles then draw to support it at right angles; that is, have his arm extended right out?
Answ. 24℔.

BOOK

BOOK II.

CHAP. I.

OF FRACTIONS.

DEFINITIONS.

1. A FRACTION is a part or parts of Unity (representing any whole which may be divided.)

2. A Fraction is expressed by two numbers placed one above the other, with a line drawn between them, as $\frac{1}{2}$, $\frac{2}{3}$; the number above the line is called the Numerator, and the number below the line the Denominator.

$$\textit{e.g.}\quad \frac{1}{2}\quad \frac{3}{5}\quad \frac{11}{12}\quad \begin{array}{l}\text{Numerator,}\\ \text{Denominator,}\end{array}$$

which are read thus, one-half, three-fifths, eleven-twelfths.

3. The Numerator denotes how many parts of the Whole the Fraction consists of.

4. The Denominator denotes how many parts the Whole is divided into.

5. A proper Fraction is that whose Numerator is less than its Denominator, as $\frac{1}{2}$, $\frac{3}{4}$.

6. An improper Fraction is that whose Numerator is equal to, or greater than its Denominator, as $\frac{2}{1}$, $\frac{4}{3}$, $\frac{4}{4}$.

7. A compound Fraction is a Fraction of a Fraction, as $\frac{1}{2}$ of $\frac{3}{4}$, or $\frac{1}{3}$ of $\frac{1}{12}$ of $\frac{1}{20}$.

8. A whole number with a Fraction annexed, is called a mixed number, as $2\frac{2}{3}$, $1\frac{1}{2}$.

9. A Fraction is said to be in its least terms, when it is expressed by the least numbers possible.

Quest. What is a Fraction?

A. A part or parts of one whole Thing.

Q. How is a Fraction expressed?

A. By two numbers placed one above the other, with a line drawn between them.

Q. What are these numbers called?

A. That above the line is called the Numerator, and that below the Denominator.

Q. What is a proper Fraction?

A. That whose Numerator is less than the Denominator.

Q. What is an improper Fraction?

A. That whose Numerator is equal to, or greater than the Denominator.

Q. What is a compound Fraction?

A. A Fraction of a Fraction, as $\frac{1}{2}$ of $\frac{3}{4}$.

Q. What is a mixt Number?

A. A whole number with a Fraction annexed, as $2\frac{2}{3}$.

CHAP. II.

REDUCTION of FRACTIONS.

1st *Preparatory Problem.*

TO find the greatest common Measure of two given Numbers.

Rule.

The greater by the less divide,
The less by what remains beside;
The last Divisor still again,
By what remains, till Nought remain:
And what divides and leaveth Nought,
Will be the common Measure sought.

Examples.

1. What is the greatest common measure of 112 and 120? *Answ.* 8.

2. What is the greatest common measure of 26 and 62? *Answ.* 2.

3. What is the greatest common measure of 279 and 403? *Answ* 31.

2nd *Preparatory Problem.*

To find the least common multiple of any given numbers.

Their product, if each prime to th' rest,
If all that they'll divide's the least.
If all not prime to all beside,
See what will two or more divide:
Divide the two or more thereby,
The like upon the quotients try;
And if thou canst divide 'em, do;
Till nought will measure any two:
Then the last quotients multiply'd,
And all the numbers which divide
Continually; the product got,
Will be the multiple that's sought.

Examples.

What is the least multiple of 3, 5, 8 and 10?

```
5) 3, 5, 8, 10
   -----------
2) 3, 1, 8,  2
   -----------
   3, 1, 4,  1
         3
         -
        12
         2
        --
        24
         5
       ---
Answ.  120
       ---
```

I survey my given numbers and discover 5 will divide two of them; viz. 5 and 10 which I divide by 5, bringing into a line with the quotients the numbers, which 5 will not measure. Again, I view the numbers in the second line, and find 2 will measure 8 and 2, and these I divide by 2, and in the third line get 3, 1, 4, 1, all prime, I multiply the numbers in the said line together with the divisors continually into each other for the number sought, and find it 120.

4. What is the least number which 3, 4, 8 and 12 will measure? *Answ.* 24.

5. What number is the least that 7, 8, 16 and 28 will measure? *Answ.* 112.

6. What is the least number which 5, 6, 12 and 16 will measure? *Answ.* 240.

Problem I.

To reduce a fraction to its least terms.

Rule.

Find the greatest common measure of the numerator and denominator of the given fraction, and divide them thereby, by, the quotients will be the least term required, *viz.* the quotient found by dividing the numerator will be the numerator, and the quotient of the denominator the denominator of the fraction required.

Examples.

Bring the following fractions to their least terms:

7. $\frac{144}{256}$	*Answ.*	$\frac{9}{16}$	10. $\frac{48}{56}$	*Answ.*	$\frac{6}{7}$
8. $\frac{91}{117}$	—	$\frac{7}{9}$	11. $\frac{182}{196}$	—	$\frac{13}{14}$
9. $\frac{744}{775}$	—	$\frac{24}{25}$	12. $\frac{216}{240}$	—	$\frac{9}{10}$

Rule II.

Take any common measure of the numerator and denominator of the given fraction, and divide them thereby, making the quotients respectively the numerator and denominator of the new fraction, which divide in like manner, and so proceed till the terms be prime to each other, and the thing proposed is manifestly effected.

Example.

Bring $\frac{56}{84}$ to its lowest term:

$$2\left\{\begin{array}{c|c|c|c} 56 & \overset{2)}{28} & \overset{7)}{14} & 2 \\ \hline 84 & 42 & 21 & 3 \end{array}\right. \quad \textit{Answ.}\ \tfrac{2}{3}.$$

Note. If the numerator and denominator be both even, 2 is a common measure to them. If one be odd (if they have any) their common measure is some odd number, as 3, 7, 11, &c. If both have for their lowest figure 5, or one 5, and the other a cypher, 5 is a common measure; and if both have cyphers in units place, &c. cut off the cyphers, 10, 100, &c. being a common measure.

Bring the following fractions to their lowest terms, *viz.*

13. $\frac{24}{28}$	*Answ.*	$\frac{6}{7}$	17. $\frac{120}{160}$	*Answ*	$\frac{3}{4}$
14. $\frac{21}{36}$	— —	$\frac{7}{12}$	18. $\frac{4800}{7650}$	—	$\frac{32}{51}$
15. $\frac{21}{35}$	—	$\frac{3}{5}$	19. $\frac{150}{300}$	—	$\frac{1}{2}$
16. $\frac{28}{56}$	—	$\frac{1}{2}$	20. $\frac{350}{400}$	—	$\frac{7}{8}$

Rule II.

To change or reduce a given fraction to another which shall be equal thereto, and have a given denominator.

Rule.

Multiply the given denominator by the numerator of the given fraction, and divide the product by the denominator thereof, and the quotient will be the numerator of the fraction sought.

Otherwise thus:

Divide the given denominator by the denominator of the given fraction, and multiply the quotient by the numerator.

Examples.

21. Reduce $\frac{2}{3}$ to a fraction whose denominator shall be 15? *Answ* $\frac{10}{15}$.

22. Bring $\frac{4}{5}$ to a fraction whose denominator shall be 15? *Answ.* $\frac{12}{15}$.

23. Reduce both $\frac{7}{8}$ and $\frac{13}{28}$ to fractions whose denominators shall be 112? *Answ.* $\frac{98}{112}$ and $\frac{52}{112}$.

24. Reduce $\frac{3}{4}$ to a fraction whose denominator shall be 100? *Answ.* $\frac{75}{100}$.

25. Reduce $\frac{5}{8}$ to a fraction whose denominator shall be 1000? *Answ.* $\frac{625}{1000}$.

Problem III.

To reduce any given fractions to others, which shall have one common denominator.

Rule.

1. Find the least number which all the denominators of the given fractions will measure, for a common denominator.

2 Reduce each fraction to another whose denominator shall be the said common denominator.

Examples

Examples.

Reduce the following fractions to others equal thereto, which shall have a common denominator.

26.	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{6}$	*Answ.*	$\frac{6}{12}$	$\frac{8}{12}$	$\frac{9}{12}$	$\frac{10}{12}$
27.	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{3}{5}$	———	$\frac{60}{120}$	$\frac{40}{120}$	$\frac{30}{120}$	$\frac{24}{120}$
28.	$\frac{3}{8}$	$\frac{4}{9}$	$\frac{6}{13}$		———		$\frac{351}{936}$	$\frac{416}{936}$	$\frac{432}{936}$
29.	$\frac{3}{4}$	$\frac{11}{12}$			———			$\frac{9}{12}$	$\frac{11}{12}$
30.	$\frac{2}{5}$	$\frac{4}{6}$	$\frac{5}{9}$	$\frac{7}{10}$	———	$\frac{36}{90}$	$\frac{60}{90}$	$\frac{50}{90}$	$\frac{63}{90}$
31.	$\frac{7}{8}$	$\frac{9}{14}$			———			$\frac{49}{56}$	$\frac{36}{56}$
32.	$\frac{5}{7}$	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{27}{28}$	———	$\frac{80}{112}$	$\frac{42}{112}$	$\frac{91}{112}$	$\frac{108}{112}$
33.	$\frac{11}{12}$	$\frac{9}{16}$	$\frac{18}{20}$	$\frac{13}{24}$	———	$\frac{220}{240}$	$\frac{135}{240}$	$\frac{216}{240}$	$\frac{130}{240}$

Problem IV.

To reduce an improper fraction to a whole or mixed number.

Rule.

Divide the numerator by the denominator, and the quotient is the whole or mixed number required.

Examples.

34. Bring $\frac{95}{19}$ to a whole number? *Anw.* 5.

35.	$\frac{45}{13}$	*Answ.*	$3\frac{6}{13}$	39.	$\frac{87}{8}$	*Anw.*	$10\frac{7}{8}$
[illegible]	$\frac{25}{7}$	———	$3\frac{4}{7}$	40.	$\frac{96}{16}$	———	6
[illegible]	$\frac{44}{18}$	———	8	41.	$\frac{600}{19}$	———	$31\frac{11}{19}$
[illegible]	$\frac{736}{31}$	———	$55\frac{21}{31}$	42.	$\frac{573}{16}$	———	$35\frac{14}{16}$

Problem V.

To reduce a whole or mixed number to an improper fraction.

1. *A Whole Number.*

1. Subscribe 1 under the given whole number, for a denominator to express it fraction-wise, 8, $\frac{8}{1}$; 17, $\frac{17}{1}$.

2. But if it be required to reduce a whole number to a fraction, whose denominator is given: multiply the said whole number by the given denominator, for a numerator to the said denominator.

As suppose 7 given to be reduced to a fraction whose denominator shall be 4; now 7 multiplied by 4 makes 28 for the numerator of $\frac{28}{4}$, the fraction required.

Examples.

43. Bring 5 to a fraction whose denominator is 19? *Answ.* $\frac{95}{19}$.

44. Reduce 8 to a fraction whose denominator is 18? *Answ.* $\frac{144}{18}$.

45. Reduce 6 to a fraction whose denominator is 16? *Answ.* $\frac{96}{16}$.

II. *A mixt Number.*

Multiply the whole number by the denominator of the annexed fraction, and add the numerator to the product for the numerator of the improper fraction required, and the denominator of the fractional part is the denominator.

Application.

Let $3\frac{2}{4}$ be given to be reduced to an improper fraction, I multiply 3 by 4 and the product is 12, to which adding 2 the numerator of the fractional part, the sum 14 is the numerator of the fraction sought, and 4 the denominator; so then $\frac{14}{4}$ is the improper fraction equal to $3\frac{2}{4}$ *Q. E. I.*

$3\frac{1}{2}$	$3\frac{2}{4}$
2	4
—	—
Answ. $\frac{7}{2}$	*Answ.* $\frac{14}{4}$
—	—

Examples.

Reduce the following mixt numbers to improper fractions.

46.	$3\frac{6}{13}$	*Answ.*	$\frac{45}{13}$
47.	$3\frac{4}{7}$	——	$\frac{25}{7}$
48.	$55\frac{21}{31}$	——	$\frac{1726}{31}$
49.	$10\frac{7}{8}$	——	$\frac{87}{8}$
50.	$31\frac{1}{19}$	——	$\frac{590}{19}$
51.	$35\frac{13}{16}$	——	$\frac{573}{16}$

Of Compound Fractions.

Problem VI.

To reduce a compound fraction to a simple fraction.

Rule.

Multiply the numerators together for a numerator, and the denominators for a denominator.

52. $\frac{1}{2}$ of $\frac{1}{12}$ of $\frac{1}{20}$ *Answ.* $\frac{1}{480}$
53. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ —— $\frac{24}{60}$ or $\frac{2}{5}$
54. $\frac{3}{4}$ of $\frac{16}{20}$ —— $\frac{48}{80}$
55. $\frac{1}{2}$ of $\frac{1}{4}$ —— $\frac{1}{8}$

Note. If any denominator of 1 member of a compound fraction be equal to the numerator of another member thereof, these equal numerators and denominators may be erased, and the other members continually multiplied, (as ꝑ Rule) will produce the fraction required in lower terms. For instance, if it be required to reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ to a simple fraction, erasing the equal numerators, they will stand thus: $\frac{2}{\cancel{3}}$ of $\frac{\cancel{3}}{\cancel{4}}$ of $\frac{\cancel{4}}{5}$ then 2 being the only numerator not erased, and 5 the only denominator, the fraction $\frac{2}{5}$ is that required, and equal to the fraction which would be produced by the continual multiplication of the numerators and denominators: for it is manifest the numerator 2 of the fraction $\frac{2}{5}$ is to be multiplied by 3 and 4, and the denominator 5 by the same, in which case the fraction produced will be equal to the fraction multiplied.

Problem VII.

To find what fraction of a higher or greater denomination, a lesser or divers lesser denominations are.

Rule.

Reduce the given denominators to the lowest mentioned for a numerator, and unity, of which they are to be the fraction to the same, for a denominator, and bring the fraction so found to its least terms,

Application.

Suppose 17*s.* 6*d.* be given to find what part of a pound it is, I bring 17*s.* 6*d.* into pence, the least denomination mentioned, and find it 210 pence for a numerator, and then 1*l.* brought to the same name makes 240*d.* for a denominator, and $\frac{210}{240}$ reduced to its least terms is $\frac{7}{8}$ of a pound equal to 17*s.* 6*d.*

	s.	*d.*	*l.*
	17	6	1
	12		20
	21|0(7		20
3)			12
	24|0(8		240
	Answ, $\frac{7}{8}$		

Examples.

		Answ.
56.	What part of a pound is 13*s.* 4*d.* ?	$\frac{4}{5}$
57.	What part of a shilling is 3*d.* ?	$\frac{1}{4}$
58.	What part of a yard is 3*qrs.* 3*na.* ?	$\frac{15}{16}$
59.	What part of a *Cwt.* is 3*qrs.* 14℔ ?	$\frac{7}{8}$
60.	What part of a pound is 7*oz.* 10*drs.* ?	$\frac{61}{128}$
61.	What part of a *l. sterl.* is 17*l.* 9½*d.* ?	$\frac{427}{480}$
62.	What part of a hogshead is 9 gallons ?	$\frac{1}{7}$
63.	What part of a day is 4*ho.* 20*mi.* ?	$\frac{13}{72}$
64.	What part of a ℔. Troy is 10*oz.* 10*dwt.* 10*gr.* ?	$\frac{505}{576}$

Problem VIII.

To find the value of a fraction, *viz.* Having given a fraction of a greater denomination, to find how many of the lesser are equal thereto.

Rule.

Multiply the numerator of the given fraction by that number which one of the greater containeth of the lesser, and divide the product by the denominator, and if any thing remain, multiply the remainder by the number which one of the denomination last found the contains of the next lesser, and so proceed till nothing remains, or the lowest be come to, and the several quotients express the denominations equivalent to the fraction given.

Application.

$$\begin{array}{rl} & 7 \\ & 20 \\ \hline 8) & 140 \\ \hline & 17 \quad 4 \text{ remains.} \\ & 12 \\ \hline 8) & 48 \\ \hline & 6 \end{array}$$

Answ. 17*s.* 6*d.*

Let $\frac{7}{8}$ of a pound be given to find its value; I multiply 7 by 20 the number of shillings in 1*l.* and the product 140 I divide by the denominator 8, and get the quotient 17, the number of twentieth parts or shillings; and 4 remaining, I multiply by 12, and again divide the product by 8, find the quotient 6 for pence, and nothing remaining, I bring the quotients together, and thereby find 17*s.* 6*d.* the lesser denominations, equal to $\frac{7}{8}$*l.*

Examples.

65. Find the value of $\frac{2}{3}$*l.*?	*Answ.* 13*s.* 4*d.*
66. How much is $\frac{1}{4}$ of a shilling?	—— 3*d.*
67. What is the $\frac{15}{16}$ of a yard?	—— 3*qrs.* 3*na.*
68. How much is $\frac{7}{8}$ of a Cwt.?	—— 3*qrs.* 14℔.
69. How much is $\frac{61}{128}$ of a ℔. Avoirdupois?	—— 7*oz.* 10*dr.*
70. How much is $\frac{427}{480}$ of a *l.* sterling?	—— 17*s.* 9$\frac{1}{2}$*d.*
71. How much is $\frac{1}{7}$ of a Hhd. of wine?	—— 9 gallons.
72. How much is $\frac{505}{576}$ of a ℔. Troy?	*Answ.* 10*oz.* 10*dwt.* 10*grs.*

Problem IX.

To reduce a fraction of a lower denomination to a fraction of a higher.

Rule.

Make it a compound fraction and reduce it to a simple one.

Application.

Application.

Let it be required to bring $\frac{3}{4}$ of a shilling to the fraction of a pound. From what hath been remarked it will be very easy to conceive that $\frac{3}{4}$ of a shilling is $\frac{3}{4}$ of $\frac{1}{20}$ of a *l.* which being reduced to a simple fraction becomes $\frac{3}{80}$ of a *l.* equal $\frac{3}{4}$*s.*

Examples.

73. What part of a *l.* is $\frac{1}{4}$*d.*? *Answ.* $\frac{1}{960}$.

74. What part of a ℔. Avoirdupois is $\frac{3}{4}$ of an ounce? *Answ.* $\frac{3}{64}$.

75. What part of a Cwt. is $\frac{7}{8}$ of 1℔.? —— $\frac{7}{896}$

76. What part of a yard is $\frac{2}{3}$ of a nail? —— $\frac{1}{24}$.

Problem X.

To reduce a fraction of a higher denomination to the fraction of a lower.

Rule.

Multiply the numerator of the given fraction, by that number which 1 of the higher contains of the lower, for a new numerator to the denominator of the given fraction.

Application.

Let it be required to bring $\frac{7}{8}$ of a *l.* to the fraction of a shilling; multiply 7 by 20, I find the product 140, *viz.* $\frac{140}{8}$ of a shilling, (in its least terms $\frac{35}{2}$*s.*) equal to $\frac{7}{8}$*l.*

77. What part of a shilling is $\frac{2}{3}$ *l.*? *Answ.* $\frac{40}{3}$.

78. What part of a penny is $\frac{1}{480}$ *l.*? —— $\frac{240}{480}$ or $\frac{1}{2}$.

Problem XI.

To find what part of a greater number any lesser number is.

Rule

Rule.

Make the lesser the numerator, and the greater the denominator of a fraction, and reduce the said fraction to its lowest terms.

So if it were required to know what part 15 is of 20, I say $\frac{15}{20}$, or its lowest terms $\frac{3}{4}$.

QUESTIONS *relating to Reduction.*

Quest. How must I reduce a fraction to its least terms?
Answ. By the measure, the greatest of the numerator,
Which likewise will measure the denominator;
Divide both the terms of the fraction; 'twill find
The terms of a fraction the least in their kind.
But rather than thus it may probably please
The least numbers to approximate by degrees:
If the numbers be even still 2 will divide,
But an odd number always in odd must be try'd;
If cyphers end,—both of like cyphers deprive;
5, or 5 and a cypher?—divide them by 5.

Q. How shall I bring a fraction to another, which shall have a given denominator?

A. Make the denominator of the given fraction the first, the numerator the second, and the given denominator the third number, of a stating in the Rule of Three, and find a fourth proportional; which will be the numerator to the given denominator.

Q. How must I reduce fractions of different denominators to others having one common denominator?

A. Find the least number which all the denominators of the given numbers will measure, for a common denominator, and reduce each fraction to another whose denominator shall be the said common denominator.

Q. How must I reduce an improper fraction to a whole or mixt number?

A. Divide the numerator by the denominator.

Q How must I make a whole number an improper fraction

A. By subscribing 1 under it for a denominator.

Q. But if the denominator be given?

A. Multiply the whole number thereby for a numerator.

Q. How must I reduce a mixt number to an improper fraction?

A. Multiply the whole number by the denominator of the annexed fraction, and add the numerator to the product for the numerator, and the denominator of the fractional part is the denominator.

Q. How must I bring a compound fraction to a simple one?

A. Multiply all the numerators together for a numerator, and the denominators for a denominator.

Q. How must I bring numbers of lesser denominations to the fraction of a greater?

A. Reduce the given denominations to the least mentioned for a numerator, and 1 of the greater to the same for a denominator.

Q. How must I find the value of a fraction, which is a part of a unit of coin, weight and measure, &c.?

A. If of coin, weight or measure the fraction is assign'd,
In the tables the fit multipliers we find,
So multiply by 20 the given numerator,
The product divide by the denominator,
The shillings contain'd in the quotient are found;
If the fraction propos'd be the parts of a pound;
Multiply next by 12 the remainder from thence,
The product divide as before for the pence;
Repeat the like process again and again,
Till the lowest name's got, or till nothing remain.

CHAP. III.

ADDITION OF FRACTIONS.

Rule I.

IF the fractions have a common denominator, add the numerators together, and under their sum place the common denominator; if the sum be an improper fraction it may be reduced to a mixt number, if not to the least terms.

Examples.

Examples.

[1]		[2]	[3]	[4]	[5]	[6]
$\frac{3}{5}$	3	$\frac{5}{12}$	$\frac{7}{24}$	$\frac{11}{20}$	$\frac{16}{24}$	$\frac{9}{16}$
$\frac{1}{5}$	1	$\frac{1}{12}$	$\frac{11}{24}$	$\frac{9}{20}$	$\frac{10}{24}$	$\frac{13}{16}$
			$\frac{13}{24}$	$\frac{17}{20}$	$\frac{17}{24}$	$\frac{15}{16}$
$\frac{4}{5}$	4					

Rule II.

If the fraction have different denominators reduce them to equivalent fractions which shall have one common denominator, and add the said equivalent fractions, (ꝑ last Rule.)

Application.

$$\begin{array}{ll} \frac{1}{2}\ldots\ldots 6 & 12 \text{ com. den.}\\ \frac{2}{3}\ldots\ldots 8 & \\ \frac{3}{4}\ldots\ldots 9 & \\ \hline 1\frac{11}{12} \quad 23 & \end{array}$$

$$\begin{array}{r} 23\\ \hline 12)23(1\\ 12\\ \hline 11\\ \hline 12 \end{array}$$

If $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$ were given to be added, I find 12 the least common denominator, and $\frac{6}{12}$, $\frac{8}{12}$ and $\frac{9}{12}$, equal to the given fractions, which having a common denominator may be added, and their sum is $\frac{23}{12}$, which brought to a mixt number is $1\frac{11}{12}$, the sum of the given fractions.

7.	Add	$\frac{2}{3}$	$\frac{5}{6}$	and	$\frac{3}{8}$			together.	*Answ.*	$1\frac{7}{8}$
8.	———	$\frac{5}{8}$	$\frac{1}{7}$	$\frac{1}{9}$	$\frac{1}{8}$			———	———	$1\frac{1}{252}$
9.	———	$\frac{3}{4}$	$\frac{11}{12}$					———	———	$1\frac{2}{3}$
10.	———	$\frac{1}{8}$	and	$\frac{5}{6}$				———	———	$\frac{23}{24}$
11.	———	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{8}$				———	———	$1\frac{7}{8}$
12.	———	$\frac{5}{32}$	$\frac{5}{64}$	$\frac{5}{128}$				———	———	$\frac{35}{128}$
13.	———	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$		———	———	$\frac{271}{280}$
14.	———	$\frac{4}{5}$	$\frac{5}{6}$	$\frac{6}{7}$	$\frac{3}{8}$ and	$\frac{8}{11}$		———	———	$3\frac{5477}{9240}$
15.	———	$\frac{13}{36}$	$\frac{11}{18}$	$\frac{7}{12}$	$\frac{4}{9}$	$\frac{5}{6}$	$\frac{2}{3}$	———	———	$3\frac{1}{2}$

Rule III.

To add mixt numbers.

Add the fractions as before, and if the sum of the fractions found be an improper fraction, reduce it to a mixt number, and add the integral part with the whole numbers of the given mixt numbers.

Application.

So $5\frac{1}{2}$, $7\frac{7}{8}$ and 15 being given to be added: I first find the sum of $\frac{1}{2}$ and $\frac{7}{8}$ to be $\frac{11}{8}$ which is found equal to $1\frac{3}{8}$ I put down $\frac{3}{8}$ for the fractional part of the sum and carry 1, the integral part to the units of the given whole numbers, and adding them the sum is found $28\frac{3}{8}$

$5\frac{1}{2}$ . . : 4 (8
$7\frac{7}{8}$ . . . 7
15

$28\frac{3}{8}$ . . 11

8)11
$1\frac{3}{8}$

Examples.

[16]	[17]	[18]	[19]		
			l.	*s.*	*d.*
$327\frac{1}{2}$	$97\frac{1}{8}$	$84\frac{5}{7}$	12	19	$2\frac{1}{6}$
$64\frac{2}{3}$	$36\frac{5}{16}$	$69\frac{3}{4}$	00	18	$6\frac{13}{84}$
$95\frac{1}{4}$	$124\frac{9}{32}$	$17\frac{5}{28}$	00	04	$7\frac{181}{336}$
	$54\frac{18}{112}$	$154\frac{13}{24}$	00	02	$3\frac{517}{672}$

QUESTIONS *for Exercises.*

1. A merchant buys 5 pieces of cloth, the first was $40\frac{3}{4}$ yards; the second $27\frac{1}{2}$ yards; the third $34\frac{7}{8}$ yards; the fourth $43\frac{3}{16}$, and the fifth $39\frac{1}{4}$ yards: I desire to know how many yards were in the 5 pieces?

Answ. $185\frac{9}{16}$ yards.

2. Bought 4 bales of spice. Nº. 1, weight $150\frac{1}{2}$ ℔.; Nº. 2, weight $139\frac{3}{4}$ ℔.; Nº. 3, weight $162\frac{2}{3}$ ℔.; Nº. 4, weight $170\frac{1}{5}$ ℔: How many ℔. weighed they together?

Answ. $623\frac{1}{24}$ ℔.

3. A grocer sold the following parcels of sugar, viz. $16\frac{1}{2}$ ℔.; $19\frac{1}{4}$ ℔.; $13\frac{3}{4}$ ℔.; $20\frac{1}{8}$ ℔.; $25\frac{1}{16}$ ℔.: $30\frac{5}{6}$ ℔.; and $11\frac{1}{4}$ ℔.: I demand how many pounds he sold in all?

Answ. $136\frac{37}{48}$ ℔.

CHAP.

CHAP. IV.

SUBTRACTION OF FRACTIONS.

SUBTRACTION of Fractions is the taking of a lesser Fraction from a greater; likewise, a mixt Number or Fraction from a greater mixt number or a whole Number.

I. *Fractions which have a common Denominator.*

Subtract the numerator of the less from the numerator of the greater, and to their difference subscribe the common denominator; so is this new fraction the difference of the given fractions.

	[1]	[2]	[3]	[4]	[5]	[6]	[7]
From	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{36}{39}$	$\frac{28}{40}$	$\frac{9}{11}$	$\frac{19}{21}$
Take	$\frac{1}{3}$	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{23}{39}$	$\frac{5}{40}$	$\frac{7}{11}$	$\frac{17}{21}$
Rem.	$\frac{1}{3}$						

II. *When they have not a common Denominator.*

Reduce them to a common denominator, and then work as the last.

From $\frac{7}{8}$ take $\frac{2}{3}$.

$$\left.\begin{array}{l} \frac{7}{8} \ldots 21 \\ \frac{2}{3} \ldots 16 \\ \hline \end{array}\right\}$$
(24 common Denominator; Numerators.

Answ. $\frac{5}{24}$ 5

	[8]	[9]	[10]	[11]	[12]	[13]	[14]
From	$\frac{11}{12}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{75}{100}$	$\frac{24}{30}$	$\frac{127}{240}$	$\frac{224}{256}$
Take	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{15}{60}$	$\frac{13}{23}$	$\frac{127}{280}$	$\frac{13}{144}$
Rem.							

III. *A Fraction from a whole Number.*

Subtract the numerator of the fraction from its denominator, and place the remainder over the denominator, for the fractional part of the difference sought; then subtract 1 from the given whole number, for the integral part of the remainder; so is a fraction or mixt number found which shall be the remainder or difference required.

Application and Reason.

Let it be required to take $\frac{1}{4}$ from 2; I take 1 the numerator of $\frac{1}{4}$ from the denominator 4, and 3 the remainder 1 put, for a numerator over the denominator, *viz.* $\frac{3}{4}$, the fraction remaining; then I take 1 from the given whole number 2 and 1 remains; so is the remainder found $1\frac{3}{4}$, Q. *E. I.*

From	$2\frac{0}{0}$
Take	$0\frac{1}{4}$
Rem.	$1\frac{3}{4}$

	[15]	[16]	[17]	[18]	[19]	[20]
From	1	3	7	8	9	12
Take	$0\frac{2}{3}$	$0\frac{3}{4}$	$0\frac{1}{10}$	$0\frac{1}{2}$	$0\frac{4}{7}$	$0\frac{5}{8}$

In like manner to subtract a mixt number from a whole number, subtract the fractional part as above, and to the lesser whole number add 1; the sum take from the greater whole number.

	[21]	[22]	[23]	[24]	[25]
From	2	3	7	11	135
Take	$1\frac{2}{3}$	$1\frac{3}{4}$	$6\frac{9}{10}$	$6\frac{17}{20}$	$87\frac{3}{11}$

IV. *A Fraction or a mixt Number from a mixt Number when the Fraction to be subtracted is the less.*

Subtract the less fraction from the greater fraction, and the less whole number from the greater.

[26]

	[26]	[27]	[28]	[29]	[30]
From	$2\frac{2}{3}$	$4\frac{3}{4}$	$9\frac{7}{8}$	$22\frac{36}{39}$	$127\frac{63}{113}$
Take	$0\frac{1}{3}$	$1\frac{1}{4}$	$9\frac{3}{8}$	$0\frac{23}{39}$	$60\frac{17}{113}$
Rem.	$2\frac{1}{3}$				

	[31]	[32]	[33]	[34]	[35]
From	$7\frac{7}{8}$ . . 21(24	$27\frac{11}{12}$	$19\frac{3}{4}$	$120\frac{75}{100}$	$185\frac{190}{250}$
Take	$3\frac{2}{3}$ . . 16	$0\frac{2}{3}$	$15\frac{1}{2}$	$23\frac{15}{60}$	$0\frac{187}{120}$
Rem.	$4\frac{5}{24}$ 5				

V. *A Fraction or a mixt Number from a mixt Number when the Fraction to be subtracted is the greater.*

Rule.

1. Reduce the given fractions to one common denominator.
2. Then subtract the numerator of the greater fraction from the common denominator, and to the remainder add the numerator of the lesser, the sum is the numerator to the common denominator, for the fractional part of the remainder.
3. Carry 1 to the lesser whole number, and subtract the sum from the greater.

Application.

Let it be required to take $2\frac{3}{4}$ from $5\frac{1}{2}$?

The given fractions being brought to one common denominator will be $\frac{2}{4}$ and $\frac{3}{4}$; I take the greater numerator 3 from the common denominator 4 and 1 remains, which added to 2 the lesser numerator, makes 3 for the numerator of the remaining fraction $\frac{3}{4}$; then I carry 1 to 2 the lesser whole number, makes 3 from 5 and 2 remains; whence the remainder sought is found $2\frac{3}{4}$.

	$5\frac{1}{2}$...2	
	$2\frac{3}{4}$...3	
Rem.	$2\frac{3}{4}$	3

Examples.

Examples.

	[36]	[37]	[38]	[39]	[40]
From	$2\frac{1}{2}$	$9\frac{5}{7}$	$19\frac{5}{11}$	$13\frac{1}{9}$	$12\frac{3}{8}$
Take	$0\frac{3}{4}$	$6\frac{6}{7}$	$0\frac{7}{15}$	$9\frac{15}{27}$	$9\frac{5}{6}$

QUESTIONS.

Quest. 1. What is the difference of $\frac{9}{14}$ and $\frac{17}{21}$?
Answ. $\frac{1}{6}$.

2. What differs $\frac{1}{19}$ from $\frac{1}{9}$? *Answ.* $\frac{10}{171}$.

3. What is the difference between $10\frac{7}{8}$ and $12\frac{1}{2}$?
Answ. $1\frac{5}{8}$.

4. What differs $\frac{3}{4}\frac{3}{5}$ from 48? *Answ* $47\frac{1}{16}$.

5. Bought a piece of cloth containing $47\frac{5}{8}$ yards, of which I cut $24\frac{13}{16}$ yards; I demand how much I have by me? *Answ.* $22\frac{13}{16}$ yards.

6. A man had 4 bags of money, containing in all *500l.* in the first was $130\frac{2}{3}$; in the second $97\frac{11}{20}$; in the third $110\frac{7}{12}$: I want to know what was in the fourth?
Answ. $161\frac{1}{5}$*l.*

CHAP. V.

MULTIPLICATION OF FRACTIONS.

Rule.

Multiply the Numerators into each other for the Numerator; and the given Denominators for the Denominator of the Product.

Application.

Application.

$$\begin{array}{ccccc}
 & M & & N & \\
a & 2 & \text{——} & 3 & c \\
\text{—} & & & & \text{—} \\
b & 3 & \text{——} & 4 & d \\
 & 6 & \text{——} & e & \\
 & \text{—} & & \text{—} & \\
 & 12 & & f & \\
 & & O & & \\
g\ 24 & & h\ 36 & &
\end{array}$$

Let the fractions M N be given to be multiplied, the numerators 2 and 3 being multiplied into each other make 6 for the numerator of the product O, and 3 multiplied into 4 makes 12 for the denominator; so $\frac{6}{12}$ or $\frac{1}{2}$ is the product found by the Rule.

If whole numbers or mixt numbers be given to be multiplied, reduce them to improper fractions, and multiply them by the Rule, and if the product be an improper fraction, it may be brought back to a mixt or whole number.

Examples.

$\frac{3}{5}$ multiplied by $\frac{7}{8}$ produces $\frac{21}{40}$.

$$\begin{array}{ccc}
3 & \text{——} & 7 \\
\text{—} & & \text{—} \\
5 & \text{——} & 8 \\
 & 21 & \\
 & \text{——} & \\
 & 40 &
\end{array}$$

Multiply the following fractions.

1. $\frac{3}{4}$ by $\frac{1}{2}$ makes $\frac{3}{8}$
2. $\frac{1}{4}$ by $\frac{2}{3}$ —— $\frac{1}{6}$
3. $\frac{6}{7}$ by $\frac{1}{3}$ —— $\frac{2}{7}$
4. $\frac{9}{15}$ by $\frac{4}{9}$ —— $\frac{4}{15}$
5. $\frac{10}{17}$ by $\frac{17}{10}$ *Answ.* 1
6. $\frac{128}{135}$ by $\frac{5}{12}$ —— $\frac{32}{81}$
7. $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ —— $\frac{1}{4}$
8. $\frac{2}{3}$ $\frac{3}{4}$ $\frac{4}{7}$ $\frac{7}{21}$ —— $\frac{2}{21}$

Note. Where several fractions are to be multiplied, if the numerator of one fraction be equal to the denominator of another, these equal numerators and denominators may be omitted.

II.

Multiply 2 by $\frac{1}{2}$? *Answ.* 1

2 expressed fractionally is $\left\{\begin{array}{ccc} 2 & \text{——} & 1 \\ \text{—} & & \text{—} \\ 1 & & 2 \end{array}\right.$

$\frac{2}{2}$, *i. e.* 1.

Multiply the following:

9. 3 by $\frac{3}{4}$	*Answ.*	$2\frac{1}{4}$	12. 9 by $\frac{1}{5}$	*Answ.*	$1\frac{4}{5}$
10. 5 by $\frac{1}{8}$	——	$\frac{5}{8}$	13. 24 by $\frac{5}{6}$	——	20
11. 12 by $\frac{1}{3}$	——	4	14. 60 by $\frac{2}{15}$	——	8

III.

Multiply $4\frac{3}{7}$ by 4. *Answ.* $17\frac{5}{7}$.

$$\begin{array}{l} 7 \\ \hline 31 \text{———} 4 \\ \hline 7 \text{———} 1 \\ \quad 124 \\ \hline 7)124 \\ \hline \quad 17\frac{5}{7} \\ \hline \end{array}$$

15. 8 by $5\frac{2}{3}$	*Answ.*	$45\frac{1}{3}$	17. 24 by $6\frac{5}{6}$	*Answ.*	164
16. $3\frac{3}{4}$ by 5	——	$18\frac{3}{4}$	18. 112 by $5\frac{1}{2}$	——	616

Multiply $8\frac{5}{12}$ by $3\frac{3}{8}$

$$\begin{array}{l} 12 \qquad 8 \\ \hline 101 \text{——} 27 \\ \hline 12 \text{——} 8 \\ \quad 2727 \\ \hline 96)2727(28\frac{39}{96} \\ \quad 192 \\ \hline \quad 807 \\ \quad 768 \\ \hline \quad\ 39 \\ \hline \end{array}$$

19.	Multiply $19\frac{4}{7}$	by	$23\frac{9}{11}$	*Answ.*	$466\frac{12}{77}$
20.	———— $8\frac{2}{3}$	by	$10\frac{4}{5}$	——	$93\frac{3}{5}$
21.	———— $75\frac{3}{4}$	by	$100\frac{2}{3}$	——	$7625\frac{1}{2}$
22.	———— $35\frac{13}{16}$	by	$11\frac{17}{25}$	——	$418\frac{29}{100}$

Remark.

Remark.

Although the Rule before delivered be universal, as appears by the foregoing examples, yet the following method will be generally more convenient in practice.

I. *To Multipy a whole Number by a Fraction.*

Rule.

Multiply the whole number by the numerator of the fraction, and divide that product by the denominator.

Application.

A B E
3c
4——3
4d

Let 4 be multiplied by $\frac{3}{4}$; 4 multiplied by the numerator 3 makes 12, which product divided by the Denominator 4, the quotient is 3, which is the product of 4 multiplied by $\frac{3}{4}$.

Cor. Since 1 doth not multiply a number, it follows, that when the numerator of a fraction is 1, the product is found by dividing by the denominator.

Let the examples II. be done by this.

Again,

To multiply a whole number by the integral part of the mixt number, and then by the fractional part, and add the products together,

Application.

8 | 8
$5\frac{2}{3}$ | $5\frac{3}{5}$
222 { 40 | 40
$5\frac{1}{3}$ | $\frac{1}{3}$ $2\frac{2}{3}$
 | $\frac{1}{3}$ $2\frac{2}{3}$
$45\frac{1}{3}$ | $45\frac{1}{3}$

As suppose it was required to multiply 8 by 5 and $\frac{2}{3}$; first, 8 multiplied by 5 is 40 and 8 into $\frac{2}{3}$ makes $5\frac{1}{3}$, or else by $\frac{1}{3}$, $\frac{1}{3}$; and again, by $\frac{1}{3}$, and lastly the sum of 40 and $5\frac{1}{3}$, is $45\frac{1}{3}$ the product required.

CHAP. VI.

DIVISION OF FRACTIONS.

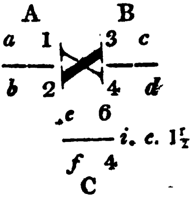
Rule.

Multiply the Numerator of the Dividend by the Denominator of the Divisor for a Numerator, and the Denominator of the Dividend by the Numerator of the Divisor for the Denominator of the Quotient.

Appli-

Application.

Let $\frac{3}{4}$ be to be divided by $\frac{1}{2}$, 3 (the numerator of the dividend) multiplying 2 (the denominaior of the devisvr) produces 6 for the numerator of the quotient, in like manner 4 multiplying 1 produces 4 for the denominator of the quotient, which is $\frac{6}{4}$ or $1\frac{1}{2}$.



Note. Whole numbers or mixed numbers must be reduced to improper fractions.

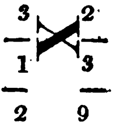
I

1. $\frac{2}{3}$ by $\frac{1}{6}$ *Answ.* 4
2. $\frac{1}{6}$ by $\frac{2}{3}$ —— $\frac{1}{4}$
3. $\frac{21}{45}$ by $\frac{3}{5}$ —— $\frac{7}{9}$
4. $\frac{5}{8}$ by $\frac{2}{7}$ —— $2\frac{3}{16}$

5 $\frac{6}{9}$ by $\frac{3}{9}$ *Answ.* 2
6. $\frac{15}{16}$ by $\frac{5}{8}$ —— $1\frac{1}{2}$
7. $\frac{107}{120}$ by $\frac{7}{15}$ —— $1\frac{51}{56}$
8. 1 by $\frac{17}{18}$ —— $\frac{18}{17}$

II.

Divide $\frac{2}{3}$ by 3



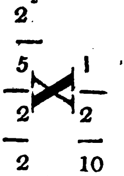
Answ. $\frac{2}{9}$

9. $\frac{1}{2}$ by 4 *Answ.* $\frac{1}{8}$
10. $\frac{2}{3}$ by 10 —— $\frac{1}{15}$
11. $\frac{5}{8}$ by 24 —— $\frac{5}{192}$
12. $\frac{11}{12}$ by 60 —— $\frac{11}{720}$

13. 4 by $\frac{1}{2}$ *Answ.* 8
14. 10 by $\frac{2}{5}$ —— 25
15. 24 by $\frac{5}{8}$ —— $38\frac{2}{5}$
16. 60 by $\frac{11}{12}$ —— $65\frac{5}{11}$

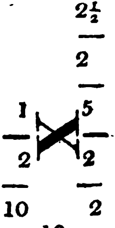
III.

Divide $\frac{1}{2}$ by $2\frac{1}{2}$



$\frac{2}{10}$ $\frac{1}{5}$ *Answ.*

Divide $2\frac{1}{2}$ by $\frac{1}{2}$



10

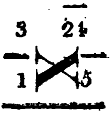
2)10(5

17. $\frac{3}{4}$ by

17. $\frac{3}{4}$ by $4\frac{1}{2}$ *Answ.* $\frac{1}{5}$
18. $\frac{1}{5}$ by $1\frac{4}{5}$ —— $\frac{1}{9}$
19. $\frac{1}{4}$ by $1\frac{5}{8}$ —— $\frac{2}{13}$
20. $\frac{7}{8}$ by $2\frac{1}{2}$ —— $2\frac{1}{2}$

21. $4\frac{1}{2}$ by $\frac{3}{4}$ *Answ.* 6
22. $1\frac{4}{5}$ by $\frac{1}{5}$ —— 9
23. $1\frac{5}{8}$ by $\frac{1}{4}$ —— $6\frac{1}{2}$
24. $2\frac{1}{3}$ by $\frac{7}{8}$ —— $\frac{3}{8}$

Divide 3 by $4\frac{5}{2}$
5



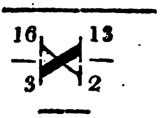
$$3\left\{\frac{15}{24}\right\}\frac{5}{8}$$

25. 7 by $8\frac{1}{8}$ *Answ.* $\frac{56}{65}$
26. $25\frac{1}{6}$ by 5 —— $5\frac{1}{30}$
27. 34 by $3\frac{7}{9}$ —— 9
28. $1\frac{1}{9}$ by 15 —— $0\frac{2}{27}$

29. 108 by $6\frac{3}{7}$ *Answ.* $16\frac{4}{5}$
30. $119\frac{1}{4}$ by 8 —— $14\frac{29}{32}$
31. 25 by $3\frac{1}{3}$ —— $7\frac{1}{2}$
32. $9\frac{3}{7}$ by 11 —— $0\frac{6}{7}$

V.

Divide $5\frac{1}{3}$ by $6\frac{1}{2}$



Answ. $\frac{32}{39}$

33. $9\frac{3}{4}$ by $13\frac{1}{2}$ *Answ.* $\frac{13}{18}$
34. $1\frac{1}{13}$ by $5\frac{5}{10}$ —— $\frac{21}{100}$
35. $6\frac{1}{2}$ by $5\frac{1}{2}$ —— $1\frac{7}{33}$

36. $10\frac{5}{6}$ by $4\frac{2}{3}$ *Answ.* $2\frac{9}{28}$
37. $60\frac{9}{16}$ by $26\frac{11}{13}$ —— $2\frac{1}{4}$
38. $1\frac{1}{2}$ by $2\frac{1}{2}$ —— $0\frac{3}{5}$

Note. When the denominator of the divisor and of the dividend are equal, the quotient may be found by common Division, *viz.* by dividing the numerator of the dividend by the Numerator of the divisor, rejecting the common denominator entirely.

As suppose $\frac{6}{9}$ to be dividend by $\frac{3}{9}$, 3 dividing 6 the quotient is 2, which is likewise the quotient of $\frac{3}{9}$, 3 dividing $\frac{6}{9}$ (See the Example, Case I.)

Now this rule may be made very extensive or universal, thus: reduce the divisor and a dividend to one common denominator, and divide the numerator of the dividend by the numerator of the divisor.

As if $\frac{2}{3}$ were to be divided by $\frac{1}{6}$; $\frac{2}{3}$ is equal to $\frac{4}{6}$, and I dividing 4, the quotient is 4, the true quotient of $\frac{2}{3}$ divided by $\frac{1}{6}$; or if $\frac{1}{6}$ be divided by $\frac{2}{3}$, *i. e.* $\frac{4}{6}$, 4 by dividing 1 the quotient is $\frac{1}{4}$.

Note. 2. If a whole number be to be divided by a fraction, multiply the whole number by the denominator of the fraction, and divide the product by the numerator thereof, thus 48 being to be divided by $\frac{4}{7}$, the quotient is

$$\frac{48 \times 7}{4} = 84.$$

To divide a fraction by a whole number, multiply the denominator of the given fraction by the given whole number for the denominator of the quotient, and make the numerator of the given fraction the numerator.

Thus let the examples of the second class be resolved.

To divide a mixed number by a whole number.

1. If the integral part of the mixt number be less than the divisor, change the mixed number into an improper fraction, and divide the said fraction according to last.

So $1\frac{4}{7}$ being to be divided by 4; the mixed number reduced to an improper fraction makes $\frac{11}{7}$, which being divided by 4, makes the quotient $\frac{11}{28}$.

2. But if the integral part of the mixed number be greater than the divisor, divide the said integral part by the said divisor, and if any thing remain, reckon it together with the fractional part, a mixed number to be divided.

So $13\frac{4}{7}$, divided by 4 quotes $3\frac{11}{28}$; for 4 in 13 is 3 times, and 1 remains, *viz.* $1\frac{4}{7}$. which divided by 4, as above, makes $\frac{11}{28}$.

If nothing remains after the divisor divides the integral part, then divide the fractional part as before.

Examples.

39. Divide $25\frac{1}{2}$ by 5 *Answ.* $5\frac{1}{10}$. 42. $9\frac{3}{7}$ by 11. *Answ.* $\frac{6}{7}$.
40. —— $1\frac{1}{9}$ by 15 —— $\frac{2}{27}$.
41. —— $119\frac{1}{4}$ by 8 —— $14\frac{29}{32}$

Here I have a mind to shew how several questions, usually solved by the Rule called Position, may more easily and intelligibly be solved. In order thereto it is only necessary to consider the contrast, already taken notice of between Addition and Subtraction; between Multiplication and Division, *viz.* that whatever is effected by the one is unravelled by the other.

One being asked how old he was answered, if my age be doubled, the $\frac{1}{2}$ and $\frac{1}{4}$ of my age added to it, more 1 year, I should be 100 years old. What is his age?

Answ. 36 years.

Here from the last number 100 given I easily discover the number sought: for being doubled, *i. e.* multiplied by 2; the $\frac{1}{2}$ and $\frac{1}{4}$ added, that is multiplied likewise by $\frac{1}{2}$ and $\frac{1}{4}$, the sum of the products more 1, makes 100; consequently the sum of the products is 100 less 1. *viz* 99. Then since the number sought multiplied by 2, by $\frac{1}{2}$; by $\frac{1}{4}$, respectively, the sum of the products is 99; if it be multiplied by $2\frac{3}{4}$, the sum of these multipliers, the product will be likewise 99. Therefore we have given the product 99 and the multiplier $2\frac{3}{4}$ to find the multiplicand.

$$\begin{array}{lr} \text{From} & 100 \\ \text{Take} & \underline{1} \end{array}$$

$$\begin{array}{llllr}
2\frac{3}{4}=\frac{11}{4} \times \frac{99}{1} & & 2\frac{3}{4} & & 99 \\
 & & & & \underline{4} \\
\underline{4} & & \underline{4} & & \\
\underline{11} & & 11 & & 11)\underline{396} \\
4 & \underline{396} & & & 36
\end{array}$$

$$\begin{array}{r} 11)\underline{396} \\ 36 \end{array}$$

CHAP. VII.

THE RULE OF THREE IN FRACTIONS.

THE Rule of Three in Fractions, is analogous to the Rule of Three in whole Numbers, both in the Stating and Operation, For

The first and third number or fraction must be of the same name or kind, and reduced to fractions of the same name or denominator.

Multiply the second and third terms together and divide the product by the first; the quotient is the fourth term required; due regard being had to the rules laid down for multiplying, dividing and reducing fractions.

Note. When the first term is 1, the fourth is found by multiplying the second and third; and when the second or third is 1, the fourth is found by dividing the other by the first.

Examples.

1. If $\frac{3}{4}$℔ of sugar cost $\frac{2}{5}$*s.* what cost $\frac{7}{8}$℔?

$$\begin{array}{ccccc} \text{℔} & & s. & & \text{℔} \\ \frac{3}{4} & \text{———} & \frac{2}{3} & \text{———} & \frac{7}{8} \end{array}$$

$$\text{Divide by} \; \frac{3}{4} \times \frac{14}{24} \; \text{the product of } \frac{2}{3} \text{ into } \frac{7}{8}.$$

2. If $\frac{3}{8}$ of a yard cost $\frac{1}{5}$5*l.* what will $\frac{2}{5}$ yard cost?
Answ. 10*s.*

3. What will $\frac{8}{9}$℔. cost, if $\frac{2}{5}$*s.* buy $\frac{7}{8}$℔.?
Answ. $4\frac{92}{105}$*d.*

4. If $19\frac{1}{5}$℔ cost $\frac{6}{10}$*l.* what quantitity can I have for $\frac{5}{8}$*s.*?
Answ. $\frac{2}{3}$℔.

5. What will $\frac{1}{2}$ *Cwt.* come to if $6\frac{3}{4}$ *Cwt.* cost 21 $\frac{3}{4}$*l.*?
Answ. 1*l.* 12*s.* $2\frac{3}{4}$*d.*

6. How many ℔ of pot-ashes can I have for $12\frac{1}{4}$ if 1℔ cost $1\frac{3}{4}$*d.* *Answ.* 7℔.

7. If for $10\frac{1}{3}$*s.* I buy one hundred of oranges; how many hundred can I have then for $105\frac{1}{12}$*s.*?
Answ. $10\frac{1}{4}$ hundred.

8. If 1℔ of any thing cost $5\frac{1}{4}$*s* what will $\frac{7}{8}$ come to?
Answ. 4*s.* $7\frac{1}{8}$*d.*

9. How much will $\frac{1}{4}$ *Cwt.* come to, at the rate of $15\frac{1}{3}$*s.* the *Cwt.* *Answ.* 3*s.* $11\frac{1}{4}$*d.*

Although the method before laid down be universally applicable, as by the foregoing examples appeareth, yet there are other methods more ready and accommodate in practice in some particular cases.

Rule

Rule I.

If the first and third terms be fractions, and the second not, reduce the said first and third to one common denominator; then rejecting the denominators, I make the numerator of the first, the first term, and the numerator of the third, the third term, and work as in whole numbers, &c.

Application.

10. If $\frac{5}{6}$ yard of linen cost 2*s.* what cost $\frac{15}{16}$ yard at that rate?

2)6, 16 $\qquad \frac{5}{6}=\frac{40}{48} \qquad$ 40——2——45
3, 8 $\qquad\qquad\qquad\qquad\qquad\qquad$ 2
3 $\qquad \frac{15}{16}=\frac{45}{48} \qquad$ 4|0)9|0
24 $\qquad\qquad\qquad\qquad$ *Answ.* 2*s.* 3*d.*
2
48 common denominator.

11. If $\frac{1}{4}$ of a pound of tea cost 1*s* 9*d.* what cost $\frac{5}{8}$ lb.?
Answ. 4*s.* 4½*d.*

12. If $\frac{3}{8}$ yard cost 9*s.* 4½*d.* what cost $\frac{2}{3}$ yard?
Answ. 16*s.* 8*d.*

13. If $\frac{1}{3}$ of a pound Troy cost 19*s.* 6*d.* what cost $\frac{11}{12}$?
Answ. 2*l.* 13*s.* 7½*d.*

Rule II.

If of the first and third terms one be 1, and the other a fraction, put the denominator of the fraction instead of 1, and the numerator in the place of the fraction, and work the question as in whole numbers as before.

Application.

14. If 1 ton of tallow cost 35*l.* what cost $\frac{3}{4}$ of a ton?

4——35——3
3
4)105
26*l.* 5*s.*

15. If 1 ounce of silver cost 5*s.* how much cost $\frac{5}{8}$ of an ounce? *Answ.* 3*s.* 1$\frac{1}{2}$*d.*

16. At 25*s.* ℔ *Cwt.* how much will $\frac{3}{4}$ of a C. cost? *Answ.* 18*s.* 9*d.*

17. If the freight of a ship be 247*l.* 16*s.* 8*d.* what must A B receive for his $\frac{5}{16}$ thereof? *Answ.* 77*l.* 8*s.* 11$\frac{1}{2}$*d.*

18. What will 1 yard come to, if $\frac{3}{4}$ yard cost 18*d.*? *Answ.* 2*s.*

19. If half a quarter of Flanders lace cost 3*s.* what is that a yard? *Answ.* 1*l.* 4*s.*

Rule III.

Of the first and third, if one be a whole number and the other a fraction, multiply the whole number by the denominator of the fraction and work as before. Or if one be a whole number and the other a mixed number, bring the mixed number to an improper fraction and put the numerator in the place of the fraction or mixed number, and multiply the whole number by the denominator of the fraction and place the product in the room of the said whole number.

Application.

20. If a piece of cambric 15 yards long cost 3*l.* 15*s.* what cost $\frac{1}{2}$ yard?

	Yds.	*l.*	*s.*	*Yd.*
	15 ——	3	15 ——	$\frac{1}{2}$
	2			
		l.	*s.*	
Half yards	30 ——	5)3	15 ——	1
		6)0	15	

Answ. 2*s.* 6*d.*

21. If a piece of linen containing 40 yards cost 30*s.* what is the price of 4$\frac{4}{5}$ yards? *Answ.* 3*s.* 7$\frac{1}{5}$*d.*

22. If 6$\frac{3}{4}$ C. of goods cost 21$\frac{3}{4}$*l.* what cost 1 ton? *Answ.* 64*l.* 8*s.* 10$\frac{2}{3}$*d.*

23. Bought a bag of wool, *wt.* neat 5$\frac{3}{4}$ C. at 9*s.* 6*d.* ℔ stone; what doth it amount to? *Answ.* 19*l.* 2*s.* 4$\frac{1}{2}$*d.*

Rule.

Rule IV.

If the second term be a fraction likewise, the first and third being brought to one denomination, multiply the first by the denominator of the second for a divisor, and the third by the numerator of the second for a dividend, divide the last by the first and the quotient is the answer.

Application.

24. If $\frac{1}{4}$ yard of Flanders lace cost $\frac{2}{3}$*s.* what cost $\frac{7}{8}$?
$\frac{1}{4}$ is $\frac{2}{8}$.

2 —— $\frac{2}{8}$ —— 7
3 2

6)14

Answ. $2\frac{1}{3}$*s.* or 2*s.* 4*d.*

25. At $\frac{5}{6}$*l.* ꝑ yard, how much will 42 yards cost?
Answ. 35*l.*

26. How much will 650℔. come to, at $3\frac{3}{4}$*s.* ꝑ ℔.?
Answ. 121*l.* 17*s.* 6*d.*

27. If $\frac{4}{7}$ of an ounce, Avoirdupois, cost $10\frac{1}{2}$*d.* what cost $\frac{5}{14}$℔? *Answ.* 8*s.* 9*d.*

28. What will $1\frac{1}{2}$ C. of pepper come to, if $15\frac{1}{4}$℔. cost $12\frac{3}{8}$*s.*? *Answ.* 6*l.* 16*s.* $3\frac{57}{61}$*d.*

QUESTIONS *for Exercise.*

1. What part of 3*d.* is $\frac{2}{3}$ of 2*d.*? *Answ.* $\frac{4}{9}$

2. What part is 176 of 368? *Answ* $\frac{11}{23}$.

3. By how much must I multiply $13\frac{2}{3}$ that the product may be $49\frac{1}{5}$? *Answ.* $3\frac{3}{5}$.

4. What differs $7\frac{3}{4}$*d.* from $3\frac{1}{3}$*s.*? *Answ.* 2*s.* $8\frac{1}{4}$*d.*

5. What number is that which when added to $\frac{1}{8}$, $\frac{1}{12}$ and $\frac{1}{19}$ together will make $\frac{25}{37}$? *Answ.* $\frac{1027}{2664}$.

6. What number is that, to which, if $\frac{3}{10}$ of $\frac{18}{7}$ of $\frac{141}{213}$ be added, the total may be 1? *Answ.* $\frac{368}{7435}$.

7. A father dying left his son a fortune, $\frac{3}{16}$ of which he ran out in 6 months; $\frac{2}{3}$ of the remainder held him a

twelvemonth longer, at which time he had bare 348*l.* left: Pray what did the father bequeath him?

Answ. 1284*l.* 18*s.* $5\frac{1}{2}$*d.*

8. A younger brother received 2200*l.* which was just $\frac{5}{12}$ of his elder brother's fortune; and $3\frac{1}{8}$ times the elder's money was $\frac{1}{2}$ as much again as the father was worth: what was that? *Answ.* 11000*l.*

9. How many stones of $1\frac{3}{4}$ foot long, $\frac{2}{3}$ foot broad, and $\frac{3}{4}$ foot thick, are equal to 50 stones of $3\frac{1}{3}$ foot long, $2\frac{1}{2}$ foot broad, and $1\frac{1}{5}$ thick? *Answ.* $571\frac{3}{7}$.

10. A merchant hath $\frac{3}{4}$ of a ship, and sells $\frac{1}{4}$ of his interest therein for 250*l.* I demand the value of the whole ship at that rate? *Answ.* 1333*l.* 6*s.* 8*d.*

11. How much will 2 bags of wool come to, N°. 1, *wt.* $94\frac{3}{16}$ stone; N°. 2, $305\frac{7}{9}$, at 10*s.* $6\frac{2}{3}$*d.* ꝑ stone; but $4\frac{2}{3}$ stone of N°. 2, are worth but $2\frac{1}{4}$ stone of N°. 1?

Answ. 127*l.* 10*s.* $4\frac{59}{84}$*d.*

12. A father devised $\frac{3\frac{1}{3}}{8}$ of his estate to one of his sons, $\frac{3\frac{3}{4}}{8\frac{1}{3}}$ of the residue to another, and the surplus to his relict, for life; the children's legacies were found to be 257*l.* 3*s.* 4*d.* different: what money did he leave the widow the use of? *Answ.* 534*l.* 2*s.* $7\frac{231}{239}$*d.*

13. If $\frac{3}{7}$ of $\frac{4}{5}$ of $\frac{7}{8}$ of a ship be worth $\frac{1}{9}$ of $\frac{6}{7}$ of $\frac{11}{13}$ of the cargo valued at 12000*l.* what did both ship and cargo stand the owners in? *Answ.* 15223*l.* 8*s.* $10\frac{306}{319}$*d.*

14. If a wedge of gold, weighing $17\frac{3}{7}$ ℔. be worth $679\frac{5}{7}$*l.* what is the value of $1\frac{3}{13}$ grain? *Answ.* 2*d.*

15. A man dying gave to his eldest son $\frac{2}{3}$ of $\frac{1}{4}$ of his estate; to his second $\frac{1}{5}$ of $\frac{1}{2}$, and when he counted their portions, the one had 40*l.* more than the other; the remainder was given to the wife and younger children; how much had each? *Answ.* The eldest son 100*l.* the second 60*l.* the wife and younger children 440*l.*

16*l.* In the year I wrote this, if to my age you add
$\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, (thereof) with $\frac{4}{5}$ more,
The number 74 will then be had:
Ingenious youths my age explore?
Answ. 36 years.

17. A in a scuffle, seized on $\frac{2}{3}$ of a parcel of sugar-plumbs, B catched $\frac{3}{8}$ of it out of his hands, and C laid hold on $\frac{3}{10}$ more: D ran off with all A had left, except $\frac{1}{7}$, which E afterwards secured slily for himself; then A and C jointly set upon B, who in the conflict, let fall $\frac{1}{2}$ he had, which were equally picked up by D and E.—B then kicked down C's hat, and to work they went anew for what it contained; of which A got $\frac{1}{4}$, B $\frac{1}{3}$, D $\frac{2}{7}$, and C and E equal shares of what was left of that stock; D then struck $\frac{3}{4}$ of what A and B last acquired out of their hands; they with difficulty recovered $\frac{5}{8}$ of it in equal shares again, but the other three carried off $\frac{1}{8}$ a piece of the same. Upon this they called a truce, and agreed that the $\frac{1}{3}$ of the whole left by A at first should be equally divided among them: How much of the prize, after this distribution, remained with each of the competitors? *Answ.* A got 2863—B 6335—C 2438—D 10294—E 4950.

BOOK II.——PART II.

CHAP. I.

OF DECIMAL FRACTIONS.

DECIMAL Fractions are a kind of fractions, which vary in the same proportion, and are managed by the same Methods of Operation as whole Numbers are.

For this purpose every Proper Fraction is supposed to be reducible to another whose Denominator shall be 10, 100, 1000, *&c. viz.* Unity with some multitude of Cyphers annexed: and Fractions with such Denominators are called Decimal Fractions: Such are $\frac{5}{10}$, $\frac{75}{100}$, $\frac{625}{1000}$.

As the denominator of a decimal fraction is always 10, or 100, or 1000, *&c*, the said denominators need not be expressed. For the numerators only may be made to express the true value of a decimal: for this purpose it is only required to write the numerator, with a point before it, to distinguish it from a whole number, when it consists of as many figures as the denominator hath cyphers annexed to unity; so $\frac{5}{10}$ may be written .5; $\frac{75}{100}$, .75; $\frac{625}{1000}$, .625.

N. B. The point prefixed is called the Separatrix.

But if the numerator hath not so many places, as the denominator hath cyphers, put as many cyphers before it, *viz.* to the left hand as will make up the defect; so write $\frac{5}{100}$ .05, $\frac{5}{1000}$ .005. And thus do these fractions receive the form of whole numbers.

We may consider unity as a fixed point, from whence whole numbers proceed infinitely increasing, and decimals infinitely decreasing towards 0, as in the following

TABLE.

Millions	C. Thousands	X. Thousands	Thousands	Hundreds	Tens.	Units	Tenth Parts	Hundredth Parts	Thousandth Parts	X. Thousandth Parts	C. Thousandth Parts	Millioneth Parts
7	6	5	4,	3	2	1	2	3	4	5	6	7

Whole Numbers. Decimals.

From this Table it is manifest that

In decimals, as well as in whole numbers, each figure taketh its value by its distance from units place: if it be in the first place after units (or the separating point) it signifies tenths; if in the second, hundredths, *&c.* decreasing in each place in a tenfold proportion.

Consequently every single figure, expressing a decimal, hath for its denominator 1, with as many cyphers as its place is distant from units place. Thus 2 in the Table is $\frac{2}{10}$, 3 $\frac{3}{100}$. 4 is $\frac{4}{1000}$, *&c.* And if a decimal be expressed by several figures, the denominator is 1, with as many cyphers as the lowest figure is distant from units place. So 234 signifies $\frac{234}{1000}$.

A cypher (or cyphers) placed at the right hand of a decimal fraction, altereth not its value, since every significant figure continueth to possess the same place. So .5 — .50—.500 are all of the same value.

But a cypher or cyphers put to the left hand of a decimal, do alter its value, every cypher depressing it to $\frac{1}{10}$ of the value it had before, by removing every significant figure one place farther from the place of units.

So .5, .05 .005, all express different decimals, *viz.* .5 $\frac{5}{10}$; .05 $\frac{5}{100}$; .005 $\frac{5}{1000}$.

Hence likewise may be observed the contrary effect of cyphers being annexed to whole numbers, and decimals: Every cypher to the right hand of a whole number encreaseth its value ten times; but cyphers to the right hand of a decimal do not alter its value. Again, cyphers put to the left hand of a whole number do not alter its value; but every cypher put to the left hand of a decimal, depresseth its value to the $\frac{1}{10}$ of what it would be without them.

So	5	is	Five	.5	is	$\frac{5}{10}$
	50		Fifty	.05		$\frac{5}{100}$
	500		Five Hundred	.005		$\frac{5}{1000}$
	5000		Five Thousand	.0005		$\frac{5}{10000}$

It is likewise manifest from the Table, that since the places of decimals decrease in a tenfold proportion from units downwards, so they consequently increase in a tenfold proportion from the right hand towards the left; as the places of whole numbers do, for ten hundredth parts make one tenth, ten tenths make 1; ten units ten; ten tens one hundred, &c. *viz.* $\frac{10}{100}=\frac{1}{10}$, $\frac{10}{10}=1$, $1\times10=10$, which proves that decimals are subject to the same law of notation and consequently of operation, as whole numbers are.

Decimal fractions of unequal denominators are reduced to one common denominator, when they are annexed to the right hand of those which have fewer places, as many cyphers as make them equal in places, with that which hath most. So these decimals. .5, .04, .125, may be reduced to the decimals .500, .040, .125, which have all 1000 for their denominator.

Of decimals, that is the greatest, whose highest figure is greatest, whether they consist of an equal or unequal number of places. Thus, .575 is greater than .395, and .5 greater than .395, for if it be reduced to the same denominator with .395, it will be .500, which is manifestly the greater.

A mixt number, *viz.* a whole number with a decimal annexed is equal to an improper fraction, whose numerator is all the figures of the mixed number, taken as one whole number and the denominator that of the decimal part. So 32.405 is equal to $\frac{32405}{1000}$, as is manifest from the method laid down to reduce a mixed number to an improper fraction, for 32 the integral part being multiplied by 1000 the denominator of the fractional part produces 32000, to which adding 405, the numerator of the fractional part, the sum 32405 is the numerator to 1000 for an improper fraction equal to the given mixed number.

32000
405
———
32405

CHAP. II.

REDUCTION OF FRACTIONS.

To Reduce a Vulgar Fraction to a Decimal.

TO the Numerator annex a competent number of Cyphers; then divide by the Denominator and the Quotient will be the Decimal required. But *note* that the Decimal must always consist of as many Places as there are Cyphers annexed to the Numerator.

Examples.

1. What decimal is equal to $\frac{1}{8}$?
2. Reduce $\frac{1}{2}$ to a decimal equal thereto?
3. What is the decimal equal to $\frac{3}{4}$?
4. What is the decimal equal to $\frac{1}{5}$?
5. What decimal is equal to $\frac{11}{16}$?
6. Reduce $\frac{12}{25}$ to a decimal?
7. What decimal is equal to $\frac{21}{32}$?
8. Bring $\frac{17}{20}$ to a decimal?

If the quotient doth not consist of as many figures as there are cyphers annexed, &c. make up the deficiency by putting cyphers to the left hand of the said quotient.

Application.

5|0)20|0
———
.04
———

Let $\frac{2}{50}$ be reduced to a decimal. I annex two cyphers, and the quotient results 4, which beingone figure less thant he cyphers annexed, I put a cypher to the left hand to make up the deficiency.

What

9. What decimal is equal to $\frac{3}{32}$?		
10. What decimal is equal to $\frac{1}{64}$?		
11. What decimal is equal to $\frac{5}{128}$		
12. Reduce $\frac{1}{3}$ to a decimal?	*Answ.*	.33333 &c.
13. Bring $\frac{11}{15}$ to a decimal?	——	.73333
14. What decimal is equal to $\frac{13}{15}$?	——	.86666
15. Reduce $\frac{3}{11}$ to a decimal?	——	.272727
16. Reduce $\frac{1}{27}$ to a decimal?	——	.037037
17. Reduce $\frac{11}{14}$ to a decimal?	——	.785785
18. Bring $\frac{1}{13}$ to a decimal?	——	.07692307 &c.
19. Reduce $\frac{5}{26}$ to a decimal?	——	.19230769 &c.
20. Bring $\frac{4}{7}$ to a decimal?	——	.57142857 &c.

Problem II.

To reduce the denominations of money, weights and measures to decimals.

Bring the given denominations first to a Vulgar fraction, and reduce said vulgar fraction to a decimal.

21. Reduce 11*s.* to the decimal of a pound? *Answ.* .55

22. What decimal is equal to 8*s.* the whole being 1*l.*? *Answ.* .4

23. Reduce 15*s.* to the decimal of a pound? *Answ.* .75

24. Reduce 8*d.* to the decimal of a shilling? *Answ.* .66666

25. Reduce 8*d.* to the decimal of a pound? *Answ.* .03333

26. What is the decimal of 5 ounces, 1℔. Troy *wt.* being the integer? *Answ.* .41666

27. What decimal of a *Cwt.* is 6℔ *Answ.* .0535714

Numbers

Numbers of Divers Denominations.

s. d.
Bring 17 6 to the decimal of a pound.
12

21|0
3)——(⅞ equal to 17 6 s. d.
24|0

8) 7.000
.875

Or
24|0) 210.00|0 (.875
192
180
168
120
120

Or thus,

Let 17*s.* 6*d.* be again given to be reduced to the decimal of a pound.

12) 6|0
2|0)17.50|0
.875,

Annex a cypher to 6*d.* and divide by 12, (the pence in a shilling) the quotient is 5, to which prefix 17, and divide 17.5 by 20 (the shillings in a pound) adding cyphers till the quotient results without remainder and the decimal required is found, .875 as before.

28. What decimal of a yard is equal to 3 *qrs.* 2 *na?*
Answ. .875.

29. Bring 1 *hhd.* 21 *gal.* 4 *pts.* to the decimal of a tun?
Answ. .335317.

30. What decimal of a pound is equal to 6*oz.* 14 *dwt.* 10*gr.* Troy? *Answ.* .56006944.

31. What decimal of a pound is 17*s.* 4½*d.*?
Answ. .86875.

32. What decimal of a *Cwt.* is equal to 3 *qrs.* 21 ℔.?
Answ. .9375.

33. What decimal of a shilling, and of a pound, is 9¾*d.*?
Answ .8125*s.* and .040625*l.*

But the decimals of sterling money may be wrote in one line by the following

Rule.

1. Write half of the greatest even number in the place of shillings, for the first decimal figure.

2. Let the farthings in the given pence and farthings possess the second and third decimal places; observing to encrease the place of hundreds by 5, if the shillings are odd: and to encrease the thousands by as many units as there are times 24 in the pence and farthings.

3. If more than three places are needful, then divide half the number of farthings in the pence and farthings, (rejecting 24 or 6*d.* if there is one) by 12 the quotient written after three places before found will give the decimal required.

Examples.

		s.	d.
1	———	10	8
2	———	13	$10\frac{1}{2}$
3	———	15	$9\frac{3}{4}$
4	———	19	$11\frac{3}{4}$
5	———	1	$10\frac{1}{2}$
6	———	0	$8\frac{3}{4}$
7	———	0	$2\frac{1}{2}$
8	———	0	[illegible]$\frac{3}{4}$

Problem. III.

To find the value of a decimal of a superior denomination in the known parts of the lesser denomination.

Multiply the decimal by that number which one of the higher denomination contains of the lesser, and from the product point off as many places to the right hand as there are in the given decimal, and the figures on the left of the separating point will be the number of the said lesser denomination, and those on the right a decimal thereof, of which find the value as before; and so from denomination to denomination till the lowest be arrived at, or till the figures cut off be all cyphers.

Application.

Application.

What is the value of .875*l.*?

I multiply the decimal .875 by 20, (the shillings in a pound) and find the product 17.500, from which I point off the three lowest figures (being the number in the given decimal) by the separatrix, and so it stands 17*s*. .500, which 500 is a new decimal part of a shilling, which I multiply by 12 (the pence in a shilling) and find its value 6*d*.; and therefore the value of the given decimal is 17*s*. 6*d*.

.875
20
s. 17.500
12
d. 6.000
Answ. 17*s*. 6*d*.

35. What is the value of .75*l*.? *Answ.* 15*s*.

36. What is the value of .66666*s*.? *Answ.* 8*d*.

37. What is the value of .033333*l*.? *Answ.* 8*d*.

38. What is the value of .875 of a yard? *Answ.* 3*qrs*. 2*na*.

39. What is the value of .335317 of a tun?
Answ. 1*hhd*. 21*gals*. 4*pts*.

40. What is the value of .56006944 of a ℔. Troy?
Answ. 6*oz*. 14*dwt*. 10*gr*.

41. What is the value of .86875*l*.?
Answ. 17*s*. 4½*d*.

42. What is the value of .9375 of a *Cwt.*?
Answ. 3*qrs*. 21℔.

But the value of the decimal part of a pound sterling may be expressed in one line thus:

Double the first figure, or primes for shillings, and if the second be 5, or exceed 5, reckon one shilling more; the figures in the second and third places (rejecting 5 from the second) are so many farthings, abating 1 for every 24.

* *Note*, where the several figures cut off are 9, the value of the decimal is one more than the figure or figures on the left hand of the separatrix.

Application.

Application.

Let it be required again to find the value of .875*l.*

.875

———

s. 17 6

I say 8 and 8 is 16*s.* and 1 (for 5 in 7) makes 17*s.* Then 5 rejected out of 7 leaves 2 in the second place which will in the third account 25 farthings, and rejecting 1, (being above 24) there remains 24 farthings or 6*d.*

Examples.

43. Find the value of .92763*l.* *Answ.* 18*s.* 6½*d.*
44. ———————— .87638*l.* 17 · 6¼
45. ———————— .09937*l.* — — — 1 „ 11¾
46. ———————— .0428*l.* — — — 10¼
47. ———————— .0035*l.* — — — 2¼

CHAP. III.

ADDITION and SUBTRACTION of DECIMALS.

TO place the Numbers to be added or subtracted.

In placing the Numbers to be added or subtracted, whether mixt or pure Decimals, take care to place the figures of the same local Value directly underneath each other (as in pure whole numbers) *viz.* hundredths under hundredths, tenths under tenths, units under units, tens under tens, *&c.* So will all the separating Points be in one perpendicular row, but this may, or may not happen to the extreme Figures of mixt Numbers either to the right hand or to the left.

Rule.

To add or subtract Decimals or mixt Numbers.

Add or subtract them as if they were pure whole numbers, and from the sum or difference point off so many decimal parts as are the most in any of the given numbers.

Examples.

[1]	[2]	[3]
25.854	.5	.61271
34.578	.666	.8752
9.076	.75	.012
13.907	.4444	.875
45.070	.7	.05

[4]	[5]	[6]
42.5	178.025	32.375
65.666	4.1385	487.25
96.875	.71683	366.66666
25.9	.03675	296.078754
240.933	.825	430.5382427
178.35	.6125	608.3075
23.275	.333333	

7. Add together .7426, .846, 7.612, 5.5 and 12.0875.

8. Add 753.0375—— 246.38246——9724.28352——67482.063750——724.00003718——and 378.2375 together.

Examples.

	[1] Years.	[2] Days.	[3] Weeks.	[4] Hours.
From	1081.761	712.10009	127.19	12.
Take	10.00012	7.121	121.	.12

	[5] Minutes.	[6] Months.	[7] Ells.	[8] Tons.
From	174.1	6100.	.172618	761.8100
Take	1.471	6.109	.0000148	18.9112

CHAP. IV.

MULTIPLICATION of DECIMALS.

WHETHER the given Numbers be mixt Numbers or pure Decimals, multiply them as if they were all pure whole Numbers and when the product is found, Point off so many Decimal parts or places, as there are in both the Multiplicand and Multiplier, accounted together.

Examples

[1]	[2]	[3]	[4]
3.024	32.12	78.546	.5745
2.23	24.3	24.36	.06757

[5]	[6]	[7]
37.025	674.4375	92.075
5.275	27.368	3.00375

N. B. It will sometimes fall out that in multiplying decimals by decimals there will not be so many figures in the product, as the rule requires decimal places; in that case supply the defect by prefixing as many cyphers as places are wanting.

[8]	[9]	[10]	[11]
.2365	.0347	.857	.007853
.2456	.0236	.025	.035
.05808440			

[12]	[13]	[14]
.03246	.83649	3.141592
.02364	.03687	52,7438
.0007673544	.0308413863	165,6995001296

To multiply by 10, 100, 1000, *&c.* remove the separating point as many places to the right hand as the multiplier hath cyphers.

$$\text{So } .125 \left.\right\} \text{multip. by} \left\{\begin{array}{r}10\\100\\1000\end{array}\right\} \text{makes} \left\{\begin{array}{l}1.25\\12.5\\125.\end{array}\right.$$

For 125 × 10 is 1 250

To multiply finite or approximate decimals, so that the product shall consist of no more than a determinate number of places.

Rule

Under that place in the multiplicand, thought necessary to be retained in the product, write the units place of the multiplier, and invert the order of all its other places; that is, write the decimals on the left, and the integers (if any) on the right; in multiplying, omit those places in the multiplicand which stand to the right of the figure multiplied by, and let the right hand place of every line stand under each other.

In each line let the lowest place be encreased by the carriage which would arise from the omitted places, carrying 1 from 5 to 15, 2 from 15 to 25, 3 from 25 to 35, *&c.* instead of carrying 1 for every 10; and the sum of these lines will give a product generally exact.

Examples.

Multiply 384.672158 by 36.8345.

Now seeing there would be ten decimal places in the product whereof the greatest part are unnecessary: therefore to keep only four decimal places in the product.

```
384.672158
    5438.63
-----------
 115401647 .
  23080329 . .
   3077377 . . .
    115402 . . . .
     15387 . . . . .
      1923 . . . . . .
-----------
14169.2065
```

15. Multiply 3.141592 by 52.7438, so as to have only four decimal places in the product? *Answ.* 165.6995.

CHAP. V.

DIVISION of DECIMALS.

DECIMALS and mixt Numbers are in effect divided as whole Numbers, as shall be set forth in the sundry cases following:

I. *General*

I. *General Rule for pointing the Quotient.*

The places of decimal parts in the divisor and quotient, counted together, must be always equal to those of the dividend, therefore point off in the quotient as many figures as are the excess of decimal places in the dividend above those in the divisor.

Application.

Let B divide A and find the quotient C, now first let B have two decimal places and A six, the excess of six above two is four the decimal places to be pointed off in C; or if B hath two and A four; the excess is two to be pointed off in C.

```
    B        A        C
 5.73)2.580219( 4503
      2.292
      -----
        2882
        2865
        ----
         1719
         1719
         ----
    B        A        C
 5.73)258.0219(45 03
```

Case 1. When the places of decimals in the dividend and divisor are equal, the quotient will be an unmixed whole number.

Examples.

[1] 8.45)295.75(

[2] .0078).4368(

[3] .7563)59062.4922

Case 2. When the places of decimals in the dividend are most, cut off the excess for decimal parts in the quotient.

[4] 24.3)780.516(

[5] .436)34246.056(

[6] .534) 30438(

Case 3. When there are not so many places of decimals in the dividend as the divisor, annex cyphers to the dividend to make them equal; then will the quotient be an unmixed whole number.

Case 4. If after the division is finished, there are not as many figures in the quotient, as there ought to be decimal places, (by the general rule) supply the defect by prefixing as many cyphers as there are places wanting.

[7] .957)7.25406([8] .525).0007875(
[9] .43).13975([10] 45.)3.9375(
[11] .00875)38.075(

Note. When decimals or whole numbers are to be divided by 10, 100, 1000, *&c. viz.* unity with cyphers, it is performed by removing the separatrix in the dividend so many places towards the left hand, as there are cyphers in the divisor.

Examples.

$$\left.\begin{array}{r}10\\100\\1000\\10000\end{array}\right\}\text{Dividing}\left\{5784\right\}\text{the qu. is}\left\{\begin{array}{l}578.4\\57.84\\5.784\\.5784\end{array}\right.$$

When the divisor doth not measure the dividend, we may approach as near the truth as we please by annexing cyphers continually, to the dividend, or the remainder: but six or seven places of decimals are generally esteemed sufficient, for the quotient differs from the truth by less than $\frac{1}{1000000}$ of a unit.

[12] .6252)14250([13] 48.25)874.86(
[14] 52.125)2.34([15] .056)82(

Division may be contracted as follows:

Rule.

Let each remainder be a new dividend, and for each such new dividend point off one figure from the right hand of the divisor, observing at each multiplication to have respect to the increase of the figures so cut off, as in multiplication.

Examples.

```
384.672158)14169.206623851(36.8345
    ...... 11540 16474
           -----------
             262904188 .
             230803295 .
             ---------
              32100893 . .
              30773772 . .
              --------
               1327121 . . .
               1154016 . . .
               -------
                173105 . . . .
                153869 . . . .
                ------
                 19236 . . . . .
                 19233 . . . . .
                 -----
[16] 9.365407)87.076326(9.297655
```

CHAP. VI.

The SINGLE RULE of THREE in DECIMALS.

REDUCE the Fractional part into Decimals of the highest name mentioned, then state the question and proceed as in the Rule of Three Direct, observing to point off the Decimal places as has been taught in Multiplication and Division of Decimals.

1. Suppose I give 6*s*. 3*d*. for 4¾ yards of cloth; what will 48½ yards of the same come to at that rate?
Answ. 3*l*. .1907 or 3*l*. 3*s*. 9¾*d*.

4.75 : 3125 :: 48.5 one L

2. If 2½ lb. of Tea cost 1*l*. 5*s*. what will 14¾ lb. come to at the same rate? *Answ*. 7*l*. .375 or 7*l*. 7*s*. 6*d*.

~~2.5~~ : ~~1.25~~ :: ~~lb.3~~ 14.75

2.5 : 1.25 :: 14.75 : 7.375

3. If 1℔. of sugar cost 11¾*d.* what will 4*hhds.* each weighing neat 4 *Cwt.* 2*qrs.* 14℔. cost at the same rate? *Answ.* 101*l.* .4417 or 101*l.* 8*s.* 10*d.*

00892858 : .489583 :: 18.5

4. A grocer buys 4 chests of tea, each weighing neat 2*Cwt.* 3*qrs.* 14℔. for 906*l.* 10*s.* what rate did he give ℘ ℔.? *Answ.* .7038 or 14*s.* 0¾*d.*

11.5 : 906.5 : 00892858

5. An oilman bought 4 tuns, 201½ gallons of Florence oil for 240*l.* 16*s.* 6*d.* but by misfortune it chanced to leak out 24½ gallons: I desire to know at what rate he may sell the remainder ℘ gallon to be no loser? *Answ.* .20322 or 4*s.* 0¾*d.* 1185 : 240.825 :: 1

CHAP. VII.

OF CIRCULATING DECIMALS.

THE following method of managing Circulating Decimals, being taken notice of in few Books of Arithmetic that I have seen, I chose to deliver it by itself, detached from the common doctrine of Decimals, before laid down.

And first it will be proper to shew how to multiply and divide by 9, 99, 999, *&c.* in a contracted and very easy way.

I. To multiply by 9, 99, 999, *&c.*

Write as many cyphers as there are nines in the multiplier to the right hand of the multiplicand, and from the result subtract the multiplicand and the remainder will be the product.

1. Let it be required to multiply 456 by 9, *&c.*
One cypher added to 456 makes 4560 $=456\times10$
From which subtract the multiplic. 456 $=456\times1$

The remainder is the product 4104 $=456\times10-1$

2. Two cyphers added to 456 make $45600 = 456 \times 100$
From which subtract — — $456 = 456 \times 1$

The remainder is the product $44144 = 456 \times 100 - 1$

468 by $9 = 4212$, $3726 \times 99 = 368874$, $7568 \times 999 = 7560432$

II. To divide by 9, 99, 999, *&c.*

Divide the given dividend into periods of as many places as there are 9's in the divisor, beginning from the left hand, and annex as many cyphers to the right hand of the number as may be wanted to complete a period.

Then write the figures of the left hand period, under the next to the right hand, add these together, and place their sum under the third period, (if the sum amount to more figures than are in a period, the highest will of course fall under the lowest place of the second period.) In like manner add this sum to the period, and place the result under the fourth and so on: Lastly, under the last figures place that figure, which would have been placed there (if any) suppose the work had been to proceed a period farther.

Add them all together; and cancel as many figures as there were cyphers annexed to the dividend; then from the figures that remain, cut off with a comma, from the right hand towards the left, as many figures as the divisor contains nines; so shall the figures to the left of the comma be the quotient, and those on the right the remainder, which if it be all nines, add 1 to the quotient.

Application.

Let it be required to divide 87325 by 99?

The dividend with a cypher to make 3 periods, 87.32.50
The first period written under the second, — 87.
The sum of 87 and 32 is — — — — 1.19
1

882 070

Under the last place I set 1, because if 50 and 119 were added, the one would be so placed, the whole sum is 882.07, the last 0 being cancelled for 0 added, *i. e.* 882 the quotient and 7 remainder.

Those decimals which are produced from vulgar fractions whose denominators measure their numerators with cyphers annexed are called finite or terminate decimals, because they consist of a determinate number of places.

Decimals, (produced from vulgar fractions, whose denominators do not measure their numerators) in which a figure is repeated continually, or in which the same figures circulate continually, are called *Circulating Decimals*, and the circulating figures are called *Repetends*, and if one figure only repeat, it is called a *Single Repetend*, as .1111. .3333.

To avoid the trouble of writing down unnecessary figures, a single repetend is denoted by a point (′) over the repeating figure, *viz.* the decimal .11111 is expressed by .1′, .3333 by .3′.

If other figures rise before the repeating figure, as 16′ $=\frac{1}{6}$ .0833, or .083′ $=\frac{1}{12}$; .06 $=\frac{1}{15}$; such decimals are called *mixed single Repetends.*

Such as have figures circulating alternately, or every third, fourth, &c. the same, are called *compound Repetends*, such as .410101, .123123123123.

And if other figures arise before the figures which circulate, then the decimal is called a *mixed compound Repetend.*

Note. Mixt Repetends, single or compound, may be called *mixt Circulates.*

Compound Repetends are distinguished by a point over the first and last repeating figure: Thus, .010101 may be written .0′1′, and .123123123.1′23′. .15656, .1′56′.

As in multiplying and dividing by these in perfect decimals, it requires frequently that the decimal must be extended to a pretty large number of places to prevent a very considerable error resulting from their imperfection; to remedy this, and to make the result perfect with less trouble, it will be useful to consider their generation.

Now as 9 in 10 is contained once and one remains, unity with cyphers annexed being divided by 9 *ad infinitum*, the quotient figures will still be 1, *i. e.* $\frac{1}{9}$, which being reduced to a decimal, will produce the circulating decimal .1′: and since .1′ is the decimal equivalent to $\frac{1}{9}$ .2′ will be equal to $\frac{2}{9}$; .3′ to $(\frac{3}{9}=)$ $\frac{1}{3}$; .4′ to $\frac{4}{9}$; .5′ to $\frac{5}{9}$; .6′ to $(\frac{6}{9}=)$ $\frac{2}{3}$; .7′ to $\frac{7}{9}$; .8′ to $\frac{8}{9}$; and .9′ to $(\frac{9}{9}=\frac{1}{1}=)$ 1.

Therefore every single repetend is equal to a vulgar fraction whose numerator is the repeating figure and denominator 9.

```
99) 1.0000 (.0101
    99
    ----
     100
      99
    ----
       1, &c.

999) 1.000000 (.001001
      999
      ----
      1000
```

Again, $\frac{1}{99}$ being reduced to a decimal makes .010101, &c. and $\frac{1}{999}$ makes .001001001, &c. or $\frac{1}{99}$=.0′1′. $\frac{1}{999}$=.0′01′; now every compound repetend of two figures will be some multiple of .0′1′ and the same multiple of $\frac{1}{99}$ the vulgar fraction equal thereto, that is the vulgar fraction whose numerator is the two repeating figures, and the denominator 99. That is $\frac{1}{99}$; = .0′1′; $\frac{2}{99}$; = .0′2′ $\frac{3}{99}=(\frac{1}{33}=)$.0′3′; $\frac{9}{99}=(\frac{1}{11}=)$.0′9′, $\frac{27}{99}=(\frac{3}{11}=)$.2′7′.

In like manner every compound repetend of three figures is shewn to be produced from a vulgar fraction, whose numerator is the three repeating figures, and denominator 999.

And so universally, we may conceive that a decimal fraction, consisting only of a repetend, is equal to a vulgar fraction whose numerator is that repetend, and the denominator a number consisting of as many nines, as there are places in the repetend.

Next to find a vulgar fraction equal to a mixt circulate, consider the next circulate as divisible into its finite and circulating parts, *viz.* the mixt circulate .16′ divisible into the finite decimal .1, and the repetend . 6′; .083′ into .08 and .3′; and .462′3′ into .46 and 2′3′.

Now .16 being thus divided into the parts .1 and 6′, the vulgar fraction = .1 is $\frac{1}{10}$ and the vulgar fraction = 6′ is $\frac{6}{9}$ provided it were a pure circulate, that is provided the circulation began immediately after units place; but as it begins after the place of tenth parts, it is $\frac{6}{9}$ of 1 of the preceding place, *viz.* $\frac{6}{9}$ of $\frac{1}{10}$ *i. e.* $\frac{6}{90}$. So the vulgar fraction = .16 is $\frac{1}{10}+\frac{6}{90}=\frac{6}{99}=\frac{15}{90}$.

Again, in .083′ .08 = $\frac{8}{100}$ and 3′ = $\frac{3}{9}$ of 1 of the preceding place, *viz.* $\frac{6}{9}$ of $\frac{1}{100}=\frac{3}{900}$. So likewise .083′ =

$\frac{8}{100}+\frac{3}{900}=\frac{72}{900}+\frac{3}{900}=\frac{75}{900}$.

And, .462′3′ may be divided into $\frac{46}{100}+\frac{23}{99}$ of $\frac{1}{100}=\frac{23}{9900}$. So the vulgar fraction = .462′3′ is $\frac{46}{100}+\frac{23}{9900}=\frac{4554}{9900}+\frac{23}{9900}=\frac{4577}{9900}$.

To reduce a mixt circulate to a vulgar fraction equivalent thereto.

Rule.

From the given mixt circulate deduct the finite part for a numerator. And the denominator of the repetend, with as many cyphers annexed as there are places in the finite part of the decimal, is the denominator.

A mixt number whose fractional part is either a pure or mixt circulate, is reduced to an improper fraction by the same rule, as will appear by the following

Examples.

1. Reduce 36.7′ to a vulgar fraction.

$$\begin{array}{r} \text{From } 36.7' \\ \text{Take } 36. \\ \hline 331 \\ \hline 9 \end{array}$$

For $36\frac{7}{8} = \dfrac{36\times9+7}{9}$ *i. e.* $\dfrac{467-36}{9}$

2. Reduce 3.8'42' to a vulgar fraction?
Answ. $\frac{3839}{999}$.

3. Reduce 57.7' to a vulgar fraction?
Answ. $\frac{520}{9}$.

4. Reduce 4275'.84 to a vulgar fraction?
Answ. $\frac{42754200}{9999}$.

Those repetends which consist of the same number of places are called *similar,* thus .1'23' and .735'14' are similar repetends.

Similar repetends which begin at the same place are said to be *conterminous.*

A single repetend may put on the form of a compound repetend, without altering its value, thus .4' may be written 4'4', or .4'44'; or .4'444' for $\frac{4}{9}=\frac{44}{99}=\frac{444}{999}=\frac{4444}{9999}$.

Hence any given repetend may put on the form of another repetend, if the number of repeating figures in the latter be a multiple of the number of repeating figures in the former; thus the repetend .4'5' may receive the forms of repetends consisting of 4, 6, 8, *&c.* figures, *i. e* .4'5'=.4'54'=.4'54545', *&c.*

Any two or more dissimilar repetends may be made similar by transforming them into other repetends, which shall consist of as many figures as the least common multiple of the several number of places found in all the repetends, contains units.

Example. I.

Dissimilar	Made similar
0, 7'	0, 7'7'
0, 5'4'	0, 5'4'

Example II.

Dissimilar	Made similar
0, 4'75'	0, 4'75475'
0, 3'24'	0, 3'242424'
0, 5'9'	0, 5'95959'
0, 327'	0, 32777777'
0, 1'	0, 1'11111'

In the last example 6 is the least common multiple and therefore the similar repetends must each consist of six places.

A pure repetend may put on the form of a mixt repetend, thus, 0, .4′5′ may be written 0, 4′545′ or 0, 4545′4′, For .4′54′ $= \frac{4}{10} + \frac{54}{99}$ of $\frac{1}{10} = \frac{4}{10} + \frac{54}{990}, = \frac{396}{990} + \frac{54}{990} = \frac{450}{990} = \frac{45}{99}$, &c.

Hence any two or more similar repetends may be made conterminous, *i. e.* may begin at the same place.

Examples.

Make 0, 40′6′, and 0,735′14′ conterminous.

This will be performed by making 0,4′06′ put on the same form with 0, 735′14′, *viz.* that of a mixt repetend having two finite places, thus, .406′40′.

Example II.

Similar Repetends	Made conterminous
0, 475475′	0, 4′7547547
0, 3′242424′	0, 324′24242′
0, 5′95959′	0, 59′595959′.
0, 327′7777′	0, 3277′7777′
0, 1′11111′	0, 11′111111′

CHAP. VIII.

ADDITION OF CIRCULATING DECIMALS.

To add Decimals which have single Repetends.

Rule.

MAKE the Repetends conterminous; then add up the right-hand column, and carry 1 for every 9 in the sum, and the overplus above the nines, put down as a Repetend in that place, the rest of the work, is the same as in *Addition of finite Decimals.*

Application.

Numbers proposed	Made conterminate.
2, 3'	2, 33'
2, 7'	2, 77'
4, 76'	4, 76'
0, 3'	0, 33'
5, 8'	5, 80'
4, 73'	4, 73'
	0, 74'

In this example the sum of the right-hand figures is 22, the 9's in 22 is twice 9 = 18 and 4 over, I put down 4 and carry 2: The reason is evident.

Other Examples.

[2]	[3]	[4]
4, 724'	3, 04'	25, 3'
28',	8, 456	18, 04
3',	23',	3',
25, 26'	0, 2'48'	29; 123'
18, 7'	33, 8'	16, 6'
5'	0', 8'	13, 02'

To add decimals which have compound repetends.

Rule.

Make the repetends similar, and conterminous (and add the decimals); then to the right-hand figure of the sum add as many units as are carried from that column of figures, wherein all the repetends begin together: Lastly point off for a repetend, as many places as were so in the numbers added together.

Numbers proposed	Made similar and conterminous
1'62'	162, 1'62162'
2, 9'3'	2, 939393
172',	172, 222222
3, 7'69230'	3, 769230
5.	5, 000000
	346, 093007
	2
	346, 0'93009'

Having made the given repetends similar and conterminous, I add the decimals together, and find the column where the repetends begin together amounts to 20, wherefore I must carry 2, which I add to the last figure of the sum and find the repetend .0'93009'.

Reason.

By making the repetends similar and conterminous their value is not altered; but the decimals properly completed, (being as it were reduced all to the same denomination) next the reason why we add to the lowest figure the number which is carried from the column, wherein all the repetends begin together, is because if the circulation were continued the very same figures would be repeated in the next succeeding column, and consequently the sum would be the same, and the same number to be carried to that which is now the lowest place.

Other Examples.

[5]	[6]	[7]
134 0'9	67 34'5'	267 .3456
97 26	8 .6'21'	33 .8'
99 .03'3	0 .2'4'	0 6'7'2
1 .5	0 .8'	44 .8'725'
0 .8'14'	75 .75	27 .3'9'

CHAP. IX.

SUBTRACTION OF CIRCULATING DECIMALS.

Rule.

MAKE the repetends similar and conterminous, and subtract them as finite Decimals, observing only when the Repetend of the number to be subtracted is greater than the Repetend it is to be subtracted from, the right-hand figure of the remainder must be made less by 1.

Note. The repentend in the remainder will consist of as many places as those of the other two numbers.

Example I.

			Made conterminous.
[1]	From	110, 6'	110, 66666'
	Take	94, 1458'3	94, 14583

Example

Example II.

Made conterminous

[2] From 5, 03' 5, 033'3
Take 3, 0416' 3, 0416

1, 9916'

Reason.

Why 1 must be subtracted from the right-hand figure of the remainder (☞ Rule). By subtracting the repetend as a finite decimal, instead of adding the common denominator of the repetends, 9, 99, *&c.* to the upper repetend (when it is less) I add 10, 100, *&c.* which is always 1 more than the real common denominator whereby the remainder results 1 too much; wherefore I deduct that 1 from the remainder.

[3]	[4]	[5]	[6]
24'.38'4'	.7'42'	6 .5'7142'8	10 .5
9 .072	.4'18'	3. 6'428'	3 .4'5'

CHAP. X.

MULTIPLICATION of Circulating DECIMALS.

To multiply a finite Decimal by a Circulate.

REDUCE the Circulate to its equivalent Vulgar Fraction, and multiply the finite Decimal thereby.

Application.

1. Multiply 48.734 by 0.04'
4 '04 = $\frac{4}{90}$

90 } 10 { 194.936
9 { 19.4936 Quot.

Answ. 2.16595'

Note, 5 the remainder, being $\frac{5}{9}$ is a circulate.

2. Multiply 48.75' by 4. *Answ.* 195.02'
3. ——— 8.47' by .68 *Answ.* 5.7648'

4. Multiply 58.76'45' by 7.

```
From 58.76'45'
Take      .587
       --------
   .587058=     .76'45' 9990
   -------
     9990
```

```
        587058
             7
     -----------
     41.0940 6=10
    ---------------
999)410.940.600
        410.350
          1   1
    ---------------
 10)4103.519
    ---------------
     410.3519'
```

5. Multiply 48.76 by 0.1'345' *Answ.* 6.5599'03'.

6. Multiply 65.7'23' by 4.6 *Answ.* 302.326'8'.

To multiply a circulate by a circulate.

Reduce them both to their equivalent vulgar fractions, then multiply their numerators into each other, and divide that product first by the denominator of one, and then by the denominator of the other.

Application.

7. Multiply 7.684' by .45'

```
      6916
        41
   -------
      6916
     27664
   -------
9) 283.556
   -------
9) 31.5062
   -------
  3.60069, &c.
```

7.684 reduced to its equivalent vulgar fraction is $\frac{6916}{900}$ and .45=$\frac{41}{90}$. I multiply the numerators and find the product 283556' I point off 3 decimal places, and then divide by 9 and by 9, the significant figures of the denominators.

8. Multiply 2.3' by 5.6' *Answ.* 13 2'.

9. Multiply 1.1' by 1.1' *Answ.* 1.2'34567900'.

10. Multiply 3.14'5' by 4.297' *Answ.* 13.5'169533'.

CHAP.

CHAP. XI.

DIVISION OF CIRCULATING DECIMALS.

I. *To Divide a Circulate by a Finite Decimal, &c.*

DIVIDE as if they were both Finite Decimals carrying on the operation by bringing down the figures of the Repetend (instead of cyphers) so oft till either the Quote Circulate; or till a sufficient number of places result.

1. Divide 195.02′ by 4.

4)195.022, &c.

48.755, &c.=48.75′

2. Divide 46.528′7 by 8. *Answ.* 5.81609′1′
3. ——— 5.7648′ by 68. *Answ.* 8.47′.
4. ——— 6.5599′03′ by 48.76 *Answ.* 13′45′.

II. If the divisor be a circulate, make the repetends of the divisor and dividend similar. Then instead of them using the numerators of their equivalent vulgar fractions, and bringing them to one common denominator, divide the numerator of the dividend by the numerator of the divisor as finite decimals.

Explanation and Reason.

5. Divide 2.16595′ by 4′.

.04)2.16595′
0 21659
4)19.4936

4.8734

The mixt number 2.16595′ having the figure 5′ a repetend is given similar to the divisor. Then by subtracting the finite part I reduce it to its equivalent vulgar fraction $\frac{194936}{90000}$. Then by making the decimal places equal in number to the cyphers of the denominator 90000, I reduce it to a mixt-fraction whose denominator is 9, the same with that of the numerator: So I divide $\frac{19.4936}{9}$ by $\frac{4}{9}$.

Note. The decimal places in the divisor and dividend will still be equal to the cyphers in their denominators.

6. Divide 54 by .17′ *Answ.* 303.75.
7. ——— 411.35′19′ by 58.76′45′ ——— 7.
8. ——— 9 by 4′5′ ——— 19.8
9. ——— 1′3.2 by 5.6′ ——— 2.3′
10. ——— 1.23456790′ by 1.1′ ——— 1.1′
11. ——— 13.516′9533′ by 4.297 ——— 3.1′45′

BOOK

BOOK III.

MERCANTILE ARITHMETIC.

CHAP. I.

PRACTICE.

PRACTICE is a compendious way of finding the price of any Quantity of goods, having the price of 1 given.

If any aliquot Part be subtracted from its Whole, I call the remainder its complement, as *5s.* subtracted from 20*s.* of which it is an aliquot Part, the remainder is 15*s.* which I call the complement of *5s.* and the correspondent parts $\frac{3}{4}$ the complement of $\frac{1}{4}$.

Case I. *Of Multiples of* 1*l.* 1*s.* *or* 2*s.*

I. When the price is pounds only, or a multiple of 1*l.* multiply the quantity by the price, and the product is the answer in pounds.

The application and reason are evident.

1. What come 110C. of hops to, at 4*l.* ⅌ C.? *Answ.* 440*l.*

2. What come 227 yards of brocade to, at 3*l.* ⅌ yard? *Answ.* 681*l.*

3. What's the amount of 56 pieces of chintz, at 5*l.* ⅌ piece? *Answ.* 280*l.*

4. What's the amount of 27 ton of tallow, at 26*l.* ⅌ ton? *Answ.* 702l.

II. When

II.

When the price is shillings only.

1. If the price of 1 be 2*s.* then the price of any quantity is discoverable at sight, *viz.* by accounting the double of the units figure shillings; and the other figures of the given quantity pounds: So 278 yards at 2*s.* ⅌ yard, will cost 27*l.* 16*s.* for the double of 8 is 16, which write down apart as shillings, then 27 the other figures of the quantity are to be esteemed pounds; so the answer is 27*l.* 16*s.*

278
£.27 : 16

2. If the shillings be any even number.

Multiply the quantity by half the number of the given shillings, and double the units of the product for shillings, the other figures of the product are pounds.

Application.

Let it be required to find the price of 736 yards. at 4*s.* ⅌ yard. I multiply 736 by 2, (the half of 4*s.*) saying twice 6 is 12; the double of 2 (*viz.* the units in the product) is 4. to be set apart for shillings, keeping 1 in mind for one ten. Again, twice 3 is 6 and 1 I carry is 7; and so proceeding as in Multiplication, I find the answer 147*l.* 4*s.*

736
2
Answ. £.147 4

Examples.

6. What cost 256 gallons of shrub, at 6*s.* ⅌ gallon?
Answ. 76*l.* 16*s.*

7. What cost 984℔. of green tea, at 8*s.* ⅌ ℔.?
Answ. 393*l.* 12*s.*

8. What cost 120 C. of beef, at 10*s.* ⅌ C.?
Answ. 60*l.*

9. What must I give for 427 reams of paper, at 12*s.* ⅌ ream? *Answ.* 256*l.* 4*s.*

10. Sold 78*C.* of cheese, at 14*s.* ⅌ *Cwt.* what come they to? *Answ.* 54*l.* 12*s.*

11. At 16*s.* ⅌ yard, what cost 526 yards of broad cloth? *Answ.* 420*l.* 16*s.*

12. What come 156*C.* of rice to, at 18*s.* ⅌ *Cwt.*? *Answ.* 140*l.* 8*s.*

3. If the shillings be odd.

Find the amount of the even number less by 1 than the given price; and then take $\frac{1}{20}$ of the given quantity and add it to the said amount.

Examples.

13. At 3*s.* ⅌ yard what cost 184 yards of linen?

	184	
	18 8	for 2*s.*
1*s.* is $\frac{1}{20}$	9 4	
	£. 27 12	

14. What cost 924 barrels of barley, at 7*s.* ⅌ barrel? *Answ.* 323*l.* 8*s.*

15. At 9*s.* ⅌ stone, what cost 347 stone of wool? *Answ.* 156*l.* 3*s.*

16. At 11*s.* ⅌ stone, what cost 833 stone? *Answ* 458*l.* 3*s*

17. What come 129 *Cwt.* of iron to, at 13*s.* ⅌ *Cwt.*? *Answ.* 83*l.* 17*s.*

4. Or otherwise, multiply the quantity by the said shillings, and the product is the answer in shillings, which reduce to pounds.

18. What is the price of 924 barrels of barley at 7*s.*?
Answ. 323*l.* 8*s.*

19. At 9*s.* ⅌ stone, what cost 347 stone of wool?
Answ. 156*l.* 3*s.* &c.

III.

When the price is pounds and shillings.

Find the price for the pounds ⅌ Case I, and for the shillings by Case II, and add the two prices together.

20. What cost 178*C.* of sugar, at 2*l.* 4*s.* ⅌ *C.*?

178
2 2

l. *s.*
356 0 for 2 0 ⅌ Case 1.
35 12 for 0 4 ⅌ Case 2.

Answ. £.391 12

What cost 234, at 1*l.* 8*s.*
93 12

Answ. £.327 12

At 5*l.* 7*s.* ⅌, what cost 327?

327
5 7

1635
114 9

£.1749 9

327 at 7
7

228|9

114 9

Otherwise thus:

If the shillings be odd, bring the pounds and shillings into shillings, and find the price ⅌ Case 2, Part 2.

327 at 5 7
107 20

2289 107
327

2|0)3498|9

£.1749 9

21. What cost 275℔. of cochineal, at 1*l.* 12*s.* ⅌ ℔.?
Answ. 440*l.*

22. At 3*l.* 8*s.* ⅌ *Cwt.* what cost 124*Cwt.* of hops?
Answ. 421*l.* 12*s.*

23. What cost 237*C*, of sugar, at 2*l.* 13*s.* ⅌ *C.*?
Answ. 628*l.* 1*s.*

Case 2. *Of Aliquot Parts.*

In order to manage *Practice* by aliquot parts more readily and easily, it will be necessary for the learner to commit to memory the following.

PRACTICE TABLES.

Aliquot Parts of 1 *Pound.*

s.	*d.*		*l.*		*s.*	*d.*
10	0	is	$\frac{1}{2}$	Complem.	10	0
6	8	—	$\frac{1}{3}$	—	13	4
5	0	—	$\frac{1}{4}$	—	15	0
4	0	—	$\frac{1}{5}$	—	16	0
3	4	—	$\frac{1}{6}$	—	16	8
2	6	—	$\frac{1}{8}$	—	17	6
2	0	—	$\frac{1}{10}$	—	18	0
1	8	—	$\frac{1}{12}$	—	18	4
1	0	—	$\frac{1}{20}$	—	19	0
0	6	—	$\frac{1}{40}$	—	19	6
0	8	—	$\frac{1}{30}$	—	19	4
0	4	—	$\frac{1}{60}$	—	19	8
0	3	—	$\frac{1}{80}$	—	19	9
0	2	—	$\frac{1}{120}$	—	19	10

Aliquot Parts of 1 *Shilling.*

				d.
6	Pence	$\frac{1}{2}$	Complement	6
4	—	$\frac{1}{3}$	—	8
3	—	$\frac{1}{4}$	—	9
2	—	$\frac{1}{6}$	—	10
1$\frac{1}{2}$	—	$\frac{1}{8}$	—	10$\frac{1}{2}$
1	—	$\frac{1}{12}$	—	11

Aliquot Parts of 2 *Shillings.*

				s.	*d.*
8	Pence	$\frac{1}{3}$	Complement	1	4
6	—	$\frac{1}{4}$	—	1	6
4	—	$\frac{1}{6}$	—	1	8
3	—	$\frac{1}{8}$	—	1	9
2	—	$\frac{1}{12}$	—	1	10

If the price be an aliquot part of 1*l.*

Rule.

Divide the quantity by the denominator of the fraction expressing the part (as in Division of Money,) and the quotient is the answer.

Application.

24. What cost 375℔. at 10s. ⅌ ℔.

10s. | 375
is ½ | £.187 10

10s. is ½l. So I divide 375 the quantity, by 2 the denominator of ½ as already taught in *Division of Money*, and find the quotient 187*l.* 10*s.*

Examples.

25. At 6*s.* 8*d.* ⅌ yard, what is the amount of 337 yards of Holland? *Answ.* 112*l.* 6*s.* 8*d.*

26. What cost 1928 hats, at 5s. ⅌ ? *Answ.* 482*l.*

27. At 4*s.* ⅌ pair, what cost 726 pair of shoes? *Answ.* 145*l.* 4*s.*

28. What come 936℔. of coffee to, at 3*s.* 4*d.* ⅌ ℔.? *Answ.* 156*l.*

29. At 2*s.* 6*d.* ⅌ ℔. what cost 224℔.? *Answ.* 28*l.*

30. What cost 1755 pair of stockings at 2*s.* ⅌ pair? *Answ.* 175*l.* 10*s.*

31. At 1*s.* 8*d.* ⅌ yard, what cost 3127 yards of dowlass? *Answ.* 260*l.* 11*s.* 8*d.*

32. Bought 1584℔. of loaf Sugar, at 1*s.* ⅌ ℔. what comes it to? *Answ.* 79*l.* 4*s.*

II. If the price be the aliquot part of a shilling, divide the quantity by the denominator of the part, the quotient is shillings which divide by 20 to bring it to pounds.

At 6*d.* ⅌ ℔. what cost 112℔. of sugar?

6*d* | 112
½ | 2|0) 5|6
| £. 2 16

The reason is manifest.

33. At 4*d.* ⅌ quart, what must I give for 504 quarts? *Answ* 8*l.* 8*s.*

34. At 3*d.* ⅌ *oz.* what come 156*oz.* to? *Answ.* 1*l.* 19*s.*

35. At 2*d.* ⅌ yard what cost 758 yards of tape? *Answ.* 6*l.* 6*s.* 4*d.*

36. At 1½*d.* ⅌ ℔ what ⅌ *Cwt.* *Answ.* 14*s.*

37. What will 976℔. come to, at 1*d.* ⅌ ℔? *Answ.* 4*l.* 1*s.* 4*d.*

If the price be an even part of 2 shillings.

1. Find the amount of the quantity at 2 shillings, which amount divide by the denominator of the part, and the quotient is the Answer.

38. At 8*d.* ⅌ ℔. what cost 326℔.?

	l.	*s.*		
8*d.*	32	12 at 2*s.* ⅌?		
$\frac{1}{3}$	£. 10	17	4	*Answ.*

39 At 6*d.* ⅌ yard what cost 560 yards? *Answ.* 14*l.*

40. At 4*d.* ⅌ ℔. what cost 504℔. *Answ.* 8*l.* 8*s.*

41. At 3*d.* ⅌ ℔. what come 156 to? *Answ.* 1*l.* 19*s.*

Note. When the pence are an even part both of a pound and shilling. (⅌ Table) as 6 $\frac{1}{40}$*l.* $\frac{1}{2}$*s.*; 4 $\frac{1}{60}$*l.* $\frac{1}{3}$*s.*; the price may be very expeditiously and conveniently discovered as follows;

d. *l.* *s.*
6 is $\frac{1}{40}$—$\frac{1}{2}$——175|5 at 6
£43 17 6

d. *l.*
4 is $\frac{1}{60}$—$\frac{1}{3}$——175|5 at 4
£29 5

Explanation.

Explanation.

I divide the given quantity by 40, which denominates the part which 6*d.* is of a pound, and thereby get the pounds, and by dividing the remainder by $\frac{1}{2}$ the part which 6*d.* is of 1*s.* I find the shillings and pence, if any, *viz.* 4|0 dividing 1755 the quotient is 43 and the remainder 35; I say $\frac{1}{2}$ 35*s.* is 17*s.* and 1 remains, which I multiply by 12 makes 12*d.* the $\frac{1}{2}$ whereof is 6*d.*: And so I find the answer 43*l.* 17*s.* 6*d.*

After this manner let the Examples be wrought.

Case III. *Of the Complements.*

I. When the price is the complement of an aliquot part of 1*l.*

Rule.

Find the amount of the aliquot part whereof it is the complement (℘ last) and subtract the same from the quantity taken as so many pounds, the remainder is the answer.

Application.

42. Let it be required to find the price of 713℔. at 13*s.* 4*d.* ℘ .?

s.	*d.*		*l.* 713	at	13	4
13	4	Comple. $\frac{1}{3}$	—237		13	4
			£.475		6	8

43. At 15*s.* ℘ *Cwt.* what cost 336*C.* of logwood? *Answ.* 252*l.*

44. How much cost 194 yards of broad cloth, at 16*s.* 8*d.* ℘ yard? *Answ.* 161*l.* 13*s.* 4*d.*

45. What cost 479 yards of velvet at 17*s.* 6*d.* ℘ yard? *Answ.* 419*l.* 2 6*d.*

46. At 18*s* 4*d.* ℘ *Cwt.* what cost 95 *Cwt.* of rice? *Answ.* 87*l.* 1*s.* 8*d.*

47. At 19s. ⅌ *Cwt.* what come 317C. of butter to?
Answ. 301*l.* 3*s.* 0*d.*

The Complement of the Aliquot part of a Shilling.

48. At 9*d.* ⅌ ℔. what cost 1784℔ of tobacco?

Here 9*d.* is the complement of ¼*s.* I take ¼ out of 1784 taken as shillings, then the remainder is 1338*s.* &c.

1784
d. *s*
9 is Comp. ¼ ——446
2|0)133|8
£. 66 18

49. At 10*d.* ⅌ yard, what cost 540 yards of flannel?
Answ. 22*l.* 10*s.* 0*d.*

50. How much cost 7644℔. at 10½*d.* ⅌ ℔.?
Answ. 334*l.* 8*s.* 6*d.*

51. What is the price of 5649℔ of pepper at 11*d.* ⅌ ℔.? *Answ.* 258*l.* 18*s.* 3*d.*

III. *Of 2 Shillings.*

First find the amount of the quantity at 2*s.* ⅌ and the said amount divide by the denominator of the aliquot part, the quotient must be subtracted from the amount at 2*s.* and the remainder is the answer.

52. What cost 524 yards of linen at 1*s.* 4*d.* ⅌ yard?

l. *s.*
s. *d.* 52 8 Amount at 2 ⅌
1 4 is Comp. ⅓—17 9 4———at 0 8*d.*

Answ. 34 18 8———at 1*s.* 4*d.*

53. What cost 1648℔ of hops, at 1*s.* 6*d.* ⅌ ℔.?
Answ. 123*l.* 12*s.*

54. At 1*s.* 9*d.* ⅌ ell, what cost 729 ells of dowlass?
Answ. 63*l.* 15*s.* 9*d.*

55. What is the price of 1738 gallons of cyder, at 1*s.* 10*d.* ⅌ gallon? *Answ.* 159*l.* 6*s.* 4*d.*

Case IV. *Of Aliquot Parts, into Complements.*

Any aliquot part (of a pound or shilling) may be divided into several aliquot parts thus. Take the greatest aliquot part, less than the given aliquant part, out of the said aliquant part, then if the remainder be likewise an aliquot part, the thing proposed is manifestly effected: So if 5*d.* be the aliquant part of a shilling, given to be divided into aliquot parts, I know, (from the Table) that 4*d.*—$\frac{1}{3}$*s.* is the greatest aliquot part less than 5*d.* and 1*d.* remains, which is likewise an aliquot part, *viz.* $\frac{1}{12}$; So 7*d.* may be divided into 6—$\frac{1}{2}$*s.* and 1*d.*—$\frac{1}{12}$; 7$\frac{1}{2}$*d.* into 6—$\frac{1}{2}$*s.* and 1$\frac{1}{2}$*d.*—$\frac{1}{8}$*s.* &c. In like manner we may divide 7*s.* 6*d.* into 5*s.*—$\frac{1}{4}$*l.* and 2*s.* 6*d*—$\frac{1}{8}$*l.*; 12*s.* 6*d*, into 10*s.*—$\frac{1}{2}$*l.* and 2*s.* 6*d.*—$\frac{1}{8}$*l.*; 11*s.* 8*d.* into 10*s.*—$\frac{1}{2}$*l.* and 1*s.* 8*d*—$\frac{1}{12}$*l.* &c.

But if the remainder be not an aliquot part, then take again the * greatest aliquot part (less than it) out of the remainder, and so on, and thus at last we shall divide the given aliquant part into several aliquot parts, whose sum shall be equal to the said aliquant part: Thus we may divide.

9$\frac{3}{4}$	8$\frac{1}{2}$	11$\frac{3}{4}$	12 7$\frac{1}{2}$
6—$\frac{1}{2}$*s*	6—$\frac{1}{2}$*s.*	6—$\frac{1}{2}$*s.*	10 0—$\frac{1}{2}$*l.*
3—$\frac{1}{4}$	2—$\frac{1}{6}$	4—$\frac{1}{3}$	2 6—$\frac{1}{8}$
0$\frac{3}{4}$—$\frac{1}{16}$	0$\frac{1}{2}$—$\frac{1}{24}$	1—$\frac{1}{12}$	1$\frac{1}{2}$—$\frac{1}{160}$
		0$\frac{3}{4}$—$\frac{1}{16}$	

Thus it is shewn how to divide any aliquant part into aliquot parts of the whole. But as the least parts may sometimes

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* In some cases it may be more convenient to take a lesser aliquot part rather than the greatest (in the remainder,) as if 4*s.* 6*d.* were to be divided into aliquot parts of 1*l.*; the greatest aliquot part less than 4*s.* 6*d.* is 3*s.* 4*d.* and it will by the rule be divided into

s.	*d.*	
3	4	$\frac{1}{6}$
1	0	$\frac{1}{20}$
0	2	$\frac{1}{120}$

but is better divided

s.	*d.*	
2	6	$\frac{1}{8}$
2	0	$\frac{1}{10}$

Note. Expertness in the most commodious division is attained by *Practice.*

sometimes have denominators greater than 12, (which should be avoided that the division may be performed) we may observe that the lesser aliquot parts of the whole, are often likewise aliquot parts of some greater aliquot part before taken, and by considering them in this light we shall generlaly avoid large divisors: Thus may the foregoing aliquant parts be divided and considered as followeth;

d.	*d*	*d*	*s.* *d.*
$9\frac{3}{4}$ into	$8\frac{1}{2}$ into	$11\frac{3}{4}$ into	12 $7\frac{1}{2}$ into
6—$\frac{1}{2}$ of 1*s.*	6—$\frac{1}{2}$ of 1*s.*	6—$\frac{1}{2}$ of 1*s.*	10 0—$\frac{1}{2}$*l.*
3—$\frac{1}{2}$ of 6*d.*	2—$\frac{1}{3}$ of 6*d.*	4—$\frac{1}{3}$ of 1*s.*	2 0--$\frac{1}{10}$*l.*
$0\frac{3}{4}$ $\frac{1}{4}$ of 3*d.*	$0\frac{1}{2}$-$\frac{1}{4}$ of 2*d.*	1 -$\frac{1}{4}$ of 4*d.*	0 6—$\frac{1}{4}$ of 2*s.*
		$0\frac{3}{4}$-$\frac{1}{8}$ of 6*d.*	0 $1\frac{1}{2}$-$\frac{1}{4}$ of 6*d.*
$9\frac{3}{4}$	$8\frac{1}{2}$	$11\frac{3}{4}$	12 $7\frac{1}{2}$

I. When the price is an aliquant part of a pound.

First divide the given price into aliquot parts of a pound; then divide the given quantity by the denominator of each aliquot part successively; Thirdly, add all the quotients into one sum, and that sum is the answer required.

Application.

Let it be required to find the price of 198 yards of broad-cloth, at 14*s.* 6*d.* ℘ yard.

198

s.	*d.*		*l.*	
10	0	is	$\frac{1}{2}$—99	
2	6	is	$\frac{1}{6}$—24	15
2	0	is	$\frac{1}{10}$--19	16
			£. 143	11

Otherwise thus:

When a lesser part is an aliquot part of a greater taken before, divide the amount or quotient of the greater, by the denominator of the less, and so will the same quotient be found as by dividing the quantity as ℘ last.

Application.

Application.

Let the same example be repeated?

				198	
	s.	d.			
	10	0 is	½	99	
of 10s.—	2	6—	¼	24	15
of 10s.—	2	0—	⅕	19	16
				£.143	11

Examples.

1. *Of a Pound.*

57. How much is the amount of 4563 cobbs, at 4s. 9d. ⅌ piece? *Answ.* 1083*l.* 14*s.* 3*d.*

58. What must I give for 127 yards of cloth, at 9*s.* 8½*d.*? *Answ.* 61*l.* 12*s.* 11½*d.*

59. At 10*s.* 6*d.* ⅌ ℔. what cost 387℔. of tea? *Answ.* 203*l.* 3*s.* 6*d.*

60. As 9*s* 10*d.* ⅌ piece, what amount 139 to? *Answ.* 68*l.* 6*s.* 10*d.*

126 = 73 . 10 . . 2

61. At 12*s.* 3*d.* ⅌ *Cwt.* what come 120Cwt. to? *Answ.* 73*l.* 10*s.*

124 @ 12 . . 3 = 7[illegible] . [illegible]–13

62. At 15*s.* 6*d.* ⅌ *Cwt.* what will 721Cwt. come to? *Answ.* 558*l.* 15*s.* 6*d.*

63. At 19*s.* 11¾*d.* ⅌ *Cwt.* what cost 95Cwt.? *Answ.* 94*l.* 18*s.* 0¼*d.*

64. At 2*s.* 8½ ⅌ piece. what come 1784 to? *Answ.* 241*l.* 11*s.* 8*d.*

2. *Of a Shilling.*

Divide the aliquant parts into several aliquot parts and proceed as before; only the sum will be shillings, &c. which must therefore be divided by 20 to give the answer in pounds.

8729 at 5*d.*

4*d.* $\frac{1}{3}$*s.*—2909 8
1*d.* $\frac{1}{4}$ of 4*d.*— 727 5

2|0)363|7 1

Answ. £. 181 17 1

66. What cost 540 yards of canvas at 7*d.* ꝑ yard?
Answ. 15*l.* 15*s.*

67. What cost 229℔ of sugar at 7$\frac{1}{2}$*d.*?
Answ. 7*l.* 3*s.* 1$\frac{1}{2}$*d.*

68. At 9$\frac{3}{4}$*d.* how much cost 329℔. *Answ.* 13*l.* 7*s.* 3$\frac{1}{2}$*d.*

Case V. When the price is 1*l.* 1*s.* or 2*s.* and an aliquot or aliquant part of the same

Find the amount of the parts, and add the quotient or quotients together with the given quantity into one sum.

Examples.

Parts. *s.* *d*
337 yards at 6 8

6*s.* 8*d.* is $\frac{1}{3}$—112 6 8 *Answ.*

198 at 14 6

10*s*: is $\frac{1}{2}$ —99
2*s.* 6*d.* $\frac{1}{4}$ —24 15
2*s.* is $\frac{1}{10}$ —19 16

£.143 11 *Answ.*

378 at 8*d.*

37 16 at 2*s.*

8*d.* is $\frac{1}{3}$—2*s.* 12 12 *Answ.*

672 at 7*d.*

6*d*: is $\frac{1}{2}$—336
1*d.* is $\frac{1}{6}$— 56

2|0)39|2

£. 19 12

1 and a part, *l.* *s.* *d.*
1*l.*———337 at 1 6 8
6*s.* 8*d.* is $\frac{1}{3}$—112 6 8

£ 449 6 8

1*l.*———198 at 1 14 0
10*s.* is $\frac{1}{2}$ —99
2*s.* 6*d* $\frac{1}{8}$ —24 15
2*s.* is $\frac{1}{10}$ —19 16

£ 341 11

378 at 2*s.* 8*d.*

37 16 at 2 0
8*d.* is $\frac{1}{3}$—12 12 at 0 8*d.*

£.50 8 at 2 8

1*s.*———672 at 1*s.* 7*d.*
6*d.* is $\frac{1}{2}$—336
1*d.* is $\frac{1}{6}$— 56

2|0)106|4

£. 53 4

1; Of

1. *Of a Pound.*

69. What is the amount of 127C. of butter at 23*s.* 4*d.*?
Answ. 148*l.* 3*s.* 4*d.*

70. How many pounds in 125 guineas?
Answ. £. 131 5 0 English currency.
142 3 9 Irish.

71. At 1*l.* 7*s.* 6*d.* ⅌ *Cwt.* what cost 327Cwt. of merchandize? *Answ.* 449*l.* 12*s.* 6*d.*

72. At 1*l.* 13*s.* 8½ ⅌ *Cwt.* what cost 315C. of madder?
Answ. 530*l.* 18*s.* 1½*d.*

73. What is the amount of 180 moydores?
Answ. £. 243 0 0 English.
263 5 0 Irish

74. At 1*l.* 17*s.* 6*d.* ⅌ *Cwt.* what cost 536C. of sugar?
Answ. 1005*l.*

2. *Of 2 Shillings.*

If the price be 2*s.* and an aliquot or aliquant part thereof; find the price for 2*s.* and then take the proper parts out of the said price and add thereto.

75. What cost 378 yards of linen at 2*s.* 2*d.* ⅌ yard?
Answ. 40*l.* 19*s.* 0*d.*

76. At 2*s.* 4*d.* ⅌ yard, what for 384 yards?
Answ. 44*l* 16*s.*

77. At 2*s.* 5*d.* what cost 974? *Answ.* 117*l.* 13*s.* 10*d.*

78. How much cost 3782℔. of chocolate, at 2*s.* 8½*d.* ⅌ ℔? *Answ.* 512*l.* 2*s.* 11*d.*

79. At 2*s.* 9½*d.* ⅌, what cost 1245℔.?
Answ. 173*l.* 15*s.* 7½*d.*

3. *Of*

3. *Of a Shilling.*

80. At 1*s.* 1*d.* what 1275?
1*d.* is $\frac{1}{12}$—106—3
2|0)138|1—3
Answ. £. 69 1 3

81. At 1*s.* 5$\frac{1}{2}$*d.* what 3284?
3284
4 is $\frac{1}{3}$—1094—8
1 is $\frac{1}{4}$— 273—8
$\frac{1}{2}$ is $\frac{1}{2}$— 136—10
2|0)478|9—2
Answ. £.239 9 2

82, At 1*s.* 3*d.* ⅌ ℔. what cost 1785℔ of pepper?
Answ. 111*l.* 11*s.* 3*d.*

83. At 1*s.* 4*d.* ⅌ ell, what cost 852 ells?
Answ. 56*l.* 16*s.*

84. What come 2782℔. to, at 1*s.* 2$\frac{1}{2}$*d.*?
Answ. 168*l,* 1*s.* 7*d.*

85. At 1*s.* 8$\frac{3}{4}$*d.* ⅌ what cost 327?
Answ. 28*l.* 5*s.* 5$\frac{1}{4}$*d.*

86. At 1*s.* 10$\frac{1}{2}$*d,* what cost 439?
Answ. 41*l.* 3*s.* 1$\frac{1}{2}$*d.*

87. At 1*s.* 11$\frac{3}{4}$*d.* how much come 3864 to?
Answ. 382*l.* 7*s.* 6*d.*

Case VI.

When the price is any number of pounds or shillings, and an aliquot or aliquant part of the same.

1. *An Aliquot Part.*

88. What cost 317C. at 4*l* 3*s.* 4*d.* ⅌ *Cwt.*?
4
4—1268———⅌ Case 1.
3*s.* 4*d.* $\frac{1}{6}$—52 16 8 ⅌ ——4.
£. 1320 16 8

[89] 3785 at 3s. 6d.

3

11355

6*d.* is ½—1892 6

2|0)1324|7 6

£.662 7 6

90. What cost 324℔ at 6s. 4*d.* ⅌ ℔?
Answ 102*l* 12*s*

91. At 2*l.* 6*s.* 8*d.* ℔ Cwt. what cost 175C.?
Answ. 408*l.* 6*s.* 8*d.*

92. At 17*s.* 3*d.* ⅌ Cwt. what cost 78 C. of cheese?
Answ. 67*l.* 5*s.* 6*d.*

93. At 19*s.* 6*d.* ⅌ piece, what amount 1784 to?
Answ. 1739*l.* 8*s.*

94. At 4*s.* 8*d.* ⅌ yard, what cost 3752 yards of linen?

3752

375 4 for 2*s.*

2

750 8——for 4*s.* ⅌ Case I.

8*d.* of 2*s.* is ⅓——125 1 4

Answ. £. 875 9 4

95. At 6*s.* 4*d.* ⅌ what cost 324? *Answ.* 102*l.* 12*s.*

96. At 18*s.* 3*d.* what amount 59 to?
Answ. 53*l.* 16*s.* 9*d.*

2. *The Complement of an Aliquot Part.*

Rule.

Multiply the quantity by 1 more than the given number of pounds or shillings; then divide the quantity by the denominator of the fraction expressing the aliquot part whose complement is given, and subtract the quotient from the product.

Examples.

Examples.

C. l. s. d.
What cost 278 at 1 16 8 ⅌ Cwt.
2

l. s. d.
s. d. 556——for 2 0 0
16 8 is comp. $\frac{1}{6}$—46 6 8——0 3 4 Subt.

Amount to £.509 13 4 at 1 16 8 ⅌ Cwt.

The reason like that of Case III.

98. At 4s. 10½d. ⅌ ℔. what cost 3185℔. of coffee?
Answ 776*l.* 6*s.* 10½*d.*

99. At 2*l.* 17*s.* 6*d.* ⅌ Cwt. what cost 424 C. of raisins?
Answ. 1219*l.*

100. What is the price of 1847 yards of cloth, at 5*s.* 8*d.* ⅌ yard? *Answ.* 523*l.* 6*s.* 4*d.*

101. At 3*l.* 15*s.* ⅌ Cwt. what cost 172C. of hops?
Answ. 645*l*

102. How much is the amount of 4563 cobbs at 4s. 9*d.* ⅌? *Answ.* 1083*l.* 14*s.* 3*d.*

103. At 9*s.* 10*d.* ⅌ piece, what amount 139 to?
Answ. 68*l.* 6*s.* 10*d.*

3. *Any Aliquot Part.*

104. What cost 54 ton of iron, at 12*l.* 10s. 6*d.* ⅌ ton?

1. Thus,
Ton l. s. d.
54 at 12 10 6
12

£. 648 ⅌ Case 1.
10s. ½ 27 } ⅌ ½.
6d. $\frac{1}{20}$ 1 27 } Case 4

£. 676 7 *Answ.*

2. Or thus,
54 at 12 10 6
250 20

250 6
2700 } for 250s
108 }
27 for 6*d.*

13527

£ 676 7

3 Or thus,
54 at 12 10 6
6

75 3 0
9

676 7 0

Examples.

Examples.

105. What cost 540 yards of broad cloth, at 17*s*. 9*d*. ?
Answ. 479*l*. 5*s*.

106. What cost 313*C*. of cheese at 14*s*. 6*d*. ?
Answ. 226*l*. 18*s*. 6*d*.

107. At 11*s*. 11*d*. ⅌, what cost 731 ells ?
Answ. 435*l*. 11*s*. 1*d*.

108. At 3*l*. 17*s*. 5*d*. ⅌ *Cwt*. what come 517 *C*. of hops to ? *Answ*. 2001*l*. 4*s*. 5*d*.

109. What cost 108 *C*. at 5*l*. 13*s*. 8*d*. ⅌ ?
Answ. 613*l*. 16*s*.

110. At 4*l*. 9*s*. 10$\frac{3}{4}$*d*. ⅌ *C*. what cost 17 *Cwt*. ?
Answ. 76*l*. 8*s*. 2$\frac{3}{4}$*d*.

111. What is the price of 129 ton of butter at 19*l*. 19*s*. 4$\frac{1}{2}$*d*. ? *Answ*. 2575*l*. 19*s*. 4$\frac{1}{2}$*d*.

112. To how much come 48756 skains of worsted, at 1*l*. 4*s*. 6*d*. ⅌ 100? *Answ*. 597*l*. 5*s*. 2$\frac{16}{25}$*d*.

CHAP. I. PART II.

OF THE QUANTITY.

TABLES.

Aliquot Parts of 1 Cwt.							*Parts of $\frac{1}{2}$ Cwt.*				*Parts of $\frac{1}{4}$ Cwt.*		
qrs.	℔.		*Cwt.*		*qrs.*	℔.	*qr.*	℔.			℔.		
2	0	is	$\frac{1}{2}$	Complem.	2	0	1	0	is	$\frac{1}{2}$	14	is	$\frac{1}{2}$
1	0	—	$\frac{1}{4}$	—	3	0	0	14	—	$\frac{1}{4}$	7	—	$\frac{1}{4}$
0	16	—	$\frac{1}{7}$	—	3	12	0	8	—	$\frac{1}{7}$	4	—	$\frac{1}{7}$
0	14	—	$\frac{1}{8}$	—	3	14	0	7	—	$\frac{1}{8}$	3$\frac{1}{2}$	—	$\frac{1}{8}$
0	8	—	$\frac{1}{14}$	—	3	20							
0	7	—	$\frac{1}{16}$	—	3	21							

Case I.

When the quantity is a fraction; or of a lesser denomination, or denominations, and the price of 1 of a higher is given.

Rule.

Divide the given price of 1, by the denominator of the fraction, or aliquot part, when the less is,

1. *An Aliquot Part.*

1. What will $\frac{1}{4}$ of 1 C. come to, at the rate of 29*s*. 6*d*. ⅌ C.

s. *d*.
29 6

1 *qr*. is $\frac{1}{4}$—0 7 4$\frac{1}{4}$ *Answ.*

Reason.

The Reason is obvious from considering the solution by the Rule of Three, *viz*. 1 *C*:——29*s*. 6*d*.——$\frac{1}{4}$. For I have multiplied 29*s*. 6*d*. by $\frac{1}{4}$, and 1 divides not.

2. If a yard of cloth cost 8*s*. 6*d*. what cost $\frac{1}{2}$ yard?
Answ. 4*s*. 3*d*.

3. What cost 16℔. of any thing, at 3*l*. 13*s*. 6*d*. ⅌ C.?
Answ. 10*s*. 6*d*.

4. At 5*s*. 6*d*. ⅌ oz. what cost a pair of silver buttons weighing 5 penny-weight? *Answ*. 1*s*. 4$\frac{1}{2}$*d*.

5. What will 14℔. of sugar come to, at the rate of 45*s*. ⅌ *Cwt*.? *Answ*. 5*s*. 7$\frac{1}{2}$*d*.

6. What will 7℔. be worth, if 1 *Cwt*. cost 26*s*. 4*d*.?
Answ. 2*s*. 3$\frac{1}{4}$*d*.

2. *The Complement.*

Rule.

Subtract the quotient (found by dividing the price by the denominator of the part whereof it is the complement) from the price, and the remainder is the answer.

7. What will $\frac{3}{4}$ of a Cwt. come to at 29*s*. 6*d*. ⅌ Cwt.?

	29	6
$\frac{3}{4}$ Comp. is $\frac{1}{4}$—	7	4$\frac{1}{2}$

Answ. 22*s*. 1$\frac{1}{2}$*d*. or 1*l*. 2*s*. 1$\frac{1}{2}$*d*.

8. At 15*s*. 6*d*. ⅌ yard what cost $\frac{7}{8}$ of a yard? *Answ.* 13*s*. 6$\frac{3}{4}$*d*.

9. What cost 3qrs. 14℔. of sugar at 2*l*. 5*s*. 6*d*. ⅌ Cwt.? *Answ.* 1*l*. 19*s*. 9$\frac{3}{4}$*d*.

10. What cost 3qrs. 20℔. at 3*l*. 12*s*. 4*d*. ⅌ Cwt.? *Answ.* 3*l*. 7*s*. 2*d*.

11. At 5*s*. 6*d*. ⅌ ounce, what cost a pair of buckles weighing 15 penny-weights? *Answ.* 4*s*. 1$\frac{1}{2}$*d*.

12. At 17*s*. 6*d*. ⅌ yard, what cost 3 qrs. 2 nails? *Answ.* 15*s*. 3$\frac{3}{4}$*d*.

3. *Any Aliquot Parts.*

Rule.

Divide the aliquant part into several aliquot parts, and divide the given price by the denominators of these several aliquot parts successively: the sum of all the quotients will be the answer.

Examples.

13. If 1 Cwt. of madder be worth 2*l*. 10*s*. what is 3qrs. 16℔. worth?

qrs.	℔.		*l*.	*s*.				
3	16	at	2	10				
qrs.	℔.						qrs.	℔.
2	0	is $\frac{1}{2}$—	1	5		— Amount of	2	0
1	0	is $\frac{1}{4}$—	0	12	6	———	1	0
0	16	is $\frac{1}{7}$—	0	7	1$\frac{5}{7}$	———	0	16
Answ.		£.	2	4	7$\frac{5}{7}$	———	3	16

14. What

14. What will 2qrs. 14℔. come to if 1Cwt. cost 50*s.*?
Answ. 1*l.* 11*s.* 3*d.*

15. If 1Cwt. cost 46*s.* 6*d.* what cost 1qr. 14℔.?
Answ. 17*s.* $5\frac{1}{4}$*d.*

16. At 17*l.* 10*s.* 8*d.* ⅌ ton, what will 3C. 3qrs. cost?
Answ. 3*l.* 5*s.* 9*d.*

17. How much will 12 ounces of silk cost, if 1℔. cost 3*l.* 10*s.* *Answ.* 2*l.* 12*s.* 6*d.*

18. What will 3 qrs. 21℔. of tallow come to, at the rate of 27*s.* 8*d.* ⅌ Cwt.? *Answ.* 1*l.* 5*s.* $11\frac{1}{4}$*d.*

19. What will $7\frac{7}{8}$ oz. of spice come to at 12*s.* 8*d.* ⅌ ℔.? *Answ.* 6*s.* $2\frac{13}{16}$*d.*

20. What cost 3 qrs. 2 nails of velvet at the rate of 17*s.* 6*d.* ⅌ yard? *Answ.* 15*s.* $3\frac{3}{4}$*d.*

21. How much will 21℔. of sugar cost, if 1 Cwt. cost 54*s.* 4*d.*? *Answ.* 10*s.* $2\frac{1}{4}$*d.*

22. At 27*l.* 10*s.* 6*d* ⅌ Cwt. what cost $14\frac{3}{4}$℔.
Answ. 3*l.* 12*s.* $5\frac{221}{224}$*d.*

Case II.

If of divers denominations the price of 1 of the lower is given.

☞ Reduce them to the said lower denomination of which the price is given, and find the amount thereof by the former Rules.

C. qrs. ℔.
23. What cost 4 3 12 of pepper, at 1*s.* 3*d.* ⅌ ℔?
4
4
484
12
544 pounds at 1*s.* 3*d.*
3*d.* is $\frac{1}{4}$ 136
2.0)68|0
Answ. 34*l.*

24. At $9\frac{3}{4}$*d.* ⅌ ℔. what cost a hogshead of tobacco weighing 12C. 1qr. 25℔.? *Answ.* 56 l. 15s. $0\frac{3}{4}$d.

25. What cost 12℔. 10 oz. of silver at 5s. ⅌ ounce? *Answ.* 38 l. 10s. 0d.

26. What will 224℔. 6 oz. of spice come to, at the rate of 3s. ⅌ ounce? *Answ.* 538 l. 10s. 0d.

27. 12 hhds. 36 gal. of brandy, at 4s. per gallon? *Answ.* 158 l. 8s. 0d.

28. If 1℔. of any thing cost 1s. 9d. what will 7C. 2qrs. 26℔. cost? *Answ.* 75 l. 15s. 6d.

Case III.

When the price of 1 of the highest is given.

For the number of the highest find the amount by the Rules of Part I. and for the lower as in Case I. of this, and add the amounts together.

Examples.

29. What will 332C. 2qrs. 22℔. come to, at the rate of 18s. 6d. ⅌ Cwt.

C. qrs. ℔. s. d.
332 2 22 at 18 6

s.
33 4 at 2
9
298 16
6d. is $\frac{1}{4}$ 8 6
0 12 $10\frac{17}{56}$
£.307 14 $10\frac{17}{56}$

qrs.
2 is $\frac{1}{2}$ 9 3
14℔. $\frac{1}{4}$ 2 $3\frac{3}{4}$
7 is $\frac{1}{2}$ 1 $1\frac{7}{8}$
1 is $\frac{1}{7}$ 0 $1\frac{55}{56}$
12 $10\frac{17}{56}$

42
49
55
146
56) 146 (2
112
$\frac{34}{56}=\frac{17}{28}$

30. At 16s. 4d. ⅌ Cwt. what cost 14 Cwt. 3 qrs.? *Answ.* 12 l. 0s. 11d.

31. What will 193 Cwt. 3qrs. cost, at the rate of 17s. $9\frac{1}{2}$d.? *Answ.* 172 l. 7s. $1\frac{5}{8}$d.

32. At 16s. $5\frac{1}{4}$d. ⅌ Cwt. what cost 203 Cwt. 3 qrs.? *Answ.* 167 l. 9s. $1\frac{11}{16}$d.

33. If a dozen of candles cost 3s. 3d. what will a box of $5\frac{1}{2}$ dozen come to? *Answ.* 17s. $10\frac{1}{2}$d.

34. What will 2061 C. 2 qrs. 7 ℔. cost at 16s. 6d. ℘ Cwt.? *Answ.* 1700 l. 15s. $9\frac{3}{8}$d.

35. At 9s. 4d. ℘ Cwt. what will 106C. 3qrs. 14℔. come to? *Answ.* 49 l. 17s. 6d.

36. How much will 26 Cwt. 2 qrs. 7℔. cost, if 1 Cwt. cost 15s. 9d.? *Answ.* 20 l. 18s. $4\frac{5}{16}$d.

37. If 1Cwt. cost 21s. 6d. how much will 306C. 3qrs. 21℔. come to? *Answ.* 329 l. 19s. $1\frac{7}{8}$d.

38. What will 951Cwt. 2qrs. 27℔. cost, at 11s. 3d. ℘ Cwt.? *Answ.* 535 l. 7s. $1\frac{5}{112}$d.

39. What will 506 Cwt. 1qr. 11℔. come to at 13s. 7d. ℘ C? *Answ.* 343 l. 17s. 10d $\frac{85}{112}$d.

40. What will 499Cwt. 3qrs. 25℔. cost, at 25s. 11d. ℘ C.? *Answ.* 647 l. 17s. $7\frac{75}{112}$d.

41. At 19s. 9d. ℘ Cwt. how much will 109C. 0qrs. 15℔. come to? *Answ.* 107 l. 15s. $4\frac{83}{112}$d.

Cost 753-1-25

42. If 1Cwt. cost 15s. 2d. hat will ~~[illegible]~~C. 1qr. 25℔. come to? *Answ.* 571 l. 7s. $8\frac{1}{8}$d. 753

43. If 1℔ of silk cost 3 l. 6s. 5d. what will 80℔. 10oz. cost? *Answ.* 267 l. 14s. $10\frac{1}{8}$d.

44. How much will 1095Cwt. 0qrs. 5℔. come to at the rate of 17s. 6d. ℘ C.? *Answ.* 958 l. 3s. $3\frac{3}{8}$d.

45. What will 231Cwt. 0qrs. 1℔. come to, if 1C. cost 18s. 0d.? *Answ.* 207 l. 18s. $1\frac{13}{14}$d.

46. If 1℔. of silver cost 3l. 4s. what will 100℔. 11oz. come to? *Answ.* 322 l. 18s. 8d.

47. How much will 20 ton 19Cwt. 3qrs. 14℔ come to, if 1 ton cost 19 l. 19s. 6d.? *Answ.* 419 l. 7s. $0\frac{3}{56}$d.

48. At 25 l. 10s. 9d. the ton, what will 10 ton 17 Cwt. 3qrs. come to? *Answ.* 278 l. 0s. $9\frac{32}{80}$d.

49. At 6s. 9d. the yard, what will 7 yds. $3\frac{1}{2}$ qrs. come to? *Answ.* 2 l. 13s. $1\frac{7}{8}$d.

50. If 1 ton cost 21 l. 15s. 6d. what cost 25 ton 15Cwt. 3 qrs. $14\frac{1}{2}$℔.? *Answ.* 561 l. 13s. $3\frac{681}{2240}$d.

51. What will 15 yds. $1\frac{1}{2}$ qrs. of cloth cost, if a piece containing 42 yards cost 38 l. 17s. $6\frac{1}{4}$d.?

Answ. 14 l. 4. $7\frac{141}{224}$d.

52. If 1C. cost 19*s.* 11$\frac{3}{4}$*d.* what will 11C. 3qrs. 27$\frac{1}{2}$℔. come to? *Answ.* 11*l.* 19*s.* 7$\frac{119}{128}$*d.*

53. At 19*l.* 19*s.* 11$\frac{3}{4}$*d.* the ton, what will 19 ton, 19C. 3qrs. 27$\frac{1}{2}$℔. come to? *Answ.* 399*l.* 19*s.* 5$\frac{16641}{17920}$*d.*

PRACTICE *performed by* DECIMALS.

I. Since 2*s.* is $\frac{1}{10}$ part of a *l.* the decimal of 2*s.* is 1; wherefore any quantity being given at 2*s.* ⅌, the price is found in pounds and decimal parts of a pound, by separating the units figure of the given quantity from the rest for a decimal, the value of which decimal in shillings is known by inspection.

Examples.

Let it be required to find the value of 278 yards at 2*s.* ⅌ yard. By pointing off the lowest figure 8 for a decimal, I find the amount *l.*27.8 which is known to be equal to 27*l.* 16*s.*

l.
27.8

II. Consequently if the price be a multiple of 2*s.* (viz. any even number of shillings) the amount of 2*s,* being first found in pounds and decimal parts as above, and that amount multiplied by the number which expresseth how often 2*s.* is contained in the given price, the product will be the amount required in pounds and decimal parts of 1*l.* &c.

What cost 256 gallons of shrub, at 6*s.* ⅌ gallon?

l. *s.*
25.6 amount at 2 ⅌ gallon
3
l. *s.*
Answ. 76.8 or 76 16

Let the Examples of Chap. I. be done by this Rule.

Likewise if the price be pounds and even shillings

l. *s,*
475 at 1 12
47.5
16 × 2 = 32*s.*
2850
475
£.760.0

Or 475
285
£.760

47.5
6
285.0

III. If the price be an aliquot part of 2 shillings, find the amount at 2*s.* and divide it by the denominator of the part.

At 8*d.* ℘ ℔. what cost 326 ℔.?

8*d.* is $\frac{1}{3}$—32.6

l. *s.* *d.*

£.10.866= 10 17 4

Let the examples of Chap. II. § 3 be done by this:

IV. If the price be an aliquant part divide it into aliquot parts.

8729 at 5*d.*

872.9

4*d.* is $\frac{1}{6}$—145.483

1*d.* is $\frac{1}{4}$— 36.37075

l. . *d.*

181.85375 or 181 17 1

524 at 1*s.* 4*d.*

52.4

8*d.* is Co. $\frac{1}{3}$—17.466

l. s. d.

34.933=34 18 8

V. If the price be pounds and shillings, or pounds, shillings and pence Reduce the lesser denominations to the decimal of a pound, and multiply, the quantity thereby.

l. *s.* *d.* *l.*

54 ton at 12 10 6=12.425

54

50100

62625

676.350

£,676 7

VI. If the quantity likewise be of divers denominations, reduce the lesser denominations to a decimal of that whereof the price is given.

So

80℔. 10 *oz.* of silk at 3*l.* 6*s.* 5*d.*?

℔. oz. *l.* *s.* *d.*
80 10=80.625 3.32088=3 6 5
80233

241875
24188
1612
64

267.739

£. 267 14 $9\frac{1}{2}$

C. qrs. s. d.
14 3 at 16·4=.816
81

14.75 =735

735

7375 900
4425
10325

9)108.4125

l. *s.* *d.*
12,04583=12 0 11

After the same manner may the examples of Case III. page 185 be done.

Otherwise

Take parts for the lower denominations, as,

80 10 3.32083
80

265.66640
$8\frac{1}{2}$ 1.660415
$2\frac{1}{2}$ .415103

267.741918

£.267 14 10

14.75 at 16.4
.8

11.800
4 $\frac{1}{60}$ .2458

12.04583

£.12 0 11

PRACTICE

APPLIED IN CASTING UP COINS.

1. Paid J D in full for T S's bill on me, 100l. *viz.*

		s.	d.	l.	s.	d.
22	at	18	3	20	1	6
12	at	18	1	10	17	0
27	at	29	3	39	9	9
26	at	22	9	29	11	6
Change				0	0	3
				£. 100	0	0

The Work :

```
     s.  d.        s.  d.              l.  s.  d.
 22 at 18  3  12 at 18  1        27 at 1   9  3 Decimally.
 ———                   12        29 for 1 -9     27
 2   4 at 2 [illegible]   ———        ———                4×2=8
     9            £.10  17          243            ———
 ————             ————               54           10.8 for 8s.
d. 19  16          l.  s.  d.     8¼  6  9   1s. ½   1.35—1s.
 3⅓  0   5  6   26 at 1  2  9      ————      3d. ¼   .3375-3d.
 ———————— 2⅒   212          2|0)78|9  9         27.      1l.
£.20  1  6 6¼d.0  13          ————            ————
 ———————— 3½d.0   6  6    £39   9   9       39.4875–1 9–3
          ————                ————          ————
          £.29  11  6                       £.39  9  9
          ————                              ————
```

2. Received of Abraham Adamson for Peter Drawer's bill, 150l. *viz.*

		l.	s.	d.	l.	s.	d.
87	at	1	2	9	98.	19.	3
127½	at	0	5	5	34.	10.	7½
17	at	0	18	3	15.	10.	3
1	at	0	19	6		19.	6
Change					0	0	4½
					£.150	0	0

3. Paid Samuel Thompson 343*l.* for 343 barrels of beef *viz.*

	s.	d.	l.	s.	d.
105 at	22	9	119	8	9
100 at	18	3	91	5	0
101 at	4	9	23	19	9
400 at	5	5	108	6	8
			343	0	2
Change returned					2
			£. 343	0	0

4. Received from J D in part payment for tobacco, *viz.*

	s.	d.	l.	s.	d.
127 at	5	5	34	7	11
478 at	3	4	79	13	4
1016½ at	1	1	55	1	2½
			£. 169	2	5½

5. Received of R F 259*l.* 7*s.* 9*d.* in part of 1500 stone of wool sold him, *viz.*

	s.	d.	l.		
300 at	5	6	84	5	0
100 at	6	0	30	0	0
150½ at	18	3	137	6	7½
199½ at	1	1	10	16	1½
			£. 259	7	9

6. Paid P. P. on account of 1000 barrels of wheat.

	l.	s.	d.	l.	s.	d.
981½ at	0	5	5	265	16	5½
314½ at	0	4	9	74	13	10½
72½ at	1	9	3	106	0	7½
94 at	1	9	1	136	13	10
27½ at	0	18	3	25	1	10½
30 at	0	17	11	26	17	6
13½ at	0	1	1		14	1½
				£. 635	18	9½

7. **Received of B G, by order and for account of S S,** *viz,*

		l.	d.	l.	s.	d.
1036½	at	1	1	56	2	10½
931	at	3	4	155	3	4
97	at	28	11	140	4	11
104	at	28	9	149	10	0
58	at	18	3	52	18	6
275	at	18	1	248	12	11
431½	at	22	9	490	16	7½
57½	at	29	3	84	1	10½
				£.1377	11	0½

BILLS OF PARCELS.

Dublin, *Sept.* 18, 1808.

1. James Bateman,

Bought of Edward Empson,

		s.	d.		l.	s.	d.
27¼ lb. of Smyrna coffee,	at	5	8	℔ lb.	7	14	5
23 lb. of Mocha ditto,	at	5	4		18	16	0
26½ lb. of imperial tea,	at	25	0		33	2	6
10¾ lb. of best Bohea,	at	14	6		7	15	10½
13 lb. of royal green tea,	at	18	8		12	2	8
21 lb. of sugar double refined,	at	1	0½		1	1	10½
					£.70	13	4

10th *Nov,* 1808.

2. Francis Pindust,

Bought of Isaac Hosier,

	s.	d.		l.	s.	d.
15 Pair of women's worsted, at	5	7	℔ P.	4	3	9
23 Pair of men's silk.	17	4		19	18	8
22 Pair of men's yarn,	3	2		3	9	8
18 Pair of Norwich hose,	4	9		4	5	6
38 Pair of thread	3	4		6	6	8
13 Pair of women's gloves, silk,	4	8		3	6	8
				£41	4	11

3. Dublin

Dublin, *Feb.* 8, 1809.

3. George Whyte,

Bought of John Brown and Comp.

			s.	d.		l.	s.	d.
10½	Yards of Yorkshire cloth,	at	6	6	℔ yd	3	8	3
7	Yards of fine Spanish black,	at	16	3	——	5	13	9
6¾	Yards of fine grey cloth,	at	15	9	——	5	6	3¾
16¾	Yards of frize,	at	3	6	——	2	18	7½
[illegible]	Yards of fine drab,	at	15	6	——	3	2	0
5⅞	Yards of superf. Span. cloth,	at	18	6	——	5	8	8¼
31	Yards of livery scarlet cloth,	at	13	0	——	20	3	0
						£. 46	0	7½

4. James Webster,

Bought of William Grocer and Comp.

	C.	qrs.	℔.		l.	s.	d.		l.	s.	d.
2 *Hhds.* of sugar,	17	2	17	at	1	13	10	℔ C	29	17	$2\frac{5}{8}$
Raisins, 3 barrels,	12	1	19	at	1	14	5	—	21	7	$5\frac{5}{16}$
Tobacco, 1 *hhd.*	4	0	12	at	4	19	4	—	20	7	$11\frac{5}{7}$
Rice, 1 barrel,	1	0	15	at	2	16	4	—	3	3	$10\frac{15}{28}$
Pepper, 1 bag,	1	3	19	at	3	12	4	—	6	18	$10\frac{1}{4}$
Brimstone,	2	1	19	at	1	19	1	—	4	14	$6\frac{13}{16}$
Bees-wax, 4 cakes,	2	2	12	at	1	18	4	—	4	19	$11\frac{2}{7}$
									£. 91	9	$10\frac{15}{28}$

5. Edward Steward,

Bought of Francis Kiernan,

	C.	qrs.	℔.		l.	s.	d.		l.	s.	d.
12 Cheshire cheeses,	5	2	24	at	1	17	4	℔ C	10	13	4
45 Gloucestershire *do*,	4	2	10	at	1	12	6	—	7	9	$1\frac{23}{28}$
40 Irish Cheeses,	5	1	18	at	0	18	0	—	4	17	$4\frac{40}{56}$
8 Firkins of butter,	7	2	0	at	1	5	6	—	9	11	3
7 Flitches of bacon,	6	1	17	at	1	4	8	—	7	17	$10\frac{13}{14}$
									£. 40	9	$0\frac{13}{28}$

6. Sir Michael Newton,

To Thomas Goldsmith, *Dr.*

			oz.	*pt.*	*grs.*		*s.*	*d.*	
May	31.	A Silver set of casters,	52	30	10	at	7	9	℔ *oz.*
June	7.	Half a doz. soup plates, - -	85	14	15	at	6	3	
	10.	A silver tea pot and lamp,	29	16	15	at	6	4	
	19.	A large punch-bowl,	67	00	16	at	16	0	
	21.	A dozen of silver spoons,	33	11	10	at	6	2	
Nov.	6.	A dozen desert knives, forks and spoons, with a shagreen case,	40*l.*						

£. 130 18 3¼

7. Thomas May,

To William Weed, *Dr.*

	C.	*qr.*	℔.		*s.*	*d.*	
1 *Hhd.* best bright tobacco,	5	2	0	at	0	10½	℔ ℔.
1 Box of Oroonoko	0	2	19½	at	0	11¾	—
5 Bags of old Spanish,	6	0	12	at	0	4⅛	—
½ Hogshead,	2	2	13	at	0	5⅜	—
2 Rolls,	0	3	10	at	1	5⅛	—

£. 55 13 5

8. Bought of James Erwin, 8 bags of Farnham hops, at 11⅛*d.* ℔ ℔. *viz.*

		C.	qrs.	lb.
N°. 1	———	2	2	18
2	———	2	3	10
3	———	2	2	17
4	———	2	0	0
5	———	3	1	8
6	———	3	1	3
7	———	3	0	20
8	———	2	2	7
		22	1	27

£.116 15 3¼

CHAP.

CHAP. II.

Of ESTIMATING IMPORTS and EXPORTS, &c.

IN casting up the amount of goods bought, imported or exported, to the prime cost of such goods we must add all the charges upon them, in order to fix the price they stand us in, as will appear from the following

Examples.

1. Suppose I import from Rotterdam 5 bales of paper, each *qt.* 10 reams, which, with charges there, amounted to 29*l.* I pay for duty here 6*d.* ⅌ ream, for freight 19*s.* porterage 1*s.* What doth it stand me in ⅌ ream: and how must I sell it ⅌ ream to gain 10 ⅌ cent.?

5 Bales,
each 10 Reams,

In all 50 Reams, at 6*d.* ⅌ *R.* duty.

6*d.* ½ 2|5

£ 1 5

	l.	*s.*	*d.*
First cost	29	0	0
Duty	1	5	0
Freight	0	19	0
Porterage	0	1	0
Whole cost £.	31	5	0

Re. 10)*l.* *s.* *Re.*
If 50 cost { 31 5 what cost 1.
50 {
{ 5)3 2 6

10)
If 100 gain 10 what 10)0 12 6 cost ⅌ ream?
0 1 3 gain by 1 ream, at 10 ⅌ cent. Wherefore I must
sell it at £. 0 13 9 ⅌ ream.

2. Suppose a bale of merchandize weighed 300℔. and cost 15*l.* 4*s.* 9*d.* pays for duty 2*d.* ⅌ ℔. for freight 25*s.* for porterage home 1*s.* 6*d.* How much will 1℔. of the said merchandize stand me in? *Answ.* 15¼*d.*

3. Imported 4 pipes of oil, containing 480 gallons, which cost me 5*s.* 5$\frac{1}{2}$*d.* ⅌ gallon; paid for freight 4*s.* ⅌ pipe, duty 6*d.* ⅌ gallon, porterage 1*s.* ⅌ pipe: what must I sell it for ⅌ gallon to gain 12 ⅌ cent.?

Answ. 6*s.* 8$\frac{16}{25}$*d.*

4. Bought 150 casks of butter, *wt.* neat 320C. 2qrs. 14lb. at 23*s.* 8*d.* ⅌ Cwt. paid custom 1*d.* ⅌ cask; porterage 2*d.* Cask ⅌; the cellarage came to 1*l.* 15*s.* cooperage 1*l.* 14*s.* 4$\frac{1}{2}$*d.* when I sell them for 416*l.* 16*s.* 3*d.* what do I gain upon the whole ⅌ Cwt. and ⅌ cent.

Answ. I gain 32*l.* 1*s.* 3*d.* upon the whole; 2*s.* ⅌ C. and 8$\frac{1}{3}$ ⅌ cent. 8$\frac{1}{3}$ Pr. Cent

5. Bought 200 carcases of beef, *wt.* neat 850C. 2qrs. at 6*s.* 6*d.* ⅌ C. and salted them up in 400 barrels which cost 2*s.* 9*d.* a piece; paid for 50 barrels of salt, at 20*s.* ⅌ barrel; paid for salting, labourers, &c. 4*l.* 0*s.* 1*d.* Now I want to know what 1 barrel stands me in?

Answ. 19*s.* 3$\frac{1}{4}$*d.*

6. Shipped for Bristol 20 packs of bay yarn, which cost me 408*l.* 10*s.* paid for 50 yards of canvas, at 7*d.* ⅌ yard; for packing 5*d.* ⅌ pack; porterage to the quay 4*d.* ⅌ pack; for licence, duty and other port charges, 5*l.* 13*s.* 4*d.* lighterage, 11*s.* 2*d.* I desire to know what it stands me in ⅌ pack on board? *Answ.* 20*l.* 16*s.* 11$\frac{1}{5}$*d.*

7. Imported from Spain 10 tun of wine, at 10*l.* ⅌ hogshead; paid duty here 1*s.* 1*d.* ⅌ gallon; freight 15*s.* ⅌ tun; the charges for lighterage, porterage, &c. came to 11*l.* 9*s.* a storm arising, one pipe containing 126 gallons was thrown overboard: what doth the remainder stand me in ⅌ gallon; how must I sell it ⅌ hogshead to gain 10 ⅌ cent. and what do I gain by the whole at that rate? *Answ.* The remainder 4*s.* 7*d.* ⅌ gallon; I must sell it at 15*l.* 17*s.* 7$\frac{1}{2}$*d.* ⅌ hogshead; and gain by the whole 54*l.* 17*s.* 3*d.*

CHAP. III.

Of ALLOWANCES *on the* WEIGHT *of* GOODS *called* *TARE and TRETT.*

TARE is the weight of the cask, barrel, box, wrapper, &c. *viz.* whatever goods are packed in, or an allowance in consideration thereof.

Trett is an allowance to retailers on account of waste, or the small draughts they sell by.

Gross weight is the weight of the commodity together with that in which it is packed without any deduction.

Neat weight is the remainder after all allowances are deducted.

I. OF INVOICE TARE.

Rule.

Add the gross weights into one sum, and the tares into another. Then subtract the total tare from the whole gross, and the remainder is the neat weight.

Examples.

1. What do 4 barrels of indigo come to, at 3*s*. 4*d*. ⅌ ℔. Nº. and *wt*. as follows, *viz*.

		C.	qrs.	℔.		℔.
Nº. 1	——	4	1	10	Tare	36
2	——	3	3	02	——	29
3	——	4	0	19	——	32
4	——	4	0	00	——	35
	Total gross	16	1	03		132 Tot. tare.

16
16
1628
3

1823℔. Gross,
132℔. Tare,

1691℔. neat at 3*s*. 4*d*.

3*s*. 4*d*. is ⅙——£. 281 16 8 *Answ*.

2. What do 6 hogsheads of tobacco come to, weight gross 35C. 0*qrs*. 9℔. tare of all 556℔. at 4*l*. 15*s*. 10*d*. ⅌ Cwt.? *Answ*. 144*l*. 6 . 1⅖*d*.

3. Bought 2 hogsheads of sugar, at 35*s.* ⅌ C. neat, *viz.*

	C.	qr.	℔.		℔	
N°. 1 ——	11	1	17	Tare	112	*Answ.* 38*l.* 18*s.* 5¼*d.*
2 ——	12	2	00	——	74	

4. Sold 10*hhds.* of tobacco at 7½*d.* ⅌ ℔ neat, N°. and *wt.* as follow, *viz.*

	C.	qr.	℔.		C.	qrs.	℔.
N°. 1 ——	6	3	17	Tare	0	3	17
2 ——	6	3	19	——	1	1	14
3 ——	6	2	14	——	1	0	02
10 ——	7	2	24	——	1	0	07
14 ——	6	3	22	——	0	3	24
16 ——	6	2	27	——	0	2	17
17 ——	7	3	04	——	1	0	09
19 ——	6	3	04	——	1	0	04
20 ——	7	2	07	——	0	3	17
25 ——	9	3	02	——	0	2	27
	73	3	0		7	2	26
Answ. £.	224	1	3				

II. When the tare or trett is so much ⅌ bag, bale, &c. multiply the tare of one by the number of bags, bales, &c. the product is the whole tare.

5. What's the amount of 16*hhds.* of tobacco, *qt.* gross 86C. 2qrs. 14℔. Tare ⅌ hhd. 100℔ at 3*l.* 15*s.* 10*d.* ⅌ Cwt.?

Gross	86	2	14	Mult. 16*hhds.*
Tare	14	1	4	by 100℔. tare of each,
Neat	72	1	10	312)1600(14 1 4 whole tare.

C. qr. ℔.
72 1 10 at 3 15 10 — 112
12 — 480
— 448
45 10 0
6 — 28)32
— 28
273 0 0
1qr. is ¼ — 0 18 11½ — 4
8℔. is $\frac{1}{14}$ — 0 5 5
2℔. is ¼ — 0 1 4¼

£. 274 5 8¾

6. 70 Bales of Smyrna silk, each 317℔. gross; tare ꝑ bale 16℔. at 16s 8d. ꝑ ℔. neat?

Answ. 17558l. 6s. 8d.

7. What's the value of 14 hhds. of tobacco, weighing gross 89C. 2 qrs. 17℔; tare 100℔. ꝑ hogshead, at $9\frac{3}{4}$d. ꝑ ℔. neat? *Answ.* 351l. 0s. $9\frac{1}{4}$d.

8. Sold 4 casks of indigo, *wt.* gross 18C. 2 qrs. tare 37℔. ꝑ cask, at 4s. 6d. ꝑ ℔. what do they amount to?

Answ. 432l. 18s. 0d.

9. What's the amount of 4 hogsheads of tallow, *viz.*

	C.	qrs.	℔.	
No. 1 ———	9	3	24	Tare 3 qrs. 6℔ ꝑ hhd.
2 ———	10	2	16	at 20l. 10s. ꝑ ton.
3 ———	11	0	12	
4 ———	9	3	1	*Answ.* 39l. 4s. $3\frac{1}{2}$d.

III. When the tare or trett is so much ꝑ Cwt. gross. Divide the pounds tare into aliquot parts of a Cwt. and take that part out of the gross, the quotient is the whole tare, which subtract from the gross, &c.

Otherwise:

If the tare or trett be a small number as 1, 2, 3, multiply the Cwt. gross by the pounds tare or trett, and for the quarters and pounds take parts of the tare or trett.

10. What cost 12 butts of currants gross 7 Cwt. 1 qr. 10℔. each butt, tare 16℔. ꝑ Cwt. at 32s. 6d. ꝑ Cwt. neat?

```
                C. qr. ℔.
                7   1  10
                       12
              ------------
Tare ꝑ C.      88   0   8 Gross,
16℔. is 1/7    12   2   9 Tare
               ------------           l.   s.  d.              Or,
               75   1  27 Nt. at      1   12   6      2qrs. 1/4 1  12  6
10s. is 1/2 l. 37  10                 -----------              ---------
2s. 6d. 1/4     9   7   6 1 qr. 1/4   0    8   1½     From 8)   16  3
                0  15  11¼ 16 1/7℔ 1/7 0   4   7¾   qr. ℔.     -------
               ------------     8 1/2  0   2   3¾   1  27Co. 1/56 7)2 0⅜
£.            122  13   5¼      2 1/4  0   0   6¾              -------
               ------------     1 1/2  0   0   3½        Take   0 3½
                                      -----------               ------
                                   £. 0   15  11¼               15 11½
```

11. What is the price of 30 barrels of figs, *wt.* gross 75C. 3qrs. 14℔ tare 14℔. ⅌ Cwt. at 18s. 6d. ⅌ Cwt. neat. *Answ.* 61l. 8s. $2\frac{23}{32}$d.

12. 8 Bags of pepper, *wt.* gross 21Cwt. 3qrs. 15℔; tare 4℔ ⅌ Cwt. at 11$\frac{3}{4}$ ⅌ ℔. neat?
Answ. 115l. 14s. $2\frac{79}{112}$d.

13. 50 Casks of butter, weight gross 202 Cwt. 3qrs. 14℔ tare 20℔. ⅌ Cwt. at 19s. ⅌ Cwt. neat?
Answ. 158l. 6s. $8\frac{33}{56}$d.

14. What come 4 hogsheads of sugar to, weight 43 Cwt. 3 qrs. 21℔. gross, tare 12℔ ⅌ Cwt. at 2l. 15s. 4d. ⅌ Cwt. neat? *Answ.* 108l. 10s. $8\frac{37}{56}$d.

OF TRETT.

TRETT in London is an allowance of 4℔. in 104℔. after the tare is deducted, on goods subject to waste which is $\frac{1}{26}$ part of the remainder.

In Cork they allow 1℔ out of every Cwt. of the gross weight of all goods, except beef, butter, hides and wool, and on these they make the following allowances:

Butter, 2℔ on every large cask;

Beef, 8℔ ⅌ carcass;

Raw hides, 4℔ ⅌ piece;

Wool 8℔ for [illegible] 20 stone, *viz.* $\frac{1}{2}$ stone in 20 stone is $\frac{1}{40}$; but in Dublin, and parts adjacent, they allow 8℔ for every 3 Cwt. or 21 stone, which is $\frac{1}{42}$ after the tare is deducted.

1. Two hogsheads of tallow weight as followeth, trett 1℔ ⅌ Cwt. out of the gross, and at 30s. ⅌ Cwt. what does it come to? *Answ.* 29l. 5s. $1\frac{17}{28}$d.

No. 1

	C.	qr.	℔.		C.	qr.	℔.
N°. 1 *wt.*	10	1	11	Tare	0	3	20
2 *wt.*	11	0	17	-------	0	3	14
Gross	21	2	00		1	3	6
1 ℘ C. Trett	0	0	$21\frac{1}{2}$				
	21	1	$6\frac{1}{2}$				
Tare	1	3	6				
Neat	19	2	$0\frac{1}{2}$				
at	30s.	℘ Ct.					
	570						
2 qrs. is $\frac{1}{2}$	15						
$\frac{1}{2}$ ℔ is $\frac{1}{112}$	0	$1\frac{17}{28}$					
	58|5	$\frac{17}{28}$					

Answ. £. 29 5 $1\frac{17}{28}$

2. Sold 4 hogsheads of sugar, weighing as follows:

N°. 1, Wt. 7C. 3qrs. 16℔. Tare 3qrs. 20℔.
2, 8Ct. 0qrs. 10℔. Tare 3qrs. 24℔.
3, 7Ct. 2qrs. 20℔. Tare 3qrs. 27℔.
4, 8Ct. 2qrs. 18℔. Tare 3qrs. 25℔.

Trett 1℔. ℘ Ct. and at 35s. 4d. ℘ Ct. how much does it amount to? *Answ.* 49l. 15s. $6\frac{193}{392}$d.

3. A Merchant has bought 4 hhds. of copperas, weight as follows, viz.

N°. 1, 10Ct 2qrs. 4℔. Tare 3qrs. 4℔.
2, 11Ct. 0qrs. 10℔. Tare 3qrs. 10℔.
3, 12Ct. 1qr. 0℔. Tare 3qrs. 14℔.
4, 11Ct. 2qrs. 14℔. Tare 3qrs. 18℔.

Trett 1℔. ℘ Ct. and at 8s. 6d. ℘ Ct. how much does it come to? *Answ.* 17l. 14s. $3\frac{72}{112}$d.

4. 4 casks of raisins, weight as follows:

N°. 1, 1Ct. 3qrs. 25℔. Tare 0qrs. 21℔.
2, 1Ct. 3qrs. 20℔. Tare 0qrs. 24℔.
3, 2Ct. 1qr. 16℔. Tare 0Ct. 1qr. 3℔.
4, 1Ct. 3qrs. 23℔. Tare 0Ct. 0qr. 25℔.

Trett 1℔. ℘ Ct. and at $30\frac{1}{3}$s. ℘ Ct. what does it amount to? *Answ.* 11l. 0s. $7\frac{15}{16}$d.

5. What will 3 bales of coffee come to, weight,
N°. 1, 2Ct. 3qrs. 12℔. Tare 16℔.
2, 3Ct. 1qr. 20℔. Tare 19℔.
3, 3Ct. 1qr. 24℔. Tare 18℔.
Trett 1℔. ⅌ Ct. and at 18l. 10s. ⅌ Ct.?
Answ. 170l. 0s. 2$\frac{2}{5}\frac{3}{6}$d.

I. When the price is given ⅌ ℔. neat and the trett 1℔. ⅌ Cwt. write the Cwt. gross three times putting the unit-figure of the second line under the tens of the first, and the units of the third under the tens of the second: Then for the odd quarters write 27¾ for 1qr. 55½, for 2qrs. and 83¼ for 3qrs. Add them into one sum, and then the trett is taken away. From that sum deduct the lbs tare (if any) and the remainder is the pounds neat.

6. To how much come the following 2 hhds. at 1s. 8d. ⅌ ℔.

C. qrs. ℔.
N°. 1 —— 2 3 10
2 —— 3 2 18

6 2
6
655 ½

721 ½
Tare 32

689 ½ at 1s. 8d.

s. d.
1 8 $\frac{1}{12}$ 57 . 8 4
. 10

£. 57 9 2

7. A merchant has sold [illegible] bags of pepper, viz.
N°. 1, weighing 3Ct. 2qrs. 0℔.
2, 4Ct. 1qr. 7℔.
3, 3Ct. 3qrs. 21℔.
Trett 1℔. ⅌ Ct. and tare 40℔. for each bag, at 14¾d. ⅌ ℔. neat, I demand how much it will come to?
Answ. 72l. 15s. 7$\frac{1}{3}\frac{1}{6}$d.

8. Sold 4 casks of indigo, *wt.* gross 18Cwt. 2qrs. 0℔. trett 1 ⅌ Ct. tare 37℔. cask, and at 4s. 6d. ⅌ ℔. what does it amount to? *Answ.* 428l. 14s. 9d.

9. Twelve casks of merchandize, *wt.* 306Ct. 2qrs. 0℔. 306 trett 1 ⅌ Ct. tare 35℔. ⅌ cask, at 15$\frac{1}{2}$d. ⅌ ℔. what does it amount to? *Answ.* 2171l. 17s. 9$\frac{3}{8}$d.

10. Three bags of cocoa, weight gross 10Ct. 1qr. 14℔. trett 1 ⅌ Ct. tare 7℔. ⅌ bag, at 9$\frac{3}{4}$d. ⅌ ℔. what does it come to? *Answ.* 45l. 18s. 8$\frac{13}{16}$d.

11. What will 4 hogsheads of tallow come to?

N°. 1 Weight 9C. 3qrs. 24℔.
2 Weight 10C. 2qrs. 16℔.
3 Weight 11C. 0qrs. 12℔.
4 Weight 9C. 3qrs. 4℔.

trett 1 ⅌ C. and tare 3qrs. 6℔. ⅌ hhd. and at 20l. 10s. ⅌ ton? *Answ.* 38l. 17s. 3$\frac{15}{112}$d.

12. How many Cwt. neat in 12C. 3qrs. 12℔. gross, tare 2℔. ⅌ C. trett 4℔. in 104?

12 2 14
4
qr. ℔.
26)50(1 26
26
24
28
196
49
686
52
166
156
10

C. qrs. ℔.
12 3 12
24 for 12C.
$\frac{3}{4}$ 1$\frac{1}{2}$ for 3qrs.
$\frac{1}{7}$ 0$\frac{3}{14}$
0 25$\frac{5}{7}$ tare,
12 2 14$\frac{2}{7}$
1 26 trett.
Answ. 12 0 16$\frac{2}{7}$

But if the ℔. neat be required.

12 3 12
12
12
1284
12
1440
25$\frac{5}{7}$ tare
1414$\frac{2}{7}$
$\frac{1}{26}$ 54 trett.
Answ. 1360$\frac{2}{7}$

26)1414(54
130
114
104
10

13. Four barrels of Spanish tobacco, *viz.*

	C.	qrs.	℔.
No. 4 ——	1	0	[illegible]
7 ——	0	3	25
9 ——	1	0	5
13 ——	0	2	26

Tare 14℔ ⅌ C. trett 4℔ ⅌ 104℔ at $9\frac{1}{2}$d. ⅌ ℔. neat.

Answ. 14l. 2s. $4\frac{4}{5}\frac{2}{2}$d.

15. Bought 3 packs of wool, weight, *viz.*

C. qrs. ℔.
No. 1—3 1 12
2—3 3 07
30 3—3 2 15
3

Tare 30℔. ⅌ pack. Trett 8℔ for every 20 stone, at 10s. 6d. ⅌ stone

Answ. 35l. 16 $7\frac{1}{2}$d.

—qrs. ℔ 10 3 6
28)90(3 6 3 6 tare
84
— 10 ℔ C.
6 7 stone at 16=1
℔ sto. —
8 of 20 is $\frac{1}{4}$|o 70 stone
1 12℔.
s. d.
68 4 at 10 6
10s. is $\frac{1}{2}$ 34 4℔ $\frac{1}{4}$ 2 $7\frac{1}{2}$
6d. is $\frac{1}{2}$|o 1 14
0 2 $7\frac{1}{2}$

Answ. £.35 16 $7\frac{1}{2}$

15. Bought 5 packs of wool, weight, *viz.* No. 1, 4C. 2qrs. 15℔ No. 2, 4C. 2qrs. 0℔. No. 3, 3C 3qrs. 21℔ No. 4, 3C. 3qrs. 14℔. No. 5, 4C. 0qrs. 14℔. tare 28℔ ⅌ pack, trett 8℔. for every 3C. at 11s. 6d. ⅌ stone; How much doth it come to?

Answ. 77l. 17s. $7\frac{17}{28}$d.

16. INVOICE of 12 bags of wool bought at Clonmel, for account of E W of Dublin, viz.

		C.	qrs.	℔.	
	Nº. 1	5	2	24	Tare 29℔. ℘ bag.
WOOL	2	2	1	00	Trett 8℔. for every 20
E. W.	3	7	1	18	stone, at 10s. 9d. ℘
	4	4	3	22	stone.
	5	6	2	13	
	6	5	1	19	*Answ.* 243l. 1s. 4d.
	7	7	2	14	
	8	5	3	24	
	9	6	1	17	
	10	4	2	26	
	11	5	3	13	
	12	6	1	19	

17. INVOICE of 15 bags of wool, bought at Mullingar, for account of A B of Cork, merchant, viz.

		C.	qrs.	℔.		C.	qrs.	℔.
WOOL	Nº. 1	4	3	16	Nº. 9	7	1	11
A. B.	2	5	1	12	10	4	3	26
	3	3	3	25	11	5	2	19
Tare 30℔. ℘ bag	4	6	1	24	12	6	1	08
Trett 8℔. for every	5	7	2	04	13	6	2	05
3 C. at 10s. 3d. ℘	6	5	3	20	14	7	1	18
stone?	7	6	2	12	15	7	2	12
Answ. £.311 13 8⅔	8	6	1	14				

18. London, 17th Sept. 1808.

Stuart, Webster and Co.

Bought of the East India Company at 4 months,

Pepper, 2 Lots, viz.

		C.	qrs.	℔.		℔.
Nº. 17	10 bags *qt.*	27	1	18	Tare	150
20	10	24	3	24		138

Gross

Tare

Neat at 10⅜d. ℘ ℔.—£. 241..2..7

S Redwood,

Redwood, 2 Lots, viz.

	Ton.	C.	qrs.
Nº. 47—120 sticks	10	13	3
48—100 ——	11	12	0

74.13.3 $\frac{2}{10}$

l. s.
at 3 7 ⅌ ton—£.

Wormseed, 3 Bales,

	C.	qrs.	lb.		qrs.	lb.
Nº. 18—	3	1	10	Tare	1	14
24—	4	2	00	——	1	16
37—	2	3	19	——	1	13

Gross
Tare

Neat at 13½d. ⅌ lb.——£. 60.12.0

Answ. £.376 8 7 $\frac{9}{10}$

19. Belfast, 23rd September, 1808.

Peter Paydown and Co.

Bought of Titus Tradewell for ready money,

Cotton, 14 bags, viz.

	C.	qrs.	lb.
No. 1—gt.	3	1	7
2—	2	3	0
3—	2	3	5
4—	3	0	15
	11	3	27
	12	1	1
	24	1	0 Total gross.
	00	3	13 Tare allow'd.
	23	1	15 Suttle 2619 lb.

	C.	qrs.	lb.
Nº. 17—	2	3	16
24—	3	1	10
28—	3	0	27
30—	2	3	4
	12	1	1

Trett 100

2519 at 14d. ⅌ lb.—£.

146..18.10

More, viz.

	C.	qrs.	lb.	
No. 30	2	3	12	Damaged.
31	3	0	10	
32	3	1	26	
33	3	2	08	
34	2	2	07	
35	1	3	10	
Gross				
Tare	0	2	17	

Suttle lb.
Trett $\frac{1}{26}$

Neat lb. at 4d. ℔ lb. — £. 30 - 3 - 2 $\frac{3}{11}$

Answ. £.177 2 0[illegible]

CHAP. IV.

Of Estimating the ALLOWANCES, PREMIUMS, *or* *RATES PER CENT, or on the 100l.*

THE Allowances or Premiums estimated at so much ℔ Cent, are Commission, Insurance, Brokerage, Interest, Discount and Exchange between England and Ireland.

Commission is a Premium allowed by the Employer to his Factor for transacting his business abroad.

Insurance is paid by one Merchant to another, or to a Company for insuring his ships, goods, &c. that is engaging to make good to him the value insured, if the same be lost or damaged.

Brokerage is an allowance to the Broker, who is a person employed to find out customers for merchandizes and the contrary.

Exchange, Interest and Discount will be treated of in their respective places.

Now in these cases we have given the rate ℘ cent. to find the allowance upon a given sum at the same rate: that is, three numbers given to find a fourth proportional, viz. *l.*

As 100: its rate : : so is the given sum : to its
rate or the proportional allowance.

Case I.

When the rate or premium is a whole number, and likewise the given sum, on which the premium is to be found.

This is already taught (see page 85.)

Case II.

When the given sum (whereof the premium is to be found) is of divers denominations, multiply it by the rate or premium of 100l. and divide by 100.

Examples.

1. What is the commission on 417l. 16s. 8d. at 2 ℘ cent.? *Answ.* 8l. 7s. $1\frac{3}{5}$d.

2. Sold goods for A B to the amount of 153l. 2s. 6d. and for commission and risque of debts am to receive 4 ℘ cent. what doth it amount to? *Answ.* 6l. 2s. 6d.

3. What is the exchange of 750l. 18s. 8d. English money, at 7 ℘ cent.? *Answ.* 52l. 11s. $3\frac{1}{2}\frac{1}{5}$d.

4. How much is the interest of 300l. 10s. at 8 ℘ cent.? *Answ.* 24l. 0s. $9\frac{3}{5}$d.

Case III.

When the rate is a fraction.

Multiply the given sum thereby, viz. divide it by the denominator of the rate, &c. as before.

What is the commission on 1026l. 17s. 4d. at ½ ℘ cent.?

```
       l.    s.  d.
     2)1026  17  4
                 ½
    ---------------
  ½—5|13     8   8
      20
    ----              l.  s.  d.
     2|68      Answ. 5   2   8 6/25
      12
    ----
    8)24|6
    ------
    100|25
```

6. How much is the brokerage of 846l. 19s. at $\frac{1}{4}$ ⅌ cent.? *Answ.* 2l. 2s. $4\frac{17}{100}$d.

7. Negociated bills for account of A C to the amount of 1538l. 13s. 6½d. what is my commission at $\frac{3}{8}$ ⅌ cent.?
Answ. 5l. 15s. $4\frac{259}{320}$d.

8. What is the brokerage of 700l. 14s. 6d. at 4s. ⅌ cent.? *Answ.* 1l. 8s. $0\frac{37}{250}$d.

9. What must I pay my broker when he sells goods to the value of 500l. 10s. 7d. at 7s. ⅌ cent.?
Answ. 1l. 15s. $0\frac{889}{2000}$d.

Case IV.

When the rate is a mixt number.

For the integral part multiply the given sum by Case 2, and for the fraction, ⅌ last: add these products together, and proceed as before.

Examples.

10. What is my commission on 199l. 3s. 6d. at 2½ ⅌ cent.?

l.	s.	d.
199	3	6
		2½
398	7	0
99	11	9
4|97	18	9
20		
19|58		
12		
7|05		

Answ. 4l. 19s. 7 $\frac{5}{100}$ or $\frac{1}{20}$d.

11. I have sold goods for account of Thomas Jones, amounting to 536l. 10s. what is my commission at 2½ ⅌ cent.? *Answ.* 13l. 8s. 3d.

12. What is the commission on 1009l. 18s. at 2¼ ⅌ cent.? *Answ.* 22l. 14s. $5\frac{23}{50}$d.

13. Insured 427l. 13s. on the ship Sea Horse, for the present voyage to Barbadoes: insurance at 8¼ ⅌ cent. what doth it amount to? *Answ.* 35l. 5s. $7\frac{47}{100}$d.

14. Bought by order and for account of Jaques Durand of Bourdeaux, 400 barrels of beef, which cost 390l. 17s. 6d. which I ensured at $5\frac{3}{4}$ ℘ cent. I demand the insurance, my commission on the same at $2\frac{1}{2}$ ℘ cent. and for insuring the same at $\frac{1}{2}$ ℘ cent.? *Answ.* Insurance 22l. 9s. $6\frac{3}{40}$d. Commission at $2\frac{1}{2}$—9l. 15s. $5\frac{1}{4}$d. at $\frac{1}{2}$—1l. 19s. $1\frac{1}{20}$d.

Method 2, by Practice.

Divide the rate into aliquot parts of 100, and divide the given sum by the denominators of the parts, &c.

Examples.

I demand the interest of 274l. 16s. 3d. at 5 ℘ cent?

l. s. d.
of 100—5l. is $\frac{1}{20}$. 264 16 3
£. 13 14 $9\frac{3}{4}$

What is the commission of 1026l. 17s. 4d. at $\frac{1}{2}$ ℘ cent?

l. s. d.
$\frac{1}{2}$ of 100 is $\frac{1}{200}$
2)1026 17 4
10)513 8 8 $\frac{1}{2}$|00
10(51 6 $10\frac{4}{10}$
5 2 $8\frac{6}{25}$

Or thus,
10|26 17 4
£.5 2 7 $\frac{1}{5}$ = 1026 ÷ 200
$1\frac{1}{25}$ = 17s.4 ÷ 200
£. 5 2 $8\frac{6}{25}$

I divide 1026 by 200 thus, cut off the two figures 26 for the cyphers in the divisor, and divide them by 10.

l. s.
100|9 18 at $2\frac{1}{4}$ ℘ cent.
2 is $\frac{1}{50}$ 20 3 $10\frac{26}{50}$ $2\frac{1}{2}$ is $\frac{1}{40}$.
$\frac{1}{4}$ -- $\frac{1}{8}$ 2 10 $5\frac{47}{50}$
£. 22 14 $5\frac{23}{50}$

In like manner may all the foregoing examples be solved, which I leave for the learner's exercise.

Method 3, by Decimals.

By the application of decimals we may easily discover the proportionable rate of 1*l.* which is a common multiplier, to any given sum at that rate.

A TABLE

Of the Rates of 1*l. at any Rate per Cent.*

Rate per Cent.	*Rate of* 1*l.*	*Rate per Cent.*	*Rate of* 1*l.*
2	.02	4	.04
2¼	.0225	4¼	.0425, *&c.*
2½	.025	5	.05
2¾	.0275	6	.06
3	.03	7	.07
3¼	.0325	8	.08
3½	·035	9	.09
3¾	.0375	10	.1, *&c.*

The Use.

These rates of 1l. are common multipliers for any sum given at the correspondent rate per cent. (the lesser denominations being first reduced to the decimal of 1l.) which will plainly appear from the following

Examples

I demand the annual interest of 264l, 16*s.* 3*d.* at 5 per cent. ?

l. s. d.
264 16 3=264.8125
·05 Rate at 1l.

13.240625

£ 13 14 9¾

What is the commission on 199*l.* 3*s.* 6*d.* at $2\frac{1}{2}$ *per cent?*

```
  199.175
     .025
 --------
   995875
  398350
 --------
 4.979375
 --------
£.4 19 7
```

Thus likewise may all the foregoing Examples be solved.

CHAP. V.

INTEREST.

ANY sum of money being lent out upon consideration of receiving the same again, with a Premium from the Borrower for the use of it a certain time, is said to be put out at Interest; and the Interest is usually rated by the 100*l.*

The sum lent out is called the Principal.

Interest is the Premium or allowance for the use of the principal.

The Rate is the interest of 100*l.* for 1 year.

The Amount is the principal and its interest together.

Interest is Simple or Compound.

Simple Interest is that which arises from the principal only: As if 100*l.* were lent for two years, the Simple Interest thereof for the two years will be 12*l.* viz. 6*l.* due at the first year's end, and 6*l.* due at the second year's end.

Compound Interest is that which arises from the amount of the preceding year, viz from the principal and the interest likewise as it becomes due: So if 100*l.* be lent and forborne two year's at Compound Interest, the amount at the first year's end 106*l.* becomes a new principal; that is, Interest must be paid the second year not only for the 100*l.* lent out, but likewise for the 6*l.* interest due thereupon.

SIMPLE INTEREST.

Case I.

To find the interest of any given sum for 1 year having the rate given.

By having the rate given we have manifestly this proportion, 100 : the rate : : the given sum : to its interest, which is solved in all cases by Chap. IV.

Examples.

1. What is the interest of 270*l.* 10*s.* 6*d.* at the rate of 5 ⅌ cent? *Answ.* 13*l.* 10*s.* $6\frac{3}{10}$*d.*

```
 l.  s.  d.
270  10  6          Or thus,            Or thus.
         5          270 10 6            270,525
———— ——. l.        ————————                ,05
13|52 12 6  5 of 100, is 1/20 13 10 6 6/20   ———————
  20                                     13, 52625
————                                  £. 13 10 6 1/4
10|52
  12
————
 6|30
```

2. What is the interest of 300*l.* 10*s.* for 1 year, at the rate of 8 ⅌ cent. ⅌ ann.? *Answ.* 24*l.* 0*s.* $9\frac{3}{5}$*d.*

3 Tell me the interest of 334*l.* 17*s.* 6*d.* for 1 year, at the rate of 6 ⅌ cent. ⅌ ann.? *Answ.* 20*l.* 13*s.* $10\frac{1}{5}$*d.*

4. What is the interest of 246*l.* 18*s.* 10*d.* for 1 year, at 3 ⅌ cent. ⅌ ann.? *Answ.* 7*l.* 8*s* $1\frac{4}{5}\frac{2}{0}$*d.*

5. What is the interest of 220*l.* for a year, at 4 ⅌ cent. per ann.? *Answ.* 8*l.* 16*s.*

Case II.

To find the interest of any sum for any number of months, years, or years and months.

Rule.

1. Find the inteerst of the given sum for 1 year.

2. For

2. *For Years.*

Multiply the interest for 1 year by the number of years given.

3. *For Months.*

Divide the months into aliquot parts of a year, and divide the interest of 1 year by the denominator or denominators, &c. as in Practice.

Or thus:

Find the rate or interest of 100*l.* for the given time.

Then it will be

100 : its interest for the given time : : any sum : its interest for the same time.

Examples.

Cast up the interest of 348*l.* 15*s.* for 18 months, at 7 ℘ cent. *Answ.* 36*l.* 12*s.* 4½*d.*

l. *s.*
348 15
7
24|41 5
20
8|25
12
3|00

l. *s.* *d.*
24 8 3 for 12 Mo.
½ 12 4 1½—6 ditto.
36 12 4½—18 Mo.

348.75
.07
Yr. 1=24.4125
½=12.20625
36.61875
36 12 4½

Or thus,

348 15 for 18 Mo. at 7 ℘ cent.
7 Rate for 12 Mo.
6 Mo. ½ 3 10 6
10 10 18
348 15
10½
3487 10
174 7 6
36|61 17 6
20
12|37
12
4|50
4
2|00

7. Find the interest of 1000*l.* for 21 months, at 8 per cent? *Answ.* 140*l.*

8. Tell me the interest of 1090*l.* for 17 months at 6 per cent? *Answ.* 92*l.* 13*s.*

9. How much is the interest of 175*l.* 10*s.* 6*d.* for 1 year and 7 months, at 6 per cent. per annum?

Answ. 16*l.* 13*s.* $5\frac{97}{100}$*d.*

10. What is the interest of 514*l.* for 1 year $7\frac{1}{2}$ months at 5 per cent. per annum? *Answ.* 41*l.* 15*s.* 3*d.*

11. Cast up the interest of 886*l.* 16*s.* for 1 year $11\frac{1}{2}$ months, at 4 per cent per ann.? *Answ.* 69*l.* 9*s.* $3\frac{2}{5}\frac{1}{5}$*d.*

12. What is the interest of 479*l* 18*s.* for 2 years, at $4\frac{1}{2}$ per cent. per annum? *Answ.* 43*l.* 3*s.* $9\frac{1}{2}\frac{1}{3}$*d.*

13. What is the interest of 571*l* 15*s.* for 8 months at 6 per cent. per annum? *Answ.* 22*l.* 17*s.* $4\frac{4}{5}$*d.*

14. What is the interest of 300*l.* for $5\frac{3}{4}$ years, at $3\frac{3}{4}$ per cent. per annum? *Answ.* 64*l.* 13*s.* 9*d.*

A Contraction.

Since 5*l.* is $\frac{1}{20}$ of 100, whenever the interest of 100 is 5; the interest of any sum at the same rate will be $\frac{1}{20}$ part of itself. But the interest of 100 is 5.

$$\text{at} \left\{\begin{matrix}5\\6\\4\\3\end{matrix}\right\} \text{℘ cent. for} \left\{\begin{matrix}12\\10\\15\\20\end{matrix}\right\} \text{Months.}$$

Wherefore when any sum at the said rates and time is given, divide it by 20, and the quotient is the interest required.

Examples.

15. What is the interest of 379*l.* 15*s*, for 12 months, at 5 per cent.?

l. *s.* 2|0) 37|9 15 *Answ.* £.18 19 9	For the odd shillings and pence value every 10*s.* equal to 6*d.* and lesser sums in proportion, *i. e.* 10*s.*=6*d.* 6*s* 8*d*=4*d.* 5*s*=3*d.* 4*d.*=2*d* 2*s.* 6*d*=$1\frac{1}{2}$*d* 1*s.* 8*d*=1*d.* 1*s.* 3*d.*=$\frac{3}{4}$*d.* 10*d.*=$\frac{1}{2}$*d* 5*d.*=$\frac{1}{4}$*d.*

16. What is the interest of 416*l.* 12*s.* 6*d.* for 10 months, at 6 per cent. per annum? *Answ.* 20*l.* 16*s.* $7\frac{1}{2}$*d.*

17. What is the interest of 427*l.* 13*s.* 9*d.* for 20 months, at 3 ℘ cent. ℘ annum? *Answ.* 21*l.* 7*s.* $8\frac{1}{4}$*d.*

18. Find the interest of 927*l.* 11*s* 3*d.* at 4 ℘ cent for 15 months? *Answ.* 46*l.* 7*s.* $6\frac{3}{4}$*d.*

Hence, to find the interest of a given principal, for any other time.

Rule.

Take parts for the time required, out of the time correspondeut to the given rate.

19. What is the interest of 279*l.* 11*s.* for 4 months, at 5 ℘ cent. per annum?

		l.	*s.*		
	2|0)	27|9	11		
of 12 Mo. 4 is	$\frac{1}{3}$	13	19	$6\frac{3}{5}$	Int. for 12 months.
Answ.	£.	4	13	$2\frac{1}{5}$	Int. for 4 months.

20. What is the interest of 197*l.* 11s. for 2 months, at 6 ℘ cent? *Answ.* 1*l.* 19*s.* $9\frac{2}{75}$*d.*

21. What is the interest of 491s. 17*s.* 10*d.* for 5 months, at 4 ℘ cent? *Answ.* 8*l.* 3*s.* $11\frac{17}{30}$*d.*

22. What is the interest of 279*l.* 11*s.* for $7\frac{1}{2}$ months, at 4 ℘ cent? *Answ.* 6*l.* 19*s.* $9\frac{3}{10}$*d.*

23. What is the interest of 571*l.* 15*s.* for 8 months, at 6 per cent per annum?

		l.	*s.*		
	2|0)	57|1	15		
Mo.					
8 Co.	$\frac{1}{5}$	28	11	9	Int. for 10 months.
		5	14	$4\frac{1}{6}$	——————2——————deduct.
		22	17	$4\frac{4}{5}$	

24. How much is the interest of 175*l.* 10*s.* 6*d.* for 1 year and 7 months, at 6 per cent per annum?

Answ. 16*l.* 13*s.* $5\frac{3}{4}$*d.* 19 months=10×2—1=$\frac{1}{10}$ of 10. or 20—1 *i. e.* $\frac{1}{10}$—$\frac{1}{20}$.

And thus may any other sums at these rates be performed, particularly 8, 9, 10, 11, 13 preceding.

Case

Case III.

Interest for Days.

By the principles assumed we have the following analogy in days.

Days : the given time : : the interest of the given sum for 1 year : to the interest required. Alternately

Days : interest of the given sum for 1 year : : given time : interest required.

25. What is the interest of 100l. from the 7th of May to the 26th of September, at 8 per cent. per annum?

From 31 days in the month of May
Take 7

May 24
June 30
July 31
Aug. 31
Sep. 26

			l.	s	d.
365 ——	8 ——	142 ——	3	2	$2\frac{70}{73}$

26. At 6 per cent. what will the interest be of 100l. from the 5th of July to the 9th of January?
Answ. 3*l.* 1*s.* $9\frac{51}{73}$*d.*

27. How much is the interest of 100*l.* from the 3rd of April to the 25th of February following, at 7 per cent?
Answ. 6*l.* 5*s.* $9\frac{51}{73}$*d.*

28. What is the interest of 100*l.* from June 1st, 1767. to March 9th, 1768, which was leap year, at 5 per cent. per annum? *Answ.* 3*l.* 17*s.* $3\frac{9}{73}$*d.*

Now if any sum be given to find the interest thereof for days, it is manifest we must first find the interest thereof for a year, per Ch. I. and then we may find the interest for the given number of days as in the foregoing questions.

Examples.

29. How much is the interest of 175*l.* for 1 year and 73 days, at 8 per cent. per annum?

l. *l.* *l.* *l.*
100——8——175——14
Days *l.* Days
365——14——73 2 16

£.16 16

And since the answer of the first stating is the second number of the second stating, these questions belong to the Double Rule of Three Direct, and may be solved by one double stating, *viz.*

$$\begin{array}{llcll} & \textit{l.} & & \textit{l.} & \\ & 100 & & \cancel{175}\ \ 35 & \\ 73 & \cancel{365} & > 8 < & 438 & \textit{Answ.}\ 16l.\ 16s. \end{array} \qquad \begin{array}{r} 365 \\ 73 \\ \hline 438 \end{array}$$

30. Cast up 240*l.* for 1 year and 135 days, at 7 ℔ cent. ℔ annum? *Answ.* 23*l.* 0*s.* $3\frac{21}{73}d.$

31. What is the interest of 200*l.* from August 14th, to December the 19th following, at 6 ℔ cent. ℔ annum? *Answ.* 4*l.* 3*s.* $6\frac{6}{73}d.$

32. What is the interest of 371*l.* for 1 year and 213 days, at 6 ℔ cent. per annum? *Answ.* 35*l.* 5*s.* $0\frac{36}{1825}d.$

33. How much is the interest of 150*l.* 15*s.* 6*d.* for 53 days, at 5 ℔ cent. ℔ ann.? *Answ.* 1*l.* 1*s.* $10\frac{2629}{3650}d,$

34. A merchant takes at interest 250*l.* at 8 ℔ cent. for 2 years, with condition to pay before the time as much of the principal as he pleases: Now at the expiration of 9 months he pays 80*l.* and 6 months after 70*l.* leaving the rest the full time of the aforesaid two years, I demand the sum said merchant is then to pay? *Answ.* 127*l.* 16*s.*

$$\begin{array}{llcll} & 100 & & 250 & \textit{l.} \\ \cancel{4} & \cancel{12} & > \cancel{8}\ 2 < & \cancel{9}\ 3\ \text{is} & 15 \end{array} \qquad 1\text{st}—\frac{\cancel{8}\times 250\times \cancel{9}}{100\times \cancel{12}}$$

Or,

$$\frac{\cancel{8}\times 250\times 3}{100\times \cancel{4}} \quad \frac{2\times 250\times 3}{100}$$

From 250
Take 80 the 1st payment.

$$\begin{array}{llcllll} & 100 & & 170 & \textit{l.} & \textit{s.} & \textit{d.} \\ 2 & \cancel{12} & > 8 < & \cancel{6}— & 6 & 16 & 2 \end{array} \qquad \frac{8\times 170\times 6}{100\times \cancel{12}}$$

$\cancel{100}|\;\cancel{8}\;\cancel{100}$
$\cancel{18}|\;2\;\;1\quad 9$——6

Interest	27	16
Principal	100	
	£.127	16

$$\frac{\cancel{8}\times170}{100\times2}\quad\frac{\cancel{2}\times170}{100}$$

$$\frac{d—8\times\cancel{100}\times9}{\cancel{100}\times\cancel{12}}$$

$$\frac{\cancel{1}\times9}{3}$$

35. Suppose I take at interest 300l. for 18 months, at 6 per cent. with condition to pay as much of the principal before the time as I please. Now after 3 months time I pay 60l. and 4 months after that 100l. and 5 months after that I pay 75l. I want to know what I have yet to pay, at the expiration of said 18 months?

Answ. 79l. 15s.

36. Given at interest 600l. the 13th of May 1757, for 1 year, at the rate of 5 ꝑ cent. ꝑ annum, with condition that the receiver may discharge as much of the principal as he pleases before the time. Now he pays the 9th of July 200l. and the 17th of September 150l. How much has he to pay for principal and interest at the expiration of the year? *Answ.* 266l. 13s. $5\frac{7}{73}$d.

Note. The pupil is recommended to work the questions in the six following Cases, by Decimals.

Case IV.

When the rate, time, and interest are given, to find the principal.

Rule.

Divide the interest by the product of the rate and time, the quotient is the principal.

.05
.3
.15)69.675

Examples.

37. I demand what principal being put to interest for 3 years, will gain 69l. 13s. 6d. at 5 ꝑ cent. ꝑ annum?

Answ. 464l. 10s. the principal.

38. I demand what principal being put to interest for $5\frac{1}{2}$ years will gain 64l. 7s. at $4\frac{1}{2}$ ꝑ cent. ꝑ annum?

Answ. 260l.

$$\frac{64.35}{5.5\times.045}\quad\frac{64.35100}{.2475}=260$$

39. I demand what principal being put to interest for 4 years, at 4 ℘ cent. will gain 67*l.* 15*s.* 9¾*d.*?

67.790625 *Answ.* 423*l.* 13*s.* 9½d.

44.04 .16

Case V.

When the amount, rate, and time are given to find the principal.

Rule.

Add 1 to the product of the rate and time, and by that sum divide the amount, the quotient is the principal.

Examples.

352.2205 40. What principal being put to interest will amount to 354*l.* 4*s* 0½*d.* in 7 years, at 3½ ℘ cent. ℘ annum?

.035×7+1 *Answ.* [illegible]. 10*s.* 3½*d.* or 5747

500.460416 41. What principal being put to interest will amount to 500*l.* 9*s* 3¼*d.* in 6 years and 5 months, at 5 ℘ cent.

6.416× ℘ annum? *Answ.* 378*l.* 17*s.* 11½*d.*

1.32083

100 42. What principal being put to interest for 7 years, 220 days, at 4¾ ℘ cent. ℘ annum will amount to 100*l.*?

7.60273× *Answ.* 73*l.* 9*s.* 4*d.* .0475+1 1.36113

Case VI.

When the principal, interest, and rate are given to find the time.

Rule.

Divide the interest by the product of the principal and rate, the quotient is the time.

Examples.

69.675

43. In what time will 464*l.* 10*s.* gain 69*l.* 13*s.* 6*d.* at

464.5×.05 ℘ cent. ℘ annum? *Answ.* 3 years.

64.35

44. In what time will 260*l.* gain 64*l.* 7*s.* at 4½ ℘ cent. ℘ annum? *Answ.* 5½ years.

260×.045=11.7

130.47916

45. In what time will 500*l.* gain 130*l.* 9*s.* 7*d.* at 6½ ℘ cent. ℘ annum? *Answ.* 4 years, 5¼ days.

.065×500=32.5

Case

Case. VII.

When the principal, amount, and rate are given to find the time.

Rule.

Take the difference between the amount and principal, and divide it by the product of the principal and rate, the quotient is the time.

284.5 X .0325 | 354.22083
-284.5

70.72083 / 9.24625 = 7.65

Examples.

46. In what time will 284l. 10s. amount to 354l. 4s. 0½d. at 3¼ ℘ cent. ℘ annum?

Answ. 7 years, 238$\frac{7}{10}$ days.

47. In what time will 672l. 5s. amount to 847l. 17s. 6d. at 4¾ ℘ cent. ℘ annum? *Answ.* 5½ years. .0475

48. In what time will 378l. 13s. amount to 500l. 9s. 3¼d. 464583 at 5 ℘ cent. ℘ annum? .9 rate .05

Answ. 6.41467, or 6 years, 5 months (nearly.)

Case VIII.

When the principal, interest, and time are given to find the rate ℘ cent.

Rule.

Divide the interest by the product of the principal and time, the quotient is the rate.

Examples.

49. At what rate ℘ cent. will 464l. 10s. gain 69l. 13s. 6d. in 3 years? *Answ.* 5 ℘ cent. .05

50. At what rate ℘ cent. will 260l. gain 64l. 7s. in 5½ years? *Answ.* 4½ ℘ cent. 64.35 / 260 X 5.5 = 1430

51. At what rate ℘ cent. will 560l. 12s. 8½d. gain 235l. 9s. 4d. in 7 years? *Answ.* 6 ℘ cent. 633476

Case IX.

When the principal, amount, and time, are given to find the rate.

Rule.

Take the difference between the amount and principal, and divide it by the product of the principal and time, the quotient is the rate.

Examples.

52. At what rate ℔ cent. will 284*l.* 10*s.* amount to 22083 354*l.* 4*s.* 0½*d.* in 7 years?
Answ. 3½ ℔ cent. or .35 or 3/2

53. At what rate ℔ cent. will 378*l.* 18*s.* amount to 4633470 500*l.* 9*s.* 3¼*d.* in 6 years?
Answ. 5*l.* 6*s.* 11¼*d.* ℔ cent. or .053//72

54. At what rate ℔ cent. will 672*l.* 5*s.* amount to 847*l.* 17*s.* 6*d.* in 5½ years?
Answ. 4¾ ℔ cent. or .0475

Case X.

The same being given to find the amount.

Rule.

The interest being found as before, add it to the principal, and that sum is the amount.

Example.

What will 1090*l.* amount to being forborn 17 months, at 6 ℔ cent. ℔ annum?

l.	*s.*	
1090	00	
92	13	Interest found as before.
£.1182	13	

Otherwise at one operation by the following proportion.

100 : 100+the rate : : any principal : the amount, *viz.*

l. *Mo.*
6 Int. of 12
$\frac{1}{3}$ 2 —— 4
$\frac{1}{4}$ 0 10 — 1

8 10 — 17 Months.
100——108½——1090
108½

8720
1090
545

1182|65 *Answ.* 1182*l.* 13*s.*
20

13|00

This Case may likewise be solved by Decimals by making 1 and its proportionate amount the first and second numbers, and then the answer is obtained by Multiplication only, *viz.*

1——1.085——1090
1.085

545
872
109

1182.65

£. 1182 13

To prevent the trouble of dividing by 365 in every computation of interest for days, the following Tables have been calculated; the first term is the interest of 1*l.* for one day at the given rate; that is, .000136986 the first number of the Table of 5 ℔ cent. found thus:

100 > 5 < 1
365 > < 1 *Answ.* .000136986.

From which the other numbers are made by *Addition.*

TABLES of *Simple Interest* for any Time at the sundry Rates following.

Days	4 ℔ Cent.	4½ ℔ Cent	5 ℔ Cent.	6 ℔ Cent.
1	.0001095890	.0001232877	.0001369863	.0001643836
2	.0002191780	.0002465753	0002739720	.0003287671
3	.0003287671	.0003698630	.0004109589	0004931506
4	.0004383561	.0004931506	.0005479452	.0006575342
5	.0005479452	.0006164383	.0006849315	.0008219178
6	.0006575342	.0007397259	.0008219178	.0009863013
7	.0007671232	.0008630136	.0009589041	.0011506849
8	.0008767123	.0009863012	.0010958904	.0013150684
9	0009863013	.0011095899	.0012328767	0014794520
10	.0010958904	.0012328767	.0013658630	.0016438356
20	.0021917808	.0024657534	.0027397260	.0032876712
30	.0032876712	.0036986301	0041095890	.0049315068
40	.0043835616	.0049315068	.0054794520	0065753424
50	.0054794520	.0061643835	.0068493150	.0082191781
60	.0065753424	.0073972602	.0082191780	.0098630136
70	.0076712328	.0086301369	.0095890410	0115068492
80	.0087671232	.0098630136	.0109589040	.0131506848
90	.0098630137	.0110958904	.0123287671	0147945206
100	.0109589041	.0123287671	0136986301	.0164383562
200	0219178082	.0246575342	.0273972602	.0328767124
300	.0328767123	.0369863013	.0410958903	0493150686
Months.				
3	.01	.01125	.0125	.015
6	.02	.0225	.025	03
9	.03	.03375	3375	.045
Years.				
1	.04	.045	.05	.06
2	.08	.090	.10	.12
3	.12	.135	15	.18
4	.16	.180	.20	.24
5	.20	.225	.25	.30
6	.24	.270	.30	.36
7	.28	.315	.35	.42

Explication.

The first Column being Time, against it stands the Interest of 1*l.* for that Time at the Rate specified on the Top of the Column; but the Interest of 1*l.* for any Time being given, the Interest of any Sum at the same Time may be found by *Multiplication, viz.* by multiplying the Interest of f 1*l.* by the given Sum; and how to find the Interest of *l.* for any Time will easily appear by these

Examples.

1. Having the interest of 1*l.* for any days less than 10; to find the interest thereof for 10 times, 100 times, *&c.* so many days.

Rule.

Remove the decimal point one place to the right-hand for every cypher in the number of days for which it is required to find the interest of 1*l.*

Thus if the interest of 1*l.* at 5 ⅌ cent. for
3 days is .0004109589, it will be for
30 ——— .004109589, and for
300 ——— .04109589

Problem II.

Having the interest of 1*l.* for any number of days, to find the interest of any given sum for the same time.

Rule.

Multiply the interest of 1*l.* and the given sum together, and the product will be the interest required.

Note. If the number be given of divers denominations, the lesser denominations must be brought to the decimal of a pound.

Application.

What is the interest of 100*l.* from the 7th of May to the 26th of September, at 6 ⅌ cent. ⅌ annum?

Time found to be 142 days.

Interest of 1*l.* for 100 days .01643836
——————— 40 ——— .00657534
——————— 2 ——— .00098630
142 ——— .024
100
2.4

Answ £.2 8.

Example II.

Unto what amounts the interest of 79*l.* 15*s.* for 16 years and 73 days, at 5 ⅌ cent. ⅌ annum?

The

				l. *Yr.*	79.75
The interest of				1 for 1 is—.05	.81
				10 is —.5	7975
				6 is —.3	63800
	l.	*s.*	*d.*	70 *Ds.*—.009589041	
Answ.	64	11	11¼	3 ——.000410959	£.64.5975
				.8099999999	or 64 11 11¼

Thus may the Examples of Case 3rd be done.

Annuities and Pensions in Arrear.

An Annuity or Pension supposed payable upon the day it becomes due; but if it be withheld or forborn, *i. e.* remain unpaid beyond the day, it is said to be in Arrears.

Now if an annuity payable yearly be in arrear, and it is required to know what the same will amount to, simple interest being computed for every particular yearly payment from the time it became due until the time it is paid, how to calculate this we shall shew in the following

Examples.

1. If an annuity of 70*l.* be forborn 5 years, what will be due for principal and interest at the end of said term, simple interest being computed at 5 ℘ cent. ℘ annum?

Rule.

1. Find the interest of the given annuity for 1 year. 2. And then for 2, 3, &c. years upon the given time less 1. 3. Multiply the annuity by the number of years given. 4. Add the several interests and the said product into one sum.

The Operation.

1. 70	*l.*	*Yr.*	*l.*	*s.*
5	Interest of 70 at 5 ℘ cent. for	1—	3	10
		2—	7	00
350		3—	10	10
320		4—	14	00
——5	Years Annuity at 70*l.* *viz.*	70×5-	350	00
1000				
	Answ. £.		385	00

By *Decimals*.

70	1 *Yr.* — 3.5	
.05	2 — — 7	
——	3 — —10.5	
3.50	4 — —14	
	350	
	£.385	

2. If the payment of a pension be omitted for 7 years, what will it amount to in that time at 6 ⅌ cent. supposing it 56*l.* ⅌ annum? *Answ.* 462*l.* 11*s.* 2⅖*d.*

3. A house being let upon a lease of 7 years, at 50*l.* ⅌ annum, and the rent being in arrear for the whole term, I demand the sum due at the end of the term, simple interest being allowed at 4 ⅌ cent. ⅌ anuum?
Answ. 392*l.*

4. Suppose a salary of 100*l.* ⅌ annum be forborn 7 years, what is the amount at 4½ ⅌ cent. ⅌ annum?
Answ. 794*l.* 10*s.*

5. If any annuity or yearly rent of 134*l.* 10*s.* 6*d.* be all forborn till the end of 4 years, what will it then amount to; simple interest being allowed at the rate of 6 ⅌ cent. ⅌ annum? *Answ.* 586*l.* 10*s.* $6\frac{96}{100}$*d.*

But if simple interest were allowed upon half-yearly or quarterly payments, it would be still more in favour of the receiver.

Find the interest of the last payment but one, and then of the other as before, *viz.*

6. If 70*l.* annuity, payable every half year, were unpaid 5 years, what will it amount to in that time at 5 ⅌ cent.?

Last payment but one, half-a-year's annuity 35*l.*

	Yr.		*l.*	*s.*	*d.*		
35	½	——	0	17	6	Or	.875
.025	1	——	1	15	0	——	1.750
——	1½	——	2	12	6	——	2.625
175	2	——	3	10	0	——	3.500
70	2½	——	4	07	6	——	4.375
——	3	——	5	05	0	——	5.250
.875	3½	——	6	02	6	——	6.125
——	4	——	7	00	0	——	7.000
17 6	4½	——	7	17	6	——	7.875
Principal 35 × 10		——	350	00	0	——	350
			£.389	07	6		389.375

CHAP. VI.

REBATE OR DISCOUNT.

REBATE or Discount is an allowance made upon any sum of Money being paid before it becomes due; or upon advancing ready Money for Bills, Notes, &c. which are payable at a future day. And this allowance is sometimes computed at so much ℔ Pound Sterling, and sometimes at so much ℔ Cent. per Annum, Simple Interest.

1. *Discount computed per Pound Sterling.*

Discount at so much per pound sterl. may be computed by aliquot parts, *viz.* dividing the given sum to be discounted by the denominator of that part which the discount of 1*l.* is of 1*l.* The quotient is the discount, which being subtracted from the given sum, the remainder is the sum to be advanced in ready money.

Examples.

1. Bought goods of A B, amounting to 127*l.* 13*s.* 4*d.* and for prompt payment am allowed 6*d.* in the pound; I want to know how much I must pay him?

	l.	*s.*	*d.*		Decimally.
d. *l.*	127	13	4		127 6
6 of 1 is $\frac{1}{40}$	3	3	10	Discount	·0 25=6*d.*
Answ. £.	124	9	6		6380
					2552
					3.1900
					£ 3 3 10, &c.

2. Changed Nathan Needy, Richard Drawer's bill on Peter Paywell of Cork, at 31 days sight, for 154*l.* at 2*d.* per *l.* discount, how much ready money must I pay for said bill? *Answ.* 152*l.* 14*s.* 4*d.*

3. How much ready money must I advance for 572*l.* 17*s.* 8*d.* discount being allowed at 4*d.* per *l.*?

Answ. 563*l.* 6*s.* 8$\frac{1}{2}$*d.*

4. Bought 6 pipes of wine, each 121 gallons, at 4*s.* 9*d.* per gallon by auction, and for prompt payment am allowed 1*s.* per pound discount, what must I pay for said wine?

Answ. 165*l.* 16*s.* 0$\frac{9}{10}$*d.*

2. *Discount computed at a given Rate per Cent. per Annum.*

If 106 : 100 : : given sum; its present worth or payment, it shall be 106 : (106 — 100)6 : : any sum to be rebated to the same sum less its present worth, *i. e.* to the rebate.

Note. The rebate subtracted from the sum discounted leaves the present worth, *et contra.*

Examples.

5. A merchant buys certain merchandizes to the value of 648*l.* payable in 12 months; how much ready money must he pay rebate at 8 ꝑ cent. ꝑ annum?

100
8 l.
108——100——648
100 l.
108)64800(600 *Answ.*
648

To Prove the Work.

Find the amount of the present payment at the time and rate given, and that will be equal to the given sum to be rebated. 100——108——600
600
648|00

6. If a debt of 100l. 6s. be payable at the end of a year to come, how much ready money will discharge the debt at the rate of 6 ꝑ cent. ꝑ ann? *Answ.* 94l. 12s. $5\frac{23}{53}$*d.*

7. What is the rebate of 45l. 15s. 6d. for 12 months at 6 ꝑ cent. ꝑ annum? *Answ.* 2l. 11s. $9\frac{45}{53}$*d.*

8. I demand how much ready money I must pay for 432l. 14s. for 12 months, rebate at 7 ꝑ cent. ꝑ annum?
Answ. 404l. 7s. $10\frac{22}{107}$*d.*

When any other time beside a year is given, find the interest of 100 for that time at the given rate, which must be added to 100 for the first number, as in the following

Examples.

9. What is the rebate of 795l. 11s. 2d. for 11 months, at 6 ꝑ cent. ꝑ annum?

12—6—11 Or thus, Or thus,
11 6 6
12)66 6 mo. is $\frac{1}{2}$ 3 11 mo. co. $\frac{1}{12}$ 0 10
4 —— $\frac{1}{3}$ 2
£.5 6 1 —— $\frac{1}{4}$ 0 10 £.5 10
20
£.5 10 interest of 100l. for 11 months.
12)120
£.10

U Then

Then 105 10——5 10——795 11 2

20 20 20

2110 110 15911

12 12

25320 190934

100

2|0)

2532|0)2100274|0(82|9 5

20256

41 9 $5\frac{2292}{2532}$

7467

5064

24034

22788

1246

12

14592

12660

2292

By decimals.

105.5 5.5 795.5583

5.5

39777916

39777916

105.5)4375.57076(41.4746

4220.0

£. 41 . 9 . $5\frac{3}{4}$

1555

1055

5007

4220

7870

7385

4857

4220

6376

6333

l.	*s.*	*d.*	
795	11	2	
41	9	6	Rebate
£.754	1	8	to pay down.

10. What ready money must I pay for 161l. 10s. due in 19 months, discount being allowed at 5 ℘ cent. ℘ annum? *Answ.* 149l. 13s. $0\frac{3}{4}$*d.*

11. Bought goods amounting to 795l. 11s. 2d. at 4 months credit, how much ready money must I pay discount at $3\frac{1}{2}$ ℘ cent. ℘ annum? *Answ.* 786l. 7s. $8\frac{1}{4}$d.

12. What is the present worth of 4000l. payable in 9 months at $4\frac{3}{4}$ ℘ cent. ℘ ann.? *Answ.* 3862l. 8s. $0\frac{1}{2}$d.

13. Tell me the rebate of 112l. 12s. for 20 months, at 7 ℘ cent. ℘ annum? *Answ.* 11l. 15s. $3\frac{27}{67}$*d.*

14. Some merchandizes being bought for 5150l. with

$4\frac{1}{2}$ months discount, at 8 ℔ cent. ℔ annum; besides 1 ℔ cent. for prompt payment. How much ready money must I pay? *Answ.* 4950l.

III.

When sundry sums are to be paid at different times, find the rebate or present worth of each particular payment separately, and when so found add them into one sum.

Examples.

15. A is indebted to B 432l. payable in 12 months, more 580l. payable in 2 years: now if A has a mind to pay both these sums immediately, rebate being allowed at 8 ℔ cent. ℔ annum, how much must he pay?

	l.	l.	l.		
In 1 year,	108 ——	100 ——	432 comes	400	
In 2 years,	116 ——	100 ——	580 ——	500	
				Answ.	£.900

16. What is the rebate of 756l. the one half payable in 6 months, and the other half payable in 6 months after that, at 7 ℔ cent. ℔ annum? *Answ.* 37l. 10s. $2\frac{1018}{2461}$*d.*

17. I have A B's notes *viz* one for 20l. payable in 3 months, and another for 36l. payable in 9 months, and having occasion to raise money upon them; I get them discounted at 6 ℔ cent. ℔ annum; what money must I receive? *Answ.* 54l. 3s. $1\frac{281}{42427}$d.

18. Discounted the following notes at 5 ℔ cent. ℔ ann.
J D's to myself 150l. 10s. payable 37 days hence.
A B's to E L 27l. 15s. payable in 15 days.
B C to T P 18l. payable in 15 months.
I want to know how much money I must receive?
Answ. 194l. 7s. $6\frac{1}{2}$*d.*

19. What is the present worth of 200l. at 4 ℔ cent. ℔ annum, payable, viz. 100l. at 2 months, 50l. at 3 months, and 50l. at 5 months? *Answ.* 198l. 0s. 6d.

20. What ready money must I pay for 1000l. of which 300l. is payable in 1 year; 300l. payable in 2 years; and the rest in 3 years, discounting at 8 ℔ cent. ℔ annum?
Answ. $858\frac{23766}{24273}$l.

In like manner we find the present worth of an annuity, rebating at simple interest for any assigned number of years; for suppose it required to find the present worth of an annuity of 100l. it is manifest we must compute the present worth of 100l due at the first year, also the present worth of 100l. due at the end of the second year, and so on, repeating the operation for every year of the term, and it is plain the sum of all these present worths of each year's rent, will be, the present worth of the annuity for the term of years assigned.

Examples.

21. How much present money is equivalent to an annuity of 100l. to continue 5 years; rebate being made at 6 ⅌ cent, ⅌ annum?

106 : 100 : : 100 : 94.33962 pres. worth of 1 year's an.
112 : 100 : : 100 : 89.28571 ——— 2 ———
118 : 100 : : 100 : 84.74576 ——— 3 ———
124 : 100 : : 100 : 80.64516 ——— 4 ———
130 : 100 : : 100 : 76.92307 ——— 5 ———

Answ. £. 425.93932 or 425l. 18s. 9¼ *d.*

22. What is the present worth of an annuity of 50*l.* to continue 6 years, at 5 ⅌ cent. ⅌ annum, simple interest?
Answ. 256l. 13s. 7d.

23. What is 80*l.* yearly rent to continue 5 years, worth in ready money, at 6 ⅌ cent? *Answ.* 340l. 15s. 1d.

24. What is a salary of 40l. ⅌ annum, to continue 7 years, worth in ready money, at 4 ⅌ cent?
Answ. 242l. 10s. 9d.

25. What is a pension of 30l. ⅌ annum, for 5 years, worth in ready money at 4½ ⅌ cent.? *Ans.* 132l. 11s. 5½d.

IV.

26. A merchant is indebted 2163l. 3s. payable at 12 months; but pays it at the expiration of 5, discounting at the rate of 6 ⅌ cent. ⅌ annum, how much should he pay? *Answ* 2090l.

From 12 months time given,
Take 5 ——— when payment was made,
Rem. 7 ——— to be discounted for
l. Mo.
6 rebate for 12

6 is ½ 3 ——— 6
1 is ⅙ 0 10 ——— 1

3 10 ——— 7
100

103 10 ——— 100 ——— 2163 3
Answ. 2090l.

27. If the aforesaid sum was paid in 3 months, how much ought he to pay? *Answ.* 2070*l.*

28. Suppose a bill drawn the 25th of September, 1792, payable * 3 months after date, for 540*l.* 15*s.* was discounted on the 18th of October following, at 6 ℔ cent. ℔ annum, what sum was received for it?

Answ. 534*l.* 10*s* $3\frac{218}{807}$*d.*

29. A merchant owes 110*l.* payable in 20 months, and 224*l.* payable in 24 months: the first he pays in 5 months, and the other 1 month after, discount at 8 ℔ cent. ℔ annum: I demand the sum he paid? *Answ.* 300*l.*

30. Dublin, the 27th Sept. 1808, discounted for Leonard Lackcash, Chr Kiteflyer's promissory note at 2 months for 20*l.* dated the 1st. inst. Edw. Empty's do. dated 17th do. at 3 months for 37*l.* 10*s.* and Thos. Trusty's bill on Peter Paywell for 50*l.* at 31 days sight, accepted by said Paywell, the 20th inst. discount 6 ℔ cent. ℔ annum: what must I pay him for said notes and bill? *Answ.* 106*l.* 13*s.*

Note. Although the foregoing method is the true and proper way of casting up discount; yet the usual method in practice is to calculate the interest that would be due upon the sum discounted in the time, which the bill, note or debt hath to run; and deducting the said interest from the sum discounted, to pay the rest as full consideration for the sum discounted: this method is readier and easier than the true method before laid down, and in small sums for a short time the difference is inconsiderable, but the difference becomes very considerable, if a large sum be to be discounted; or if the time be long for which the discount is to be allowed.

Find the different sums to be paid for 100*l.* by both methods for 1 month, 1 year, and 10 years, discount being allowed at 6 ℔ cent. ℔ annum?

	True method.				Common method.		
	l.	*s.*	*d.*		*l.*	*s.*	
Answ.	99	10	$0\frac{120}{201}$	——	99	10	for 1 month,
	94	6	$9\frac{54}{106}$	——	94	0	for 1 year,
	62	10	0	——	60	0	for 10 years,

---

* *Note.* Three days are allowed beyond the day of date when a bill or note becomes payable, which are called *Days of Grace;* so these three days must be added, as this bill becomes due according to date 25th December, but according to custom on the 28th.

From the above it appears that the difference is less than $\frac{3}{4}$*d.* for 100*l.* for 1 month; but becomes more considerable as the time is longer; for by the common method the sum to be paid is 6*s.* 9$\frac{1}{2}$*d.* too little for a year, and 2*l.* 10*s.* in 10 years.

However as the last method is that most generally used in business, it may be proper that the learner be set to work some of the preceding questions thereby.

EQUATION OF PAYMENTS

Is when several debts are payable at different times, but is mutually agreed between debtor and creditor, that all those several sums be paid at once, and at such a time as that neither party may be wronged thereby; this is called Equating the time of payment, for which this is the

Rule.

Multiply the sum of each particular payment by its time, then add the products together, and divide the sum by the whole debt, the quotient (by this Rule) is the equated time for the payment of the whole.

Examples.

1. B owes C 600l. whereof 200l. is to be paid at 3 months, 150l. at 4 months, and the rest at 6 months; but they afterwards agreed the whole should be paid at once; required the time? *Answ.* 4 months, 15 days.

2. A bought of B a quantity of goods which came to 460*l.* to be paid in the following manner, viz. 260*l.* at 5 months, and the rest at 7 months, but afterwards they agree to make one payment of the whole; I demand the equated time? *Answ.* 5 months, 26$\frac{1}{23}$ days.

3. C. owes D a certain sum which is to be discharged in the following manner, viz. $\frac{1}{2}$ at 8 months, $\frac{1}{3}$ at 4 months, and $\frac{1}{6}$ at 9 months. but they afterwards agree to have but one payment of the whole; the equated time is required? *Answ.* 4 months, 10 days.

4. A debt is to be discharged thus, viz. $\frac{1}{4}$ at present, $\frac{1}{4}$ at 4 months, and $\frac{1}{4}$ at 5 months, and the rest at 6 months: what is the equated time for the whole? *Answ.* 5 mo.

5. E is indebted to F 240*l.* which by agreement is to be paid at 5 months hence, but E is willing to pay 40*l.* down, provided he will give him a longer time for the payment of the remainder, which is agreed on, the time of payment is required? *Answ.* 6 months.

CHAP. VII.

EXCHANGE.

THE doctrine of *Exchange,* taken in its full extent, would far exceed the bounds of a Chapter in such a Treatise as this; but as far as it is usually considered as a Rule of Arithmetic, it is chiefly comprehended in this Problem, *How to Reduct the Money of one Country into that of another, according to any given Rate or Proportion.*

In most countries they have real and imaginary monies, the real monies are the coins made, or current, in the country. The imaginary are those whereby they keep their accounts and calculate their payments.

Par, in exchange, is a supposed equality between the monies of one country and those of another, *i. e.* when the money received for a * Bill of Exchange is equal in value to the money paid for it, then exchange is said to be at par.

The course of exchange is the value which the coin of one country (taken for the standard of exchange) will yield in another.

So the par is fixed, being the supposed real value of foreign money in any country; but the course of exchange is variable and fluctuating; being sometime above and sometime below par: For bills of Exchange are a kind of commodity, which rise and fall in price according as there is a greater or less demand for them.

SECT. I.

Of Exchange *between* Dublin *and* London.

Dublin and other places of Ireland, exchange immediately with London only, and draw their foreign as well as English demands by that channel, so we shall shew first how to reduce English money into Irish, and *contra;* and then how to reduce foreign money into English, and the contrary.

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* *A Bill of Exchange* is a written order delivered in one place for value received there, for the like value, according to a rate of exchange agreed upon, to be paid in the place on which the bill is drawn.

The exchange between Ireland and England is calculated at a certain rate ⅌ cent. *viz.* Ireland gives 100*l.* + the rate of exchange for 100*l.* English.

In both kingdoms accounts are kept in pounds, shillings and pence: the coins or real monies of England are guineas, half guineas, crowns, shillings, &c. all which are current in Ireland, *viz.*

An English shilling is current in Ireland at 1s. 1d. and the other English coins in the same proportion, according to which proportion 100 English are equal to $108\frac{1}{3}$ Irish; So that the par of exchange between England and Ireland is $8\frac{1}{3}$ ⅌ cent.

The course of exchange is generally from 5 to 12 ⅌ cent.

I. *To reduce* English *Money to* Irish *at Par.*

Rule.

To the English money add $\frac{1}{12}$ thereof, the sum is the Irish money equal thereto.

50*l.* English, how much in Irish currency?

	50	0	0	
$\frac{1}{12}$	4	3	4	*Reason.* 1 *Eng.* = $1\frac{1}{12}$ *Irish.*
				Therefore 50 *Eng.* = $50+\frac{50}{12}$
£.	54	3	4	

2. How much in Irish currency will 15*l.* 17*s.* 6*d.* amount to? *Answ.* 17*l.* 3*s.* $11\frac{1}{2}$*d.*

3. Suppose I have a legacy of 150*l.* left me in England, and have it transmitted in specie, how much Irish currency am I to receive? *Answ.* 162*l.* 10*s.*

4. Sent by Daniel Draper to sell for me at Chester fair, 50 dozen of silk handkerchiefs, which he sold at 4*s.* 8*d.* ⅌ piece; now admit I allow him $2\frac{1}{2}$ ⅌ cent. commission, what sum Irish am I to receive? *Answ.* 147*l.* 17*s.* 6*d.*

II. *To reduce* Irish *Money into* English *at Par.*

Rule.

From the Irish money subtract $\frac{1}{13}$ thereof, the remainder is English.

	l.	*s.*	*d.*	
	54	3	4	Irish money how much English?
$\frac{1}{13}$	4	3	4	
Answ.	£.50	0	0	

6. How much English money will 17*l.* 3*s.* 11$\frac{1}{4}$*d.* Irish amount to? *Answ.* 15*l.* 17*s.* 6*d.*

7. I sent to my correspondent, at Liverpool, in specie, 162*l.* 10*s.* Irish, how much must he give me credit for on this account? *Answ.* 150*l.*

To reduce English *Money to* Irish *at any given rate of Exchange.*

1. The exchange is found in all cases by Chap. IV. of this.

2. Add the exchange to the English money, the sum is Irish.

Examples.

8. A merchant in Dublin, draws on his correspondent at London, for 120*l.* and passes his bill at 5 ℘ cent. how much money is he to receive? *Answ.* 126*l.*

1st. Method.	2d. Method by Aliquot Parts.		3d. by Decimals.
	l.	*l.*	
120	120 Eng.	120	120
5	6 Exch.	$5\frac{1}{20}$ 6	.05
6|00	126 Irish.	126	6.00

9. 270*l.* English, at 7 ℘ cent. how much Irish? *Answ.* 288*l.* 18*s.*

10. 510*l.* English, at 8 ℘ cent. what will it come to? *Answ.* 550*l.* 16*s.*

11. 955*l.* at 6 ℘ cent. what will it come to? *Answ.* 1012*l.* 6*s.*

12. 750*l.* 18*s.* 8*d.* at 7 ℘ cent. how much is it?

1st.	*l.* *s.* *d.*	
750 18 8	750 18 8—Eng.	
7	52 11 $3\frac{17}{25}$ Exch.	3d. Method.
		750.933
52|56 10 8	803 9 $11\frac{17}{25}$	.07
20		
	2d. Method	52.56531
11|30	*l.* *s.* *d.*	
12	750 18 8	52 11 $3\frac{3}{4}$ Exch.
3|68|17	$5\frac{1}{20}$ 37 10 $11\frac{1}{5}$	
	$2\frac{1}{50}$ 15 0 $4\frac{12}{25}$	
100|25	£.52 11 $3\frac{17}{25}$ Exch.	

	l.	s.	d.			l.	s.	d.
13.	901	1	11	at 8 ℘ cent.	*Answ.*	973	3	$8\frac{1}{25}$.
14.	300	15	6	at 9	———	327	16	$10\frac{37}{50}$.
15.	199	19	11	at 11	———	221	19	$10\frac{89}{100}$.
16.	999	19	9	at 3	———	1029	19	$8\frac{91}{100}$.

17. 309l. 15s. 4d. English, at $4\frac{1}{4}$ ℘ cent. advance, how much is it Irish? *Answ.* 322l. 18s. $7\frac{31}{50}$d.

l. s. d.
309 15 4
$4\frac{1}{4}$
1239 1 4
77 8 10
13|16 10 2
20
3|30
12
3|62 | 31
100 | 50

l. s. d.
309 15 4
13 3 $3\frac{31}{50}$
322 18 $7\frac{31}{50}$

2nd Method.
5) 309 15 4
$4 - \frac{1}{25}$ 5) 61 19 $0\frac{4}{5}$
$\frac{1}{4} - \frac{1}{16}$) 12 7 $9\frac{12}{25}$
0 15 $5\frac{43}{50}$
13 3 $3\frac{31}{50}$

3rd Method.
309.766
.0425
1547830
619532
1239064
13.1649550
13 3 $3\frac{1}{2}$,

Or shorter thus,
309.766
$.04\frac{1}{4}$
12 39064
$\frac{1}{4}$ - 77441
13.16505

	l.	s.	d.			l.	s.	d.
18.	709	0	0	at $7\frac{3}{4}$ ℘ cent.	*Answ.*	763	18	$11\frac{1}{5}$.
19.	250	10	0	at $3\frac{7}{8}$	— —	259	13	9
20.	103	0	11	at $5\frac{1}{3}$	— —	108	10	$9\frac{296}{300}$
21.	300	15	0	at $7\frac{1}{2}$	— —	323	6	$1\frac{1}{2}$
22.	460	19	9	at $5\frac{3}{4}$	— —	487	9	$10\frac{251}{400}$
23.	999	19	0	at $3\frac{7}{8}$	— —	1038	13	$11\frac{107}{200}$
24.	150	18	$6\frac{3}{4}$	at $5\frac{1}{2}$	— —	159	4	$7\frac{1}{800}$
25.	380	15	$9\frac{1}{2}$	at $5\frac{3}{4}$	— —	402	13	$8\frac{317}{800}$
26.	400	0	$6\frac{1}{4}$	at $7\frac{1}{8}$	— —	428	10	$6\frac{89}{128}$
27.	519	19	$11\frac{3}{4}$	at $6\frac{3}{4}$	— —	555	1	$11\frac{1173}{1600}$
28.	100	10	$6\frac{3}{4}$	at $5\frac{1}{8}$	— —	105	13	$6\frac{461}{640}$

To reduce Irish *Money into* English.

29. A merchant in London has a bill of exchange for 100l. Irish money, which he sells or negociates at 5 ℘ cent. that is, 105l. in Ireland is worth 100l. in England. I demand how much English money he is to receive?

Irish English Irish
105——100——100
Answ. 95l. 4s. $9\frac{1}{2}$d.

13.

	l.	s.	d.				l.	s.	d.
30.	160	10	0	at 7	ꝑ cent. ?	*Answ.*	150	0	0
31.	545	14	0	at 7	—	—	510	0	0
32.	1093	13	3	at 7$\frac{3}{4}$	—	—	1015	0	0
33.	323	6	1$\frac{1}{2}$	at 7$\frac{1}{2}$	—	—	300	15	0
34.	62	2	0	at 3$\frac{1}{2}$	—	—	60	0	0
35.	100	0	0	at 5$\frac{1}{8}$	—	—	95	2	5$\frac{8}{8}$44
36.	314	5	0	at 4$\frac{3}{4}$	—	—	300	0	0
37.	259	13	9	at 3$\frac{7}{8}$	—	—	250	0	0

SECT. II.

Of Exchange between England, Holland *and* Flanders.

England exchanges with Holland and Flanders at so many schellings and groot or deniers Flemish ꝑ pound sterling.

Books of accounts are kept in Holland sometimes in pounds, schellings and groot Flemish; but more frequently in guilders, stivers and pennings.

The par of exchange between England and Holland, is about 36.59s. ꝑ pound sterling, and between London and Antwerp, 35l. 17s.

8 Pennings, 12 Groots, 20 Schellings } make { 1 Groot, 1 Schell, 1 Pound
2 Groots, 6 Stivers, 20 Stivers, 6 Guild. } make { 1 Stiver. 1 Schell. 1 Guild. 1 Pound.

8 Pennings	make	1 Groot
12 Groots	make	1 Schell
20 Schellings	make	1 Pound
2 Groots	make	1 Stiver.
6 Stivers	make	1 Schell.
20 Stivers	make	1 Guild.
6 Guild.	make	1 Pound.

To reduce English *Money into* Flemish pounds:

38. Reduce 482l. 18s. sterling, into Flemish pounds, exchange at 35s. 10d.

	l.	s.			s.	d.
l: s.	482	18	at 1	15	10
of 1—10$\frac{1}{2}$	241	9			
of 10s. 5$\frac{1}{2}$	120	14	6		
of 5 10d. $\frac{1}{6}$	20	2	5		
£.	865	3	11	*Answ.*	

*** For 33 4 multiply the sterling money by 5 and divide the prod. by 3. For 400 (pence in 33s. 4d.) : 240 (p. in 20s.) : : 5 : 3 & contra 240 : 400 : : 3 : 5.

39. A merchant in Antwerp hath a bill of exchange for 100l. payable in London, which he negociates at 33s. 4d. Flemish ꝑ pound sterling; how much Flemish money must he receive? *Answ.* 166l. 13s. 4d.

d. Sterl.
8) 24|0 : : 40|0 : : 100 : : 166$\frac{2}{3}$, or 13s. 4d.
3 5

0. A merchant in Rotterdam hath a bill drawn on him for 673*l.* 16*s.* 8*d.* sterl. exchange at 33*s.* 4*d.* Flemish ⅌ pound sterling; how much must he pay there?

Answ. 1123*l.* 1*s.* $1\frac{1}{3}$*d.*

41. One in Antwerp delivering money by exchange for London, at 35*s.* 6*d.* Flemish ⅌ pound sterling; how much must he pay there to receive here 597*l.* sterling?

Answ. 1059*l.* 13*s.* 6*d.* Flemish.

42. How many pounds Flem. are in 169*l.* 10*s.* $1\frac{1}{2}$*d.* sterl. at 33*s.* 3*d.* ⅌ pound sterl. *Answ.* 281*l.* 17*s.* $4\frac{63}{80}$*d.*

To reduce Flemish *Money into* Sterling.

43. Reduce 865*l.* 3*s.* 11*d.* Flem. into sterl. at 35*s.* 10*d.* Flem. ⅌ pound sterling?

```
 s.  d.     l. Sterl.   l. Fl.   s.   d.
 35  10.————1————————165     3    11
 12                       20
 ———                   ————
 430                   17303
                          12
                       —————        l.   s.
              43|0)  20764|7   (482  18 Answ.
                     172
                    ————
                      356
                      344
                     ————
                      124
                       66
                     ————
                      387
                       20
                     ————
                      774|0
                      43
                     ————
                      344
                      344
                     ————
```

44. If 1*l.* sterling is 33*s.* 4*d.* Flemish, how much sterling is equal to 166*l.* 13*s.* 4*d.* Flemish? *Answ.* 100*l.*

45. How much sterling is equal to 1123*l.* 1*s.* $1\frac{1}{4}$*d.* Flem. at 33*s.* 4*d.* ⅌ pound sterling? *Answ.* 673*l.* 16*s.* 8*d.*

46. How much English money is 1059*l.* 13*s.* 6*d.* Flem. exchange 35*s.* 6*d.* Flem. ⅌ pound sterl.? *Answ.* 597*l.*

To reduce Sterling *Money into* Guilders.

47. How many guilders bank are equal to 852l. 12s. 6d. sterling, exchange at 34s. $4\frac{1}{2}$d. ꝑ pound sterling?

1st Method by the Rule of Three.

l. sterl. s. *d.* *l.* s. *d.*

1 —— 34 $4\frac{1}{2}$ —— 852 12 $6=\frac{5}{8}$

12 825

412

2

825

$852\frac{5}{8}$

1650

4125

6600

$\frac{4}{8}$ $\frac{1}{2}$ — $412\frac{1}{2}$

$\frac{1}{8}$ $\frac{1}{4}$ — $103\frac{1}{8}$

Guild. *Sti.* *Pen.*

Answ. 8792 13 $13\frac{1}{2}$

8|0)70341|$5\frac{5}{8}$ Half pence or half groot.

8792 4) $55\frac{5}{8}$ Half groot.

$13\frac{29}{32}$=13 stiv. 29 half pen.

i. e. $14\frac{1}{2}$ pennings.

2nd Method. Reduce the pounds sterling to pounds Flemish, and then the Flemish pounds and shillings into guilders and stivers, by multiplying by 6.

s.	d.		l.	s.	d.		s.	d.
			852	12	6	at	31	$4\frac{1}{2}$
10	0	— $\frac{1}{2}$	426	6	3			
4	0	— $\frac{1}{5}$	170	10	6			
0	4	— $\frac{1}{12}$	14	4	$2\frac{1}{2}$			
	$\frac{1}{2}$	— $\frac{1}{8}$	1	15	$6\frac{5}{16}$			
			1465	8	$11\frac{13}{16}$			
					$3\frac{15}{16}$			
			1465	8	$15\frac{3}{4}$			
					6			
			8792	13	$14\frac{1}{2}$			

To reduce Flemish pounds, shillings and pence to guilders, stivers and pennings, add $\frac{1}{3}$ of the pence to the number of pence, and then multiply the pounds, shillings and pence so encreased $\frac{1}{3}$ by 6.

Method 3. Reduce the rate of exchange into guilders, stivers and pennings, then multiply the sum English by the guilders, and for the stivers and pennings take the parts of a guilder, &c. The English money is pounds, shillings and pence, which are something different from the form of guilders, stivers and pennings. Yet I may put them into the form thereof by adding $\frac{1}{3}$ to the pence of the given sum, and multiplying and dividing the denominations as guilders, stivers and pennings; but as we are more used to the form of pounds, shillings and pence, we may multiply and divide them in that form, and by adding $\frac{1}{3}$ of the resulting pence to themselves, we reduce them to pennings.

	l.	*s.*	*d.*		*l.*	*s.*	*d.*
	852	12	6 sterl. at		1	14	$4\frac{1}{2}$ Flem.
			10				6 guilders 1*l.*
Sti. Pen.	8526	5	0		10	6	3
5——$\frac{1}{4}$	213	3	$1\frac{1}{2}$				
1——$\frac{1}{5}$	42	12	$7\frac{1}{2}$				
0 3—$\frac{1}{4}$	10	13	$1\frac{7}{8}$				
	8792	13	$10\frac{7}{8}$				
			$3\frac{5}{8}=\frac{1}{3}$ of $10\frac{7}{8}$ add.				
Answ.	8792	13	$14\frac{1}{2}$				

48. How many guilders are in 1409l. sterl. if 1*l.* sterl. is 33s. 8d. Flemish? *Answ.* 14230 guild. 18 stiv.

49. In 100*l.* 10s. sterl. at 33s. 9d. Flemish ℘ *l.* sterl. how many guilders is it? *Answ.* 1017 guild. $14\frac{1}{4}$ stiv.

50. Tell me how many guilders are in 1941l. 15s. 6d. sterling the exchange at 33s. 11d. Flemish ℘ *l.* sterling?

Answ. 19757 guilders, 11 stivers, $3\frac{96}{240}$ pennings.

51. At 34s. 5d. Flemish, ℘ *l.* sterling, I would know how many guilders there are in 1597l 9s. 11d. sterling?

Answ. 16494 guilders, 2 stivers, $14\frac{56}{240}$ pennings.

To reduce Guilders *into* Sterling *Money.*

52. If the exchange between London and Amsterdam be 34s. $4\frac{1}{4}$d. Flemish ℘ *l.* sterling, how much English money will be equal to 8792 guilders, 13 stivers, $14\frac{1}{2}$ pennings?

s. *d.* *l. Sterl.* *G.* *St.* *Pen.*
34 4½——————1——————8792 13 14½
12 20

412 Groot 175853 Stivers
8 16

3300 Penning 1813662 Penning
2 2
l. *s.* *d.*
6600 66|00) 36273|25 (852 12 6 *Answ.*

53. How many pounds sterl. are equal to 1680 guilders at 33s. 3d. Flem. ℘ *l.* sterl.? *Answ.* 168l. 8s. 5$\frac{21}{339}$d.

54 In 2080 guilders, 15 stivers, at 34s. 9d. Flem. ℘ *l.* sterling, how many pounds sterling is it?
Answ. 199l. 11s. 10$\frac{66}{417}$d.

55. At 33s. 11d. Flemish ℘ *l.* sterling, I demand how many pounds sterling are there in 6048 guilders?
Answ. 594l. 7s. 11$\frac{215}{407}$d.

56. 2048 guilders, 15 stivers, at 34s. 5d. Flemish ℘ *l.* sterling, how many pounds sterling is it?
Answ. 198l. 8s. 6$\frac{114}{413}$d.

57. In 1000 guilders, at 33⅓s. Flemish ℘ *l.* sterl. how many pounds is it? *Answ.* 100l.

To convert Bank Money at Amsterdam *into current Money.*

In Holland the money of exchange is called bank money, and is better than current money by 4 or 5 ℘ cent. *i. e.* 100 bank money makes 104 or 105l. current money: The difference between the bank and current money, is called the agio, as when 100l. bank is worth 104l. current, they say the agio is 4 ℘ cent.

Therefore bank money is reduced to current just as English money is reduced to Irish, and current to bank as Irish to English.

Example.

Guild. Stiv.
In 14230 18 bank money, how much current money, agio 5 ℘ cent.

```
 14230  18       Or,  14230    18           Or,  14230.9
         5            ____________                    .05
 _________       5 - 1/20 711   10 9/10             _______
 711|54  10                                         711.545
     20                                             _______
    ____                G.    St.                 . 711.10 9/10
   10|90          Bank 14230  18
      10          Agio   711  10  14
   _____                ____________
   14|40 Current       14942   8  14 Answ.
```

SECT. III.

Of Exchange between England *and* Hamburgh.

England exchanges on Hamburgh as on Holland and Antwerp, from 32 to 35s. gross or Flem ⅌ *l.* sterl.

Books of accounts are kept in marks, sous and deniers-lubs, and by some in rixdollars, sous or schellings, and deniers, and by others in *l. s. d.* Flemish.

The par of exchange between London and Hamburgh is about 35.17*s.* Flem. ⅌ *l.* sterl.

12 Deniers-Lubs 2 Deniers-Gros	make	1 Sous-Lubs,
16 Sous-Lubs		1 Mark Lubs,
3 Marks-Lubs		1 Rixdollar,
7½ Marks-Lubs 2½ Rixdollars		1 Pound Gros or Flem.
6 Deniers-Lubs		1 Denier Gros
6 Sous-Lubs		1 Shilling Gros,
8 Shillings Gros		1 Rixdollar,
30 Shillings Gros		1 Pound Gros.

To reduce *l.* sterl. into *l.* gros, differs nothing from § 2.

To reduce l. Sterling *into* Marks Lubs.

Reduce the rate of exchange into sous-lubs and multiply the *l.* sterling thereby, and for the odd shillings and pence sterl. take parts of the exchange, the sum of said product and parts will be the answer in sous-lubs, which reduce to marks by the Table.

58. A bill of 254l. 6s. 8d. sterl. being drawn at London upon Hamburgh, at 32s. 4d. gros ⅌ *l.* sterl. how many marks lubs are to be paid for that bill?

Answ. 3083 marks. 12 so. 8 den.-lubs.

l. s. d. l. s. d.
1 ——— 32 4 ——— 254 6 8
6 + 4de. = 2so. 194
——— ———
194 1016
2286
254
6s. 8d. is $\frac{1}{3}$l. $64\frac{2}{3}$
———
16)$49340\frac{2}{3}$ or 8d.
———
Answ. 3083 12 8

59. How many marks-lubs are equal to 200l. 10s. at $32\frac{1}{2}$ gros ⅌ l. sterling? *Answ.* 2443 marks, $9\frac{1}{2}$ stivers or sous-lubs.

60. How many marks-lubs are in 550l. 15s. sterling, at 32s. 9d. gros ⅌ l. sterling? *Answ.* 6763 marks, $14\frac{3}{8}$ stivers-lubs.

61. In 1000l. sterling, at 32s. 3d. gross ⅌ l. sterl. how many mark-lubs? *Answ.* 12093 mar. 12 stiv. lubs.

62. In 345l. 10s. 6d. sterling, at 32s. 6d. gros ⅌ l. sterling, how many marks-lubs? *Answ.* 4211 marks, $1\frac{3}{8}$ stivers-lubs.

To reduce Marks *to Pounds* Sterling.

63. 3083 marks, 12 so. 8 den. lubs, how much sterling money, exchange at 32s. 4d.?

s. d. Mks. So. Den.
32 4 ——— 1 ——— 3083 12 8
12 16
——— ———
388 Den. Gross 18500
6 3084
——— ———
2328 Den. Lubs. 49340 So. Lubs.
12
———
2328)592088(254 6 8

Otherwise thus:

32 4 ——— 1 ——— 3083 $12\frac{2}{3}$
6 16
——— ——— l. s. d.
194 So Lubs. 194)$49340\frac{2}{3}$(254 6 8

64. In 2443 marks, 9½ stivers-lubs, how many l. sterling, at 32s. 6d. gros ⅌ l. sterling? *Answ.* 200l. 10s.

65. In 6763 marks, 14⅜ stivers-lubs, at 32s. 9d. gros ⅌ l. sterling, how many l. sterling does it make?
Answ. 550l. 15s.

66. In 12093 marks, 12 stivers-lubs, at 32s. 3d. gros ⅌ l. sterling, how many pounds sterl? *Answ.* 1000l.

67. How many pounds sterling are in 4211 marks, 1⅜ stivers lubs, at 32s. 6d. gros ⅌ pound sterling?
Answ. 345l. 10s. 6d.

SECT. V.

Of Exchange between England *and* France.

England exchanges with France on the crown of 3 livres tournois or 60 sols, and gives pence sterling more or less for this exchange crown.

Accounts are kept in France in livres, sols and deniers.

12 Deniers —— 1 Sol.
20 Sols —— 1 Livre.
3 Livres —— 1 Crown.

The par of exchange between England and France, is about 29⅜ sterling, ⅌ crown, according to Dowling, Postlewait puts it after Sir Isaac Newton, 31$\frac{16}{17}$.

To reduce French *Crowns to Pounds* Sterling.

68. How many l. sterl. are equal to 1800 French crowns, exchange at 32½d ⅌ crown? *Answ.* 243l. 15s.

1800 at 32½ or 2 8½

s. d.
2 6 —— ⅛l. | 225
2½ —— $\frac{1}{12}$ | 18 15
£. 243 15

Or, 1800
s. d.
180 at 2 0
⅓ 60 — 0 8
$\frac{1}{16}$ 3 15
£. 243 15

But if the sum to be exchanged be given in livres, sols and deniers, divide them all by 3, which reduces them to crowns, sols and deniers d'or, or to crowns, and twentieths, and two hundred and fortieths of a crown, which denominations are of the same form as pounds, shillings and pence, and therefore proportionable to them. Divide these crowns, sols and deniers by the parts which 1 crown is of 1l. sterling, and the quotient or sum of the quotients will be the sterling money required.

69. In 5929 livres, 4 sols, how many pounds sterling, $32\frac{1}{2}$d. sterl. ⅌ crown?

l. s.
3) 5929 4
1976 8 at 2s. $8\frac{1}{2}$d.
2s. 6d.—$\frac{1}{8}$ 247 1
$2\frac{1}{2}$—$\frac{1}{12}$ 20 11 9
Answ. 267 12 9

70. How many pounds sterl. are in 8468 liv. 8 s. 4 de. at 36d. sterl. ⅌ crown? *Answ.* 423l. 8s. 5d.

71. How many pounds sterling are equal to 1000 French crowns at $33\frac{1}{4}$d. sterl. ⅌ crown?

Answ. 138l. 10s. 10d.

72. In $1263\frac{1}{2}$ crowns, at $32\frac{1}{2}$d. sterl. ⅌ crown, how many pounds sterl. is it? *Answ.* 171l. 1s. $11\frac{3}{4}$d.

73. How many pounds sterl. are equal to $1037\frac{3}{4}$ crowns at 33d. sterl. for every crown? *Answ.* 142l. 13s. $9\frac{3}{4}$d.

74. In 1976 crowns, 1 livre, 4 sols, at $32\frac{1}{2}$d. sterl ⅌ crown, how many pounds sterl. is it?

Answ. 267l. 12s. 9d.

75. At $33\frac{3}{4}$d. sterl. ⅌ crown, how many pounds sterl. are there in 1000 crowns? *Answ.* 140l. 12s. 6d.

76. How many pounds sterling are in 2822 crowns, 2 livres, 8 sols, 4 deniers, at 36d. sterl. ⅌ crown?

Answ. 423l. 8s. 5d.

To reduce English *Money into* French.

77. A bill being drawn in London upon Paris, for 243l. 15s. at $32\frac{1}{2}$d. ⅌ crown? I want to know how many crowns are to be paid for that bill?

d. Cr. l. s.
$32\frac{1}{2}$——1——243 15
2 20
65 4875
12
58500
2
117000

——Crow.

```
  ———Crow.    Liv.
65) 117000 (1800 or 5400
    65
    ——
     520
     520
     ——
```

Note. If the answer be required in livres, multiply the crowns by 3.

78. How many crowns are equal to 138l. 10s. 10d. sterling, at $33\frac{1}{4}$d. sterling ⅌ crown?

Answ. 1000 crowns, or 3000 livres.

79. In 171l. 1s. $11\frac{3}{4}$d. sterling, how many crowns does it make, at $32\frac{1}{2}$d sterl. ⅌ crown?

Answ. $1263\frac{1}{2}$ crowns, or 3790 livres, 10 sols.

80. How many crowns are in 142l. 13s. $9\frac{3}{4}$d. sterling, at 33d. sterling ⅌ crown?

Answ. 1037 crowns, 2 livres, 5 sols.

81. At $32\frac{1}{2}$d. sterling ⅌ crown, how many crowns are there in 267l. 12s. 9d sterling?

Answ. 1976 crowns, 1 livre, 4 sols.

82. In 140l. 12s. 6d. sterling, how many crowns, at $33\frac{3}{4}$d. sterling ⅌ crown? *Answ* 1000 crowns.

83. How many crowns Tournois, are equal to 423l. 8s. 5d. sterling, exchange at 36d. sterling ⅌ crown?

Answ. 2822 crowns, 2 livres, 8 sols, 4 deniers.

The foregoing method is universal; but may be contracted in some cases, viz.

When the rate of exchange is 30d. or 2s. 6d. ⅌ crown, 2s. 6d. is $\frac{1}{8}$l. consequently in this case every pound sterling contains 8 crowns, wherefore the English money multiplied by 8, will produce the crowns required.

Note. If the English money be pounds, shillings and pence, the answer will result crowns, sols and deniers d'or, which being multiplied by 3 throughout, will give the answer in livres, sols and deniers tournois.

And if the rate exceed 30d. by an even part thereof, as $32\frac{1}{2}$d. exceeds it by $2\frac{1}{2}$, $\frac{1}{12}$ of 30, then $2\frac{1}{2}$ is contained just once more in the given rate $32\frac{1}{2}$ than in the assumed rate 30, viz. 13 times; and $32\frac{1}{2}$ is $\frac{13}{13}$ of itself and $30\frac{12}{13}$ of the same. Therefore $32\frac{1}{2} : 30$ ($\frac{1}{8}$l. :) : : $\frac{13}{13}$ or $1 : \frac{12}{13}$, or $1-\frac{1}{13}$: so that if $\frac{1}{13}$ be deducted, the remainder will manifestly be in the ratio to the crowns required as $\frac{1}{8}$ thereof. And the same is shewn of any other rate which exceeds 30d. or 2s. 6d. by an aliquot part thereof, according to the following

Rule

Rule.

If the rates exceed 30d. by an even part of itself, deduct that part of the sterling money from itself, which is denominated by one more than the excess is of 30d. and multiply the remainder by 8.

Examples.

How many crowns in 243 15 at $32\frac{1}{2}$?

	243	15
$\frac{1}{13}$	18	15
	225	0
		8
Answ. £.	1800	0

Accordingly let such of the foregoing as fall under this Rule be done.

For $31\frac{1}{2}$	Excess	$1\frac{1}{2}$	is $\frac{1}{20}$	deduct	$\frac{1}{21}=\frac{1}{3}\times\frac{1}{7}$
32	—	2	— $\frac{1}{15}$	—	$\frac{1}{16}=\frac{1}{4}\times\frac{1}{4}$
$32\frac{1}{2}$	—	$2\frac{1}{2}$	— $\frac{1}{12}$	—	$\frac{1}{13}$
33	—	3	… $\frac{1}{10}$	—	$\frac{1}{11}$
$33\frac{1}{3}$	—	$3\frac{1}{3}$	— $\frac{1}{9}$	—	$\frac{1}{10}$
$33\frac{3}{4}$	—	$3\frac{3}{4}$	— $\frac{1}{8}$	—	$\frac{1}{9}$
35	—	5	— $\frac{1}{6}$	—	$\frac{1}{7}$
36	—	6	— $\frac{1}{5}$	—	$\frac{1}{6}$

SECT. V.

Of Exchange between England *and* Spain.

England exchanges with Spain on the piastre or piece of $\frac{8}{8}$ for an uncertain number of pence sterling.

In Madrid, Cadiz, Malaga, and all the places of trade in the Straits, Mediterrean, Africa, and the West Indies, the Spaniards keep their accounts generally in piastres or dollars, rials, half rials and quartiles; 16 quartiles =1 rial, and 8 rials=1 dollar: Or in dollars, rials and maravadies, reckoning 34 maravadies to the rial, and 8 rials to the dollar.

The old piastre is valued at 8, and the new at 10 rials plate.

To reduce Spanish *Money into* English.

84. How much English money is equal to 4384 dollars 3 rials and 8 quartiles, at $40\frac{1}{2}$d.?

s.	d.		*Dol.*	*Ri.*	*Qu.*	*d.*
			4384	3	8 at	$40\frac{1}{2}$
3	4	$\frac{1}{6}$	730	13	4	
	$\frac{1}{2}$	$\frac{1}{80}$	9	2	8	
2 *Ri.*		$\frac{1}{4}$	0	0	$10\frac{1}{8}$	
1		$\frac{1}{2}$	0	0	$5\frac{1}{16}$	
8 *Qu.*		$\frac{1}{2}$	0	0	$2\frac{17}{32}$	
Answ.		£.	739	17	$5\frac{23}{32}$	

85. Suppose Cadiz remits to London 3537 dollars, 6 rials, at $40\frac{7}{8}$d. ꝑ dollar, what will the remittance amount to in England? *Answ* 602l. 10s. $5\frac{17}{32}$d.

English *into* Spanish.

86. How many dollars or pieces of $\frac{8}{8}$ are equal to 739l. 17s. $5\frac{3}{4}$d. sterl. exchange at $40\frac{1}{2}$ sterl. ꝑ dollar?
Answ. 4384 dol. 3 ri. 8 quart.

d.		*dol.*		*l.*	*s.*	*d.*
$40\frac{1}{2}$	———	1	———	739	17	$5\frac{3}{4}$

87. How much Spanish currency is 602l. 10s. $5\frac{1}{2}$d. sterl. exchange at $40\frac{7}{8}$ ꝑ dollar?
Answ. 3537 dollars, 6 rials.

SECT. I.

Of Exchange between England *and* Portugal.

England exchanges with Portugal on the milrea; accounts are kept in milreas and reas, 100 reas to the milrea.

The par of exchange is about 67.166d. ꝑ milrea.

To reduce Portugal *Money into* Sterling *Money.*

Take parts out of the milreas for the given rate; and if there be odd reas, divide the milreas and reas as 1 number, and the answer will be got in pounds and thousandth parts of a pound; in the usual way of practical operations, which may be changed into shillings, pence and farthings.

Oporto remits to London 4366 milreas, 183 reas, at 5s. $5\frac{5}{8}$d. ꝑ milrea, how much sterling is it?

Mils. Rs. *s. d.*
4366 .183 at 5 5$\frac{5}{8}$

s. d.
5 0 is $\frac{1}{4}$ 1091 .545|75
5 -- $\frac{1}{12}$ 90 .962|14
$\frac{5}{8}$ -- $\frac{1}{8}$ 11 .370|17

1193 .87806

1193 .17 6$\frac{3}{4}$

Note. If the reader is not versed in Decimals, let him divide the milreas and reas as 1 whole number and cut off the three last figures from the sum (which divides it by 1000,) the figures cut off are the remainder, viz. so many thousandths of 1l. of which remainder the value may be found.

88. How many pounds sterling in 1000 milreas, at 5s. 6d. ⅌ milrea? *Answ.* 275l.

89. In 2000 milreas, at 5s. 8$\frac{1}{2}$d. ⅌ milrea, how many pounds sterling? *Answ.* 570l. 16s. 8d.

90. At 5s. 6$\frac{3}{4}$d. ⅌ milrea, how many *l.* in 2060 mil. 380 reas? *Answ.* 573l. 0s. 10$\frac{73}{200}$d.

91. Tell me how many pounds in 1056 mil. 999 reas, at 5s. 4$\frac{1}{2}$d. ⅌ milrea? *Answ.* 284l. 1s. 4$\frac{871}{2000}$d.

To reduce Sterling *Money into* Portugal *Money.*

92. How many milreas are equal to 1193l. 17s. 6$\frac{3}{4}$d. at 5s. 5$\frac{5}{8}$d. ⅌ milrea?

s. d. *Re.* *l.* *s.* *d.*
5 5$\frac{5}{8}$——1000——1193 17 6$\frac{3}{4}$=$\frac{6}{8}$
12 20

65 23877
8 12

525 286530
8

Mil. Rs.
525) 2292246.000 (1366 183 nearly.

93. How many milreas are equal to 226l. 16s. the exchange at 5s. 4d. ⅌ milrea?
Answ. 850 milreas, 500 reas.

94. In 275l. at 5s. 6d. ⅌ milrea, how many milreas?
Answ. 1000 milreas.

95. How many milreas are in 570l. 16s. 8d. the exchange at 5s. 8$\frac{1}{2}$d. ⅌ milrea? *Answ.* 2000 milreas.

96. At 5s. 10d. ⅌ milrea, how many milreas are there in 106l. 5s. 6$\frac{1}{4}$d.? *Answ.* 364 milreas, 375 reas.

SECT.

SECT. VII.

Of Exchange between England *and* Italy.

England exchanges on Leghorn for the dollars of 6 livres, and gives pence sterling for the same, they reckon 12 deniers to the soldi, and 2 soldi to the dollar.

On Genoa for the piastre of 5 livres, they keep accounts in livres, sols, and deniers, 12 deniers=1 sol, and 20 sols=1 livre.

On Venice for the ducat of 24 gross banco for pence sterling, accounts are kept in livres, sols, and gross current, 12 gross=1 sol, 20 sols=1 livre.

The bank and bankers keep their accounts in livres, sols and grosses banco; 1 livre=10 ducats bank, or 12 current, so that current money is of less value than bank money by 20 ⅌ cent.

From which it is easy to conceive that these exchanges are calculated on the like principles as the French, which having explained, it doth not appear necessary to enlarge farther on this head.

PART II.

ARBITRATIONS OF EXCHANGE.

UNDER the head of *Arbitrations of Exchanges* are comprehended first the calculation of the proportional exchange of two places by means of one or more intermediate exchanges given, and secondly, having two or more different routes of exchange, to compute which is more gainful, or whether it is more advantageous to remit money or bills, or to draw or order a remittance. *et contra.*

Since Ireland draws all its foreign demands by London, there is always one intermediate exchange whereby to compute the value of any foreign sum in Irish money according to the present rates of exchange, wherefore we begin with this problem.

To reduce Foreign Money into Irish.

Examples.

1. How much will 100l. Flemish amount to Irish currency, exchange between London and Amsterdam being 33s. 4d. ⅌ *l.* and between London and Dublin 8 ⅌ cent. advance.

In

1st. *Method.*

s. *d.*	*s.* Sterl.	*l.*	*l.* Sterl.
If 33 4	20	100	*Answ.* 60
l. Eng.	*l.* Irish	*l.* Eng.	
If 100	108	60	*Answ.* $64\frac{4}{5}$.

But in this case where a comparison is made between one rate and a second, between that second and a third, between the third and a fourth, &c. instead of stating each comparison ℔ *Rule of Three*, there is a more ready and contracted way called the *Rule of Conjunction*, for which this is the Rule.

1. Place the antecedents on the left, and the consequents on the right hand.

2. The first antecedent and last consequent must be of one name or species; so must the first consequent and second antecedent, the second consequent and third antecedent, which order must be continued through the whole.

3. The terms being thus disposed, multiply all the antecedents for a divisor, and all the consequents for a dividend, if the place of an antecedent be blank: and all the consequents for a divisor, and all the antecedents for a dividend, if the place of a consequent be blank.

s. *d.*			
33 4			
12			
		d	
Flemish 400	=	240	English.
English 100	=	108	Irish.
Irish - -	=	100	
Divisor 40000		10800	
		240	*Answ.* $64\frac{4}{5}$ *l.* or 16 *s.*
		432	
		216	
4|0000)		259|2000	
		$64\frac{3}{4}\frac{2}{5}\|\frac{1}{8}$	

And this operation may be further contracted in the following cases.

I. If any antecedent and consequent are equal, erase them; for then the quotient or answer will result the same.

l. *l.*
Flemish $1\frac{2}{3}$ = 1 Eng.
English $\cancel{100}$ = 108 Irish.
Irish ? = $\cancel{100}$ Flemish.
$1\frac{1}{2}$
3
—

5 108
— —
3 1
324
—
5) 324 (64$\frac{4}{5}$ *Answ.*

II. If any antecedent and consequent be composite to each other, divide them by the greatest common measure, and take the quotients instead of the numbers themselves.

5 $\cancel{400}$ = $\cancel{240}$ 3
$\cancel{100}$ = 108 s,
? = $\cancel{100}$ *Answ.* 64$\frac{4}{5}$ = 16

III. If any term have a fraction annexed, multiply the integral part by the denominator of the fraction (adding in the numerator) and place the result in the room of the mixt number and the denominator on the opposite side below.

5 $\cancel{1}\frac{2}{3}$ = 1
$\cancel{100}$ = 108
? = $\cancel{100}$
3

2. What is a guilder worth at Cork, if the exchange between London and Rotterdam is at 35s. Flemish ⅌ l. sterl. and the exchange from Cork to London at 9 ⅌ cent. advance? *Answ.* 2$\frac{8}{103}$*s.*

3. When the exchange between London and Dublin is 6 ⅌ cent. advance and the exchange between London and Hamburgh at 33*s.*.4*d.* gross ⅌ l. sterling; what is then a marks lubs worth in Dublin? *Answ.* 20$\frac{44}{125}$*d.*

4. Received from Jean le Fevre of Bourdeaux, acct. sales of 400 casks of butter, neat proceeds amounting to 4326 livres. The exchange between London and Bourdeaux being 36*d.* ⅌ crown, and between London and Dublin 5 ⅌ cent. what sum Irish currency must I debit him for?
Answ. 227l. 2*s.* 3$\frac{3}{5}$*d.*

5. What is a guilder of 20 stivers worth at Cork, when the exchange between London and Rotterdam is at 33s. 4d. Flemish per l. sterl. and the exchange between London and Cork, at $8\frac{1}{3}$ per cent. advance for Cork? *Answ.* 26d.

6. My correspondent at Lisbon writes that he hath remitted A B of London for my account, 537 mil. 432 reas, exchange at 5s. $3\frac{1}{2}$d. per milrea, for value whereof I draw on A B at $7\frac{3}{4}$ per cent. advance, quere the sum Irish I must receive? *Answ.* 153l. 4s. $3\frac{1}{2}$d.

2. Irish *into* Foreign.

7. When 105l. in Dublin is worth 100 in London, and the exchange between London and Paris at 36d. per crown, how much then in Paris is 12d. of Dublin worth?

Answ. $19\frac{1}{21}$ sols.

8. When the exchange between London and Dublin is $8\frac{1}{4}$ per cent. and between London and Rotterdam 33s. 4d. Flemish per pound sterling; how much is 12d. of Dublin worth in Rotterdam? *Answ.* $9\frac{1}{87}$ stivers.

9. Remitted George Dawson of London 160l. 10s. Irish, exchange at 7 per cent. with orders to remit the value to Gerrard Lepanto of Cadiz at 40d. sterling per dollar; I want to know what sum Spanish I am creditor for on account of this remittance? *Answ.* 900 dollars.

3. Foreign exchanges may be calculated by intermediate rates in like manner.

10. London exchanges with Amsterdam at $38\frac{1}{3}$s. Flemish per pound sterling; Amsterdam with Middleburgh at 2 per cent. advance: How stands the exchange between London and Middleburgh? *Answ.* 34s. per pound sterling.

11. Amsterdam exchanges with London at 34s. 10d. and London with Paris at 33d. per crown; what is the arbitrated price then of a crown at Amsterdam?

Answ. $57\frac{1}{4}$d. Flemish per crown.

12. Amsterdam exchanges with London at 34s. 4d. per pound sterling; and on Lisbon at 52d. Flemish for 400 reas: How then ought the exchange to go between London and Lisbon? *Answ.* $75\frac{75}{103}$ *d.* per milrea.

13. Exchange between London and Amsterdam 34s. 6d. Flemish per pound sterling; between London and Paris $31\frac{3}{4}$d. sterling per crown; what is the proportional arbitrated price between Amsterdam and Paris?

Answ. $54\frac{123}{160}$d. Flemish per crown.

14. Paris on Amsterdam $54\frac{12}{16}\frac{3}{8}$d. Flemish per crown and on London $31\frac{3}{4}$d. sterling; How then ought the exchange to go between London and Amsterdam?

Answ. 34s. 6d. Flemish ⅌ pound sterl.

15. Amsterdam exchanges on London at 34s. 6d. Flemish ⅌ pound sterling: and on Paris at $54\frac{12}{16}\frac{3}{8}$d. Flemish per crown: How then ought the exchange to go between London and Paris? *Answ* $31\frac{3}{4}$d. per crown.

IV. Foreign weights and measures, &c. may be compared and arbitrated by the same rule of conjunction.

16. Suppose 4lb. of A are worth 3lb. of B, and 5℔ of B equal to 4℔ of C, and 6℔ of C to 5℔ of D; how many pound of D are equal to 240℔ of A?

$$\begin{array}{llcll} A & 4 & = & 3 & B \\ B & 5 & = & 4 & C \\ C & 6 & = & 5 & D \\ D & ? & = & 240 & A \end{array}$$

$$12|0)\ \underline{1440|0}$$

Contracted.

$$\begin{array}{rrr} & \cancel{4} & \cancel{3} \\ & \cancel{5} & \cancel{4} \\ 2 & \cancel{6} & \cancel{5} \\ ? & 2) & \underline{240} \\ & & 120 \end{array}$$

Answ. 120℔.

17. If 3 yards of cloth cost as much as $3\frac{3}{4}$ of ratteen and $4\frac{1}{2}$ yards of ratteen are worth 5 yards of drugget, how many yards of drugget are then worth 27 yards of cloth?

Answ. $37\frac{1}{2}$ yards.

18. If 3 yards in London are equal to 4 ells in Holland, and 50 ells in Holland cost 26l. Flemish; how many pounds Flemish will 40 yards in London come to?

Answ. $27\frac{11}{15}$l. Flemish.

19. What will 1℔ of pepper cost, if 3℔. of cloves cost as much as 6℔ of pepper, and $2\frac{1}{2}$℔. of cinnamon cost as much as 4℔. of cloves, and 3℔. of cinnamon cost 8 shillings? *Answ.* 10d. ⅌ ℔.

20. If 10℔. at London be equal to 9℔ at Amsterdam; 45℔. at Amsterdam to 49℔. at Bruges, and 98℔ at Bruges to 116℔. at Dantzick; how many pounds at Dantzick are equal to 112 at London? *Answ.* $129\frac{97}{100}$

21. Suppose a merchant at Hamburgh hath orders to procure 81 yards of English cloth, so that 7 ells of Hamburgh may be procured for 3l. sterling: Now if 7 ells of France make 9 yards of London, and 7 ells of Holland make 4 ells of France: and 1 ell of Holland make $1\frac{1}{5}$ el of Hamburgh; how much will the said cloth amount to at Hamburgh, the exchange being at 33s gros ⅌ L. sterl?

Answ. 701 ms. 10 S. $7\frac{1}{15}$ den.

5. *Concerning the Gain or Loss by Exchange.*

Note. When the exchange is so many pence sterling for some piece of foreign money the gain is the more, the lower the exchange: for it is evident I can receive more French crowns for 100l. when the exchange is at 30d. than when it is at 33d.

But when the exchange is so much per pound sterling, the higher the exchange, the more the gain; for I can receive more Flemish money for 100l. sterling when the exchange is at 35s. than at 33s. Flemish per pound sterling.

22. When the exchange between London and Dublin is 7 per cent. which is it better for a merchant here to draw a bill on London or have specie sent him, and what is the difference per cent.? *Answ.* Specie better by 1$\frac{1}{3}$ per cent.

From the exchange of 100l at par 8$\frac{1}{3}$
Take the current exchange 7
Remains 1$\frac{1}{3}$ Gain per cent.

23. A merchant in Lisbon owes a merchant in Dublin 850 milreas. Which is most advantageous to Dublin to have the same sent over in moydores of 4 milreas each, which pass here for 29s. 3d per piece; or to have it remitted to London, at 6s. 6d. per milrea, and from London hither at 8 per cent. advance? *Answ.* Moydores better by 4$\frac{1}{6}$ per cent.

24. A at Paris draws on B of London 1200 crowns, at 31d. per crown: for value whereof B draws again on A at 33d. per crown, besides reckoning $\frac{1}{2}$ per cent. for his commission. Did A get or lose by this transaction, and what?

Answ. He gains 30 cro. 1 liv. 12 sol. 9 den.

25. A of Amsterdam owes B of Paris 1000 gilders of current specie, which he is to remit him; by order, the exchange at 60$\frac{1}{2}$d. Flemish de Banco per crown, the agio being four per cent. but when this is to be negociated, the exchange suppose fallen to 59$\frac{1}{2}$d. per crown, and the agio risen to 5 per cent. Did B get or lose by this turn of affairs, and what? *Answ.* 9 cro. 0 liv. 3 so. 4 den. he gets.

26. Exchange between London and Dublin, being at 7$\frac{1}{2}$ per cent. in Dublin, and 8$\frac{1}{4}$ in London, suppose A B of London, owes me 560l. 15s. English: I want to know the difference between my drawing and his remitting me the said sum? *Answ.* Remittance better by 4l. 4s. 1$\frac{7}{30}$*d.*

27. Suppose A of Bourdeaux owes B of Dublin 1036 crowns, and the exchange between Paris and London is

31½d. per crown, between Paris and Amsterdam 56d. Flemish per crown, and between Amsterdam and London at 34s 8d. per pound sterl. The question is, whether B had better have the money remitted immediately from France, or by way Amsterdam; and what advantage he gets, admitting his correspondent at Amsterdam is to have ½ per cent. for negociation? *Answ.* By Amsterdam better by 2l. 15s. 9½d. English.

INVOICES

Intended for Exercises of sundry preceding Rules.

1. INVOICE of 49 barrels of beef, and 10 hogsheads of white biscuit, shipped by me Isaac Sharp on board the Dublin Merchant, Nicholas Tory, master, and consigned to Thomas Gunston, merchant, at Port Loyal in Jamaica, for account and risque of Richard Allom of London, being marked and numbered as per margin, contents cost and charges, viz.

49. barrels of beef, at 15s. 6d. per barrel——£.

		C.	qrs.	lb.	
	10*hhds.* of white biscuit, *wt.*	20	0	26	at 10s.

	Charges.	*l.*	*s.*	*d.*
ꝯ	Custom of the beef, -	2	9	0
T. G.	Dit. of the biscuit, -	0	10	0
N° 1 to 59.	Entry and fees of cocket,	0	5	6
	Searcher and warfinger,	0	7	0
	Carriage of the whole,	0	9	5
	For 10*hhds.* 25s. cooperage, hoops and heading, 16s.	2	1	0
	Porters and shipping,	0	3	4
				£.
	To my commission, at 2½ per cent.			
				£.

Errors excepted,
Dublin, Sept. 5, 1808.
ꝑ Isaac Sharp.

Supposing the exchange 10 per cent. advance in favor of England, what sum may I. Sharp draw for on London? *Answ.* 54l. 17s. 3d.

2. IN-

Amsterdam, January 10, 1809.

2. INVOICE of 10 pieces of Holland, 9 pieces of cambrick, 9 pieces of Ghentish cloth, laden by me Jonas Diligent, on board the Josiah, Thomas Cook, master, for the proper risque and account of Henry Porter, merchant in London; marked as ⅌ margin; contents, cost and charges, viz.

10 Pieces of Holland,

No.	*qt.*	No.	*qt.*
	$31\frac{1}{2}$		$33\frac{1}{4}$
1	$33\frac{1}{4}$	6	32
to	32	to	34
5	31	10	$31\frac{1}{2}$
	$30\frac{1}{2}$		$32\frac{1}{4}$

		G.	Sti.
In all	Ells, at	1	11 ⅌
9 Pieces of cambrick, *qt.* $124\frac{1}{2}$ ells, at		1	3
9 Pieces of Ghenting, *qt.* $105\frac{1}{2}$ —— at		0	19

Charges,

H. P.

	G.	Sti.
To custom and brokerage of Hollands, 3 guil. ⅌ piece,	30	0
To charges in buying - -	2	3
To custom of cambrick and Ghenting, - - - -	19	11
To sledage and boatage - -	3	16
To warehouse room, - - -	4	3
To average and porterage -	1	11

To my commission at $2\frac{1}{2}$ ⅌ cent.

Errors excepted,

⅌ Jonas Diligent.

Exchange at 34s. 6d. Flemish ⅌ l. sterling, what doth it amount to English? *Answ.* 79l 9s. $6\frac{1}{4}\frac{3}{5}$d.

Oporto, December 1, 1808.

3. INVOICE of wine, laden ⅌ Nicholas Strong and Owen Jamyn, on board the Savannah, John Snap, master, for account of P. Lilly and company, and consigned to Paul Ludulph and company, in Dantzick.

To

		Mil.	*Rea.*
	To cost of 10 pipes of wine bought of Matthew de Minis, at 16 mil ⅌ pipe,		
P. L.	To custom, at 1055 reas ⅌ pipe, - -		
	To trimming, &c. at 400 reas ⅌ pipe, -		
	To primage, at 60 reas ⅌ pipe, - - -		
N°. 1	To brokerage, at ½ ⅌ cent. - - - -		
to 10	To commission, at 3 ⅌ cent. - - -		
	To port charges of the said ship, - -		
		6	380

Errors excepted,

⅌ Nich. Strong and Owen Jamyn.

Exch. 40 reas for 3d, what sterling? *Answ.* 58l. 13s.

Cadiz, Oct. 17, 1808.

4. INVOICE of 1 barrel, containing 1 seron of cascarilla, shipped on board the Sevilla Merchant, captain Jonathan Braddle, commander, and consigned to Matthew Roan, merchant in London, viz.

	Ry.	*Fl.*
1 Seron, *qt.* nett 209¼℔. of cascarilla, at 9 pcs. $\frac{8}{8}$ ⅌ ℔.	-	-
To dispatch, 4 pieces $\frac{8}{8}$, is - - - - -	32	—
To porterage to the house and boats, -	4	—
To boat hire abroad, - - - - - - - -	8	—
To brokerage at 1½ ⅌ cent. - - -	39¼	—

To my commission at 2½ ⅌ cent.

Errors excepted,

⅌ Samuel Dickson.

At 3 ryals of plate ⅌ piece of $\frac{8}{8}$. what does the whole amount to, exchange at 52d. sterling ⅌ piece of $\frac{8}{8}$?
Answ. 420l. 10s. 10⅜d.

5. INVOICE of 6 hhds. of tobacco, and 3 bales of woollen cloth, shipt on board the Dublin Trader, Nich. Dun, master, for the proper account and risque of Robert Merchant, of Dublin, and consigned to himself, marked as ⅌ margin, contents, cost and charges, viz.

N°.

Best bright tobacco, viz

	C.	qrs.	℔.		qrs.	℔.
N°. 1	2	3	7	Tare	2	14
2	3	1	10	——	2	20
3	3	3	0	——	2	0
4	4	1	27	——	3	4
5	2	2	20	——	2	10
6	5	2	10	——	3	12

Tot. gr.
Tare

℔. Trett 4℔. ⅌ 104.

d.
℔. Nt. at 7½ ⅌ ℔.—£.

Cloth, 3 bales, viz.
N°, 27, 28, 29, *qt.* each 10 short cloths, at 12l. ⅌,

	Charges.	l.	s.	d.
	To custom of all, - -	53	18	6
⊕	To cost of 3 wrappers,	0	10	6
T. M.	Storage, - - - - -	1	00	0
	Cartage and porterage,	0	10	6
	Brokerage, at ½ ⅌ cent.			
				£.

To my commission, at 2½ ⅌ cent.

Errors excepted in London,
November 4, 1808, ⅌
George Trusty.

Now suppose said Trusty draws upon a Merchant for the value of this factory, at 7¾ ⅌ cent. how much doth it come to? *Answ.* 531l. 5s. 9d. nearly.

CHAP. VIII.

BARTER.

BARTER or Truck is the exchange of wares, value for value, according to the rates or prices agreed upon.

Questions relating thereto are solved by the Rule of Three, or the contracted method of Practice instead thereof, viz. 1st, If a given quantity of goods at a given price are proposed to be bartered for other goods at a given price, it is plain, I must find the price of those goods, of which the quantity is given; and 2ndly, what quantity of the goods to be received in exchange, at the price they are rated at, will amount to the same value, &c.

Examples.

1. A merchant barters 10 pieces of drugget, each 25 yds. at 18½d. ⅌ yard, against pepper, at 15d. the ℔. The question is, How many pounds of pepper he must receive?

Answ. 308⅓℔.

Solution.

10 pieces each 25 yards are 250 yds. (166) which at 18½ come 4625d.

Therefore he must receive pepper, at 15d. ⅌ ℔. to the value of 4625d. So then say 15d. : its value 1℔. : : 4625 : its value, viz. 308⅓℔.

10 pieces,
each 25 yards,

is 250 yds. at 1s. 6½d.
6d. is ½ 125
½ — $\frac{1}{12}$ 10 5
℔.
15——1——385 5
12

15)4625(308⅓
45

125
120

$\frac{5}{15}|\frac{1}{3}$

2. How much sugar at 8d. ⅌ ℔. must be delivered for 20 cwt. of tobacco, at 3l. ⅌ Cwt.?

Answ. 16C. 0qrs. 8℔.

3. A gives B 250 yards of drugget at 18½d. ⅌ yard, for 308⅓℔. of pepper; I demand what the pepper stands him in ⅌ ℔.? *Answ.* 15d.

4. A merchant hath 1000 yards of canvas at 9½d. the yard, which he barters for serge at 10¼d. ⅌ yard; how many yards must he receive? *Answ.* $926\frac{34}{41}$ yards.

5. A hath 1200 stone of tallow at 2s. 3¼d. ⅌ stone: B has 110 tanned hides, *wt.* 3994℔. at 5¾d. ⅌ ℔. and they barter at these rates. Tell me how much money A is to receive of B besides the hides? *Answ.* 40l. 11s. 2½d.

6. B delivered 3 hogsheads of brandy, at 6s. 8d. ⅌ gallon, to C for 126 yards of cloth, what was the cloth ⅌ yard? *Answ.* 10s.

7. A has silk of 14s. ⅌ ℔. B has cloth of 10s. ⅌ yard, which he barters at 12s. 6d. the yard, at how much must A put his silk to make his profit equal with B?

Answ. 17s. 6d.

s. s. d. s.
If 10——12 6——14

Or thus,
s. d.
12 6
10 0

If 10——2 6——14 comes 3 6
14

Answ. 17 6

8. A hath linen cloth worth 20d. ⅌ ell ready money; but in barter he will have 2s. B has broad cloth worth 14s. 6d. ⅌ yard, ready money; at what price ought the broad cloth to be rated in barter? *Answ.* 17s. 4$\frac{4}{5}$d.

9. C has candles at 6s. ⅌ dozen ready money; but in barter he will have 6s. 6d. ⅌ dozen; D has cotton at 9d. ⅌ pound ready money: what price must the cotton be at in barter; and how much cotton must be bartered for 100 dozen of candles? *Answ.* The cotton 9$\frac{3}{4}$d. ⅌ ℔. in barter, and 7 Cwt. 16℔. of cotton must be given for 100 dozen of candles.

10. A has coffee, which he barters with B at 10d. ⅌ ℔ more than it cost him, against tea, which stands B in 10s. ⅌ ℔. but puts it to 12$\frac{1}{2}$s. I would know how much the coffee did cost at first? *Answ.* 3s. 4d.

11. B has 5 tons of butter, at 25l. 10s. ⅌ ton; and 10$\frac{1}{2}$ ton of tallow, at 33l. 15s. ⅌ ton: which he barters with C, thus, to have 150l. 1s. 6d. in money, and the rest in beef, at 21s. ⅌ barrel. How many barrels is he to receive? *Answ.* 316 barrels.

	l.	l.	s.	d.
5 ton at	25$\frac{1}{2}$ is	127	10	0
10$\frac{1}{2}$ ton at	33$\frac{3}{4}$ is	354	7	6
	£.	481	17	6
	In cash	150	1	6

s. Bar.
If 21———1———331 16 0 *Answ.* 316 Bar.

21. A

12. A and B barter, A hath 41C. of hops, at 30s. ℘ Cwt. for which B gives him 20l. in money, and the rest in prunes, at 5d. ℘ ℔. what quantity of prunes did B give A? *Answ.* 17C. 3qrs. 4℔.

13. Two merchants barter, A has 20C. of cheese, at 21s. 6d. ℘ Cwt. B has 8 pieces of Irish cloth, at 3l. 14s. ℘ piece: whether of them must receive money, and how much? *Answ.* A must receive 8l. 2s.

14. A has 5 bales of pepper, *wt.* neat 1600℔. at 17d. ℘ ℔. which he barters with B for two sorts of goods, the one at 5d. the other at 8d. ℘ ℔. to have $\frac{1}{3}$ in money, and of each sort of goods an equal quantity: I demand how many ℔. of each sort of goods he is to receive, and how much in money? *Answ.* 139$\frac{34}{39}$℔. and 37l. 15*s.* 6$\frac{2}{3}$d. in money.

15. Two merchants barter; C has 10 hogsheads of madder, *qt.* neat 90C. 3qrs. 14℔. which cost 38s. 6d. ℘ Cwt. and he puts it at 42s. ℘ Cwt. and he receives of D $\frac{1}{4}$ in ready money, and the rest in hemp; which cost 34s. 10d. ℘ Cwt. Tell me how much money and hemp C is to receive, and what D ought to rate his hemp ℘ Cwt. to make the barter equal? *Answ.* C must receive in money 47l. 14s. 2$\frac{1}{4}$d. and 73C. 0qrs. 5$\frac{75}{289}$℔. of hemp, at 38s. ℘ C.

Note. The above question is taken from Voster, who makes the quantity of hemp 75C. 1qr. 9$\frac{1}{33}$℔. but according to this solution the barter doth not appear equal to me, since C according thereto, hath a profit upon the whole, and D only on $\frac{3}{4}$ of the value.

CHAP. IX.

PROFIT AND LOSS.

1. BOUGHT a piece of cloth; long 53 yards, at 12s. ℘ yard. Sold the same at 14s. the yard, what is the profit upon the whole piece? *Answ.* 5l. 6s.

Sold for ——— 14s. ℘ yard.
Bought for ——— 12s. ℘
Whereby is gain'd 2 ℘

Yd. s. Yds. s. l. s.
Therefore 1 ——— 2 ——— 53 : 106 or 5 6

2. Bought 18Cwt. of cheese, at 28s. ℘ cwt. which I sell out again at 3$\frac{1}{2}$d. ℘ pound: what is the profit in the whole? *Answ.* 4l. 4s.

3. If 980℔ of merchandises are bought for 61l. 5s. and sold for 69l. 8s. 4d. I demand the profit upon each ℔?

Answ. 2d.

4. Bought 12hhds. of wine, at 10l. 5s. the hogshead, paid for cellarage and other charges 4s. 6d. ⅌ hogshead, sold the hogshead for 37 crowns, each, (at 5s. 5d.) I demand the profit or loss? *Answ.* 5l. 9s. loss.

SECT. II.

5. If a ton of tallow cost 20l. and is sold for 22l. 10s. I demand the profit ⅌ cent? *Answ.* $12\frac{1}{2}$l.

Sold for——$22\frac{1}{2}$
Bought for——20

[illegible]

If 20 gain : $2\frac{1}{2}$——100 *Answ.* $12\frac{1}{2}$.

6. If I buy a pound of cloves for 6s. 3d. and sell it for 6s. how much ⅌ cent. loss is it? *Answ.* 4 ⅌ cent.

7. Bought 1000 barrels of wheat, at 10s ⅌ barrel, paid for divers charges 10l. how much ⅌ cent. is gained if it is sold at $11\frac{1}{4}$s. ⅌ barrel? *Answ.* $10\frac{5}{17}$ ⅌ cent.

8. If 1℔ of pepper is sold for $10\frac{1}{2}$d then there is lost 2d. ⅌ ℔ how much is the loss ⅌ cent.?

Answ. 16 ⅌ cent.

9. When I sell a pound of silk for 26s. and 8d. I gain 2s. 6d. Now I want to know how much I would gain if I sold a bale of silk which cost 120l.? *Answ.* 12l. 10s.

SECT. III.

10. If 1 tun of wine cost 40l. for how much must it be sold to gain $6\frac{1}{4}$ ⅌ cent.? *Answ.* 42l. 10s.

100 $106\frac{1}{4}$ 40 *Answ.* 42l. 10s.

Or thus,

100 $6\frac{1}{4}$ 40 *Answ.* 2 10
40

£. 42 10

11. If 100℔ weight of any commodity cost 30s. at what price must 1℔ weight of that commodity be sold to gain after the rate of 10l. for every 100 laid out?

Answ. $3\frac{24}{25}$d.

12. Having bought 18 gallons of brandy for 12l. how may I sell 1 gallon and gain at the rate of 8 ⅌ cent.?

Answ. 14s. $4\frac{4}{5}$d.

13. Having sold 10 yards of cloth for 4l. 16s. and thereby gained at the rate of 10 ⅌ cent. what was the prime cost of 1 yard? *Answ.* 8s. $8\frac{8}{11}$d.

14. At what rate must I sell a Cwt. of madder, which cost 3. 15s. to gain 10 ⅌ cent.? *Answ.* 4l. 2s. 6d.

15. At what rate must the aforesaid madder be sold to lose 10 ⅌ cent. *Answ.* 3l. 7s. 6d.

16. Bought 12 pieces of white cloth for 6l. 10s. ⅌ piece; paid 20s. 10d. a piece for dying; for how much must I sell them a piece to gain 20 ⅌ cent.? *Answ.* 9l. 1.

17. Suppose I buy 28 pieces of stuff, at 4l. ⅌ pieces, and sell 10 pieces at 6l. and 8 at 5l. at what rate must I sell the rest to gain 10 ⅌ cent. by the whole?

Answ. 2l 6s. 5d. nearly.

18. Having sold a yard of cloth for 11s. 6d. I gained at the rate of 15 ⅌ cent. but if I had sold it for 12s. what should I gain ⅌ cent.? *Answ.* 20.

19. Bought 7 tuns of wine, at 17l. ⅌ hhd. which I sell at 1s. ⅌ pint: what is the whole gain, and how much ⅌ cent.? *Answ.* The whole gain 229l. 12s. and 48l. 4s. 8½d. ⅌ cent.

SECT. IV.

20. Bought a tun of wine for 50l. ready money; sold it for 54l. 10s. payable in 8 months time. I demand how much ⅌ cent ⅌ annum I gain?

$$\begin{array}{r} 54\frac{1}{2} \\ 50 \\ \hline \end{array}$$

$$\left.\begin{matrix}50\\8\end{matrix}\right\rangle 4\frac{1}{2} \left\langle\begin{matrix}100\\12\end{matrix}\right. \qquad \textit{Answ. } 13\frac{1}{2}.$$

The work may be contracted in like manner; see Case IV. page 95, *viz.*

$$\begin{matrix}1\\2\end{matrix}\;\left.\begin{matrix}\cancel{50}\\\cancel{8}\end{matrix}\right\rangle 4\frac{1}{2} \left\langle\begin{matrix}\cancel{100}\\\cancel{12}\end{matrix}\right.\;\begin{matrix}2\\3\end{matrix}$$

21. Having bought a parcel of goods for 18l. and sold the same immediately for 25l. with 4 months credit: what is gained ⅌ cent. ⅌ annum? *Answ.* 116⅔.

22. Bought 300℔. of coffee, at 4s. 2d. ⅌ ℔. ready money, and sold for 5s. ⅌ lb. payable in 8 months; I want to know how much was gained upon the whole, allowing discount at 6 ⅌ cent. ⅌ annum: and how much ⅌ cent. ⅌ annum was gained? *Answ.* 9l. 12s. 3d $\frac{1}{3}$d, upon the whole, and 30 ⅌ cent.

23. Bought 40 gallons of brandy, at 3s. ⅌ gallon; by accident 6 gallons of it is lost, at what rate may I sell

the rest, with 8 months credit, and gain upon the whole prime cost at the rate of 10 ℘ cent. ℘ annum?

Answ. 3s. 9d $\frac{3}{17}$d. ℘ gallon.

24. Bought 100 yards of cloth, at 14s. ℘ yard, which I propose to sell for ready money, at 25 ℘ cent. profit; and if I sell upon time to have moreover 5 ℘ cent. ℘ annum for forbearance: how must I sell it ℘ yard, at 6 months, to make both these gains? *Answ.* 17s. 11$\frac{1}{4}$d.

25. Bought 100 casks of raisins, at 50s. ℘ Cwt. payable in 9 months, sold them for 52s. ℘ Cwt. payable in 15 months; what is the profit ℘ cent. ℘ annum at that rate?

Answ. 8 ℘ cent.

15 Mo.	52s.
9 Mo.	50s.
6 Mo.	

$$\begin{array}{l} \text{s. } 5 \\ \text{Mo. } 6 \end{array} > 2 < \begin{array}{l} 100 \\ 12 \end{array}$$

Answ. 8 ℘ cent.

26. A vintner buys a parcel of brandy at 5s. 3d. ℘ gallon, payable in 6 months: sells the same at 5s. 10$\frac{1}{2}$d. ℘ gallon, payable in 10 months. How much ℘ cent. does he gain at that rate ℘ annum?

Answ. 35$\frac{5}{7}$ ℘ cent.

27. If 1 Cwt. of merchandizes are bought for 56s. payable in 9 months time, and sold for 6$\frac{1}{4}$d. the pound, payable in 12 months time. I demand what he gains ℘ cent. in a year at that rate? *Answ.* 16$\frac{2}{3}$l.

28. A tobacconist buys 10hhds. of tobacco, *wt.* neat 30C. 3qrs. 14℔ at 9$\frac{1}{4}$d ℘ ℔ payable in 3 months, sells it for 11d. half farthing, payable in 7 months. Tell me how much ℘ cent. ℘ annum is gained at that rate?

Answ. 60$\frac{30}{37}$.

SECT. V.

29. Received from London, 11300℔ of pewter, cost 706l 5s. English money, the charges here are 10l. 6s. 8d. at what rate must I sell it a pound to gain 15 ℘ cent. allowing 12d. in England to make 13d. in Ireland?

Answ. 19d very near, or exact 18$\frac{42483}{45200}$d.

s.	d.	l.	s.			l.	s.	d.
1 ——	1$\frac{1}{12}$ ——	706	5		comes	765	2	1
		58	17	1	Charges	10	6	8
		765	2	1	whole cost	775	8	9
100 ——	15 ——	775	8	9	comes	116	6	3$\frac{3}{4}$
					£.	891	15	0$\frac{3}{4}$

11300 —— 891 15 0$\frac{3}{4}$ —— 1 *Answ.* 18$\frac{42483}{45200}$d.

30. A merchant receives from Lisbon 180 casks of raisins, which stand him in here 16s. each; trucks them against other merchandizes, at 28s. ℘ Cwt. finds to have gained 25 ℘ cent. profit. I demand the weight of each cask, one with another? *Answ.* 80℔.

31. I sell barley at 6s. ℘ kilderkin, and gain 20 ℘ cent. now if I sell a parcel of the same barley, amounting to 8l. 16s. and find I gain 30 ℘ cent. tell me how many barrels were in the last parcel? *Answ.* 10; and at what rate I sold them? *Answ.* 7½s.

32. Received 3 pieces of Holland, *qt.* No. 1, 42 ells, at 15¼ stivers the ell; No. 2, 52 ells, at 19½ stivers the ell; No. 3, 64½ ells, at 20¼ stivers the ell. At how much must I sell it the yard, one with another to gain 20 ℘ cent. allowing 10½ stivers, for 1s. sterling, and that 3 yards make 4 ells? *Answ.* 2s. 10[illegible]d.

33. A merchant sends to Barbadoes 300 firkins of butter, *wt.* 175 Cwt. 2 qrs. 19℔ at 18s. 8d. ℘ Cwt. pays for duty and other charges 11l. 0s. 10d. his correspondent at Barbadoes sells the butter at 6d. the pound, (weight as above) pays for freight, &c. 24l. 17s. 1d. takes 5 ℘ cent. for his commission; I demand the loss or gain, if 135l. Barbadoes is worth but 100l. in Ireland?

Answ. 152l. 14s. 5$\frac{2}{7}$d. profit.

C.	qr.	℔.		s.	d.		l.	s.	d.
175	2	19	at	18	8	℘ C.—	163	19	2
						Charges	11	0	10
						First cost	175	0	0

C.	qr.	℔.		l.	s.	d.
175	2	19	at 6d. the ℔.—	491	17	6
Charges	24	17	1			
Commission	24	11	10½			
				49	8	11½
			The neat proceeds,	442	8	6½

If							l.	s.	d.
135	— 100	— 442	8	6½	comes		327	14	5$\frac{2}{7}$
					The first cost,		175	0	0
					Profit,	£.	152	14	5$\frac{2}{7}$

34. Sent to Rotterdam 500 salt ox hides, *wt.* neat 400C. 2 qrs. 14℔ cost 20s. 4d. ℘ Cwt. paid for duty and other charges 15l. 10s. 3d. when the hides arrived, they weighed 30370℔. and my correspondent sells them for 15 guil-

ders the 100℔ deducts for freight and other charges 650 guilders, 17¼ stivers, and for his commission 2 ⅌ cent. Now tell me whether I gain or lose, if I value each guilder at 2*s.*? *Answ.* I gain 90l. 16. 8½d.

35. A merchant sends to Bourdeaux 200 barrels of beef, which cost him 20s. ⅌ barrel, and 400 casks of butter, wt. neat, 600C. 2qrs. at 20s. 4d. ⅌ Cwt. pays for duty and other charges 45l. 10s. 6d. the correspondent in Bourdeaux sells the beef for 16 livres the barrel, and the butter for 20 livres the 100℔ (*wt.* 67850℔) deducts for charges 1500 livres, and 2 ⅌ cent. for his commission. Now if the merchant values each livre at 16d. to know whether he gets or loses? *Answ.* He gains 139l. 12s. 1¾d.

36. A merchant in Ireland sent to Lisbon 500 barrels of wheat cost 10½s. ⅌ barrel; 200 firkins of butter, wt. 110C. 2qrs. 14℔ at 18s. 8d. ⅌ C. and 400 tanned hides, wt. 106C. 3qrs. at 5¾d. ⅌ ℔ paid for duty and other charges 35l. 16s. 1d. and pays premium of insurance of 688l at 16 ⅌ cent. The wheat at Lisbon measured 5000 alquires, and is sold for 300 reas the alquire; the butter, *wt* 837 arobes and 6℔ and is sold at 80 reas the ℔ the leather *wt.* 373 arobes and 20℔ and sells for 130 reas the ℔ the factor pays for duty and other charges, 832 mil. 62 reas, and charges for his commission 3 ⅌ cent. the factor remits the neat proceeds to London, at 5s. 6d. ⅌ milrea, and the merchant in Ireland, draws it from London at 5 ⅌ cent. advance. I demand whether he gets or loses by this cargo? *Answ.* He gains 161l. 14s. 6d.

Note. Every aroba is 32℔.

37. A merchant in Ireland sends to Ostend 20hhds. of tallow, *wt.* 10 ton, 17C. 3qrs at 30l. 10s. 9d. the ton: pays for duty and other charges, 25l. 10s. and for premium of insurance 35l. 10s. The tallow arriving at Ostend, *wt.* 24388℔, and the merchant there sells it at 18 guilders the 100℔ pays for freight, duty and other charges, 595 guilders, 10 stivers, 4 pennings, and reckons for his commission 2 ⅌ cent. and remits the nea proceeds to London, at 33s. 9d. Flemish ⅌ *l.* sterling, and the Irish merchant draws it from London at 4 ⅌ cent. advance, to know whether he gets or loses? *Answ.* He loses 32l. 15s. $1\frac{86529}{2350086}$d

CHAP. X.

FELLOWSHIP.

THE intent of this Rule is principally to divide the gain or loss made by trade in Partnership (where the stocks are unequal) proportionably among the partners, viz. in proportion to their respectiue stocks.

Rule.

Add all the given stocks into one sum, then it will be as the whole stock is to the whole gain or loss, so is each man's particular stock, to his particular gain or loss.

Examples.

1. A and B buy certain merchandizes, amounting to 80l. of which A pays 30l. and B 50l. and they gain by said goods 20l. Now I would know each man's share in the profit in proportion to the sum he puts in?

Answ. A's 7l. 10s. B's 12l. 10s.

A's stock 30
B's ——— 50

1st If 80 gain 20 what 30 l. *Answ.* 7 10
2dly —— 80 —— 20 —— 50 l. —— 12 10

Profit £ 20 0

But since the operation must be repeated for every man's share, the work may be often much contracted and facilitated by applying decimals and thereby finding a common multiplier, by which if we multiply each man's stock, his gain or loss will be produced, viz.

l. s. l.
As 80 gain 20 what 1
8|0)20|0

25 Common multiplier being the gain of 1l.

30	50
.25	.25
7.51 or 7 10	12.50 or 12 10

The same may be effected by vulgar fractions, thus:

80 —— 20 —— 1. *Answ.* $\frac{1}{4}$ $\frac{30}{7\ \ 10}$ $\frac{1}{4}$ $\frac{50}{12\ \ 10}$

2. A and B have gained by merchandize 182l. A did put in 300l. and B 400l. I demand each of their shares of the profit? *Answ.* A 78, and B 104l.

3. A merchant being deceased, it is found he owes to A 500l. to B 900l. though he left but 1100l. behind him: I demand how much each is to have in proportion of his debt? *Answ.* A 392$\frac{6}{7}$, and B 707$\frac{1}{7}$.

4. A, B and C, freight a ship from the Canaries, for England with 108 tuns of wine, of which A had 48, B 36, and C 24; the mariners meeting with a storm at sea, were constrained for the safety of their lives, to cast 45 tuns thereof overboard: Here the question to be resolved is, how many of the 45 tun every particular merchant has lost according to the rate of his adventure?

Answ. A 20, B 15, and C 10 tun.

5. A chapman breaking owes unto 4 men the following sums of money, *viz.*

		l.	*s.*	*d.*
To	A	21	9	6
	B	72	19	3
	C	114	13	9
	D	264	17	6
Sum		474	0	0

His whole estate is found to be but 148l. 2s. 6d. what must each have of the same, and what will it be ꝑ *l.*

		l.	*s.*	*d.*
Answ.	A	6	14	2$\frac{1}{2}$
	B	22	16	0
	C	35	16	9$\frac{3}{4}$
	D	82	15	5$\frac{3}{4}$

at 6s. 3d. ꝑ £.

6. Three butchers pay among them 40l. for a grass inclosure, into which they put 30 cows, whereof A had 80, B 100, and C 120; how much had each to pay?

Answ. A 10l 13s 4d. B 13l 6s. 8d. and C 16l.

7. A father left his estate of 100 l. among 3 sons, in such manner that for every 2L. that A gets, B shall have 3, and C 5; how is the estate to be divided?

Answ. A's share 200l. B 300l. C 500l.

8. Four men traded with a stock of 800l. and they gained in 2 years time twice as much and 40l. more; A's stock was 140l. B's 260, C's 300l. I demand D's stock, and what each man gained by trading?

Answ. D's stock was 100l. and A gain'd 287l. B 533, C 615, and D 205l.

9. A, B and C put in money together; A put in 20*l.* B and C put in 85; they gain'd 63*l.* of which B took up 21*l.* What did A and C gain, and B and C put in?

Answ. A gain'd 12*l.* and C 30*l.*
B put in 25*l.* and C 50*l.*

10. A, B and C put in money together; A put in 20*l.* B 30*l.* C a sum unknown; they gained 36*l.* C took up 16*l.* what did A and B gain, and C put in?

Answ. A gain'd 8*l.* and B 12*l.* and C put in 40*l.*

11. Two merchants have gained 450*l.* of which A is to have 3 times as much as B. How much is each to have?

Answ. A 337*l.* 10*s* B 112*l.* 10*s.*

SOLUTION.

It is manifest from the question that 450*l.* is to be divided in proportion as 3 to 1. Therefore

3
1
—
4——450 $<$ 3 *Answ.* 337 10 (*l.* *s.*)
           1 ——— 112 10

12. Three persons are to share 600*l.*; A is to have a certain sum, B as much again as A, and C three times as much as B, I demand each man's part?

Answ. A $66\frac{2}{3}$*l.* B $133\frac{1}{3}$*l.* and C 400*l.*

13. Three persons have gained 1320*l.*; now when A takes 6*l.* B takes 4*l.* and C 2*l.* I demand how much each gets? *Answ.* A 660; B 440, and C 220*l.*

14. A, B and C shipped off sundry goods in company to have their gains upon the whole in proportion to their inputs, *viz.* A put in 80 pieces of serge, valued at 5*l.* 10*s.* ℔ piece; B 70 pieces of frize, at 4*l.* ℔ piece: and C 90 pieces of stuff, at 2*l.* 10*s.* C paid charges of shipping on the whole 27*l.* 10*s.* and consigned them to his correspondent at Lisbon; by whose account of sales the neat proceeds appear to be 3416 mil. 600 reas, which he remits to C's correspondent at London at 5*s.* 6*d.* ℔ millrea; and C draws on London at $7\frac{3}{4}$ ℔ cent. It is required to divide the neat amount rateably among the adventurers?

	l.	*s.*	*d.*
Answ. A	458	9	$10\frac{428}{778}$
B	291	9	$7\frac{16}{773}$
C	262	17	$1\frac{113}{773}$

15. A ships off 400 barrels of beef worth 20s. ⅌, belonging to B; and 250 casks of butter of his own, wt. 313C. 0qrs. 21℔. at 21s. 6d. ⅌ C. and it was agreed between them to run equal risques in the whole, the loss or gain to be divided in proportion to their outsets; A paid 23l. 6s. 5⅝d. charges on them, which is to be reckoned as so much stock. The goods he consigned to his correspondent in Bourdeaux, who returns account of the neat proceeds amounting to 11425 livres, and by his order sends in return 30 hhds. of wine, which cost 80 livres ⅌, and 4 casks of indigo, *wt.* gross 2078℔. Tare 37℔. ⅌ cask, at 2¼ livres ⅌ ℔. neat; when these goods arrive A pays duty and charges on them 40l. 10s. 0¼d. and afterwards sells the wine at 9l. ⅌ hhd. and the indigo at 5s. 10d. ⅌ ℔. (100℔. at *Bourdeaux* being 110 here) and the balance his correspondent in Bourdeaux remits to Amsterdam, at 54d. ⅌ crown, which his correspondent at London draws for at 35s. Flemish ⅌ *l* sterl. and remits to him here at 8½ ⅌ cent. deducting ½ ⅌ cent. for his commission; so that it comes here 194l. 6s. 6d. neat. It is required to find what A must pay B for his share of the neat amount of this adventure, being allowed 2½ ⅌ cent. commission on the sale of the returns, and what each gained thereby?

Answ. A must pay B 590l. who gains 96l.
A gains 64l. 16s. and commission 22l. 4s. 7½d.

N. B. The following questions usually solved by position may be more easily effected by the proceeding problem, *viz.*

16. The sum of the ages of A, B and C is 154 years; B was as old as A and ½ as old again; and C twice as old as B: I demand each man's age?

By considering this question, I discover that A's age is in proportion to B's as 1 to 1½, and to C's as 1 to 3; Therefore it is required to divide 154 in proportion to 1, 1½, and 3, *viz.*

1
1½
3

5½ : 154 : : 1 : 28 A's age.
1½ : 42 B's
3 : 84 C's

17. A gamester loses in 4 turns of dice 160 shillings, and trebled each turn the sum he put in. How much did he play for the first and last time?

Answ. 4s. and 108s.

18. Forty yards of drugget and 50 yards of cloth cost 21l. 10s. but every yard of cloth double as much as a yard of drugget: I demand what the drugget cost ⅌ yard? *Answ.* 3s. 6d.

19. Divide 15 into two such parts, that when the greater is multiplied by 4 and the lesser by 16, their product must be equal? *Answ.* 12 and 3.

I discover that the numbers required must have the same proportion to each other, the less to the greater, as 4 to 16, consequently 15 must be divided into parts in proportion to 4 and 16, *viz.* 3 and 12.

20. A master hires a journeyman on this condition, that he shall have 12d. a day and his diet for every day he works; but for every day he doth not work he is to pay him 6d. for his diet. Now on the 30th day they come to account, but neither of them receives or pays any money: How many days did he work and how many was he idle?

Answ. He worked 10 days and was idle 20.

21. What numbers are they which when added, will make 266$\frac{2}{3}$, and when one is multiplied by 3 and the other by 5 the sum shall be equal? *Answ.* 166$\frac{2}{3}$ and 100.

FELLOWSHIP WITH TIME.

WHEN stocks continue unequal time in company, so that consideration is made of the time as well as stock, this is called Fellowship with Time; for which this is the

Rule.

Multiply each man's stock by his time, and add the products into one sum; then divide the total gain or loss in proportion to these products.

Application.

1. Suppose two merchants A and B to be partners in traffick, and that A permits his stock of 100l. to be employed in their joint traffick 3 months, and B his stock of 50l. 8 months, and they gain 28l. I demand how much hereof belongs to each?

100 A's

	100 A's Stock	50 B's Stock,
Multiply by	3 — Time.	8 — Time.
Produce	300	400
	400	
	700——28 < 300 / 400	*l.* *Answ.* 12 A's Gain. —— 16 B's Gain.

2. Two merchants made a company; A put in 100l. for four months; and B put in 136l. for three months, and they gain 50*l.* I demand each man's share of the profits?

Answ. A 24l. $15\frac{5}{101}$s. and B 25l. $4\frac{95}{101}$s.

3. Three graziers hired a piece of land for 60l. 10s. A put in 5 oxen for $4\frac{1}{2}$ months; B put in 8 oxen for 5 months; and C put in 9 oxen for $6\frac{1}{2}$ months: I demand how much each must pay?

Answ. A 11l. 5s. B 20l. and C 29l. 5s.

4. Three persons have received 665l. interest, A had put in 4000l. for 12 months, B 3000l. for 15 months, and C 5000l. for 8 months: how much is each man's part of the interest? *Answ.* A 240l. B 225l. and C 200l.

5. Three persons hired a piece of land for 12l. 10s. 6d. A put in 20 sheep for 5 days, B put in 16 sheep for 7 days, and C put in 25 sheep for 4 days: I demand how much each must pay? *Answ.* A 4l. 0s. $3\frac{6}{13}$d. B 4l. 9s. $11\frac{1}{13}$d. and C 4l. 0s. $3\frac{6}{13}$d.

6. Three merchants lost by some dealings 45l. 10s. A's stock was 100l. for $6\frac{1}{2}$ months, B's stock 100l. for $9\frac{1}{2}$ months, and C's 150l. for $8\frac{2}{3}$ months; what is each man's part of this loss? *Answ.* A 10l. $3\frac{28}{29}$s. B 14l. $18\frac{3}{29}$s. and C 20l. $7\frac{27}{29}$s.

7. Three merchants put in a stock, *viz.* A puts in the 1st of January 120l. until the 23d of March; B puts in 176l. the 10th of February, until the 12th of April; C puts in 295l. the 2d of February, until the 25th of April, and they gain 800l. I want to know each man's part of the gain, allowing 28 days to the month of February?

Answ. A 174l. 3s. $4\frac{37200}{44646}$d. B 192l. 7s. $6\frac{6180}{44646}$d. and C 433l. 9s. $1\frac{1266}{44646}$d.

CHAP.

CHAP. XI.

ALLIGATION.

IS that Rule whereby we resolve Questions concerning the mixing of several Simples, or commodities into one Compound Quantity.

Alligation is either *Medial* or *Alternate*.

Alligation Medial, is when having the several quantities and rates of divers simples proposed, we discover the rate of a mixture compounded of these simples.

Rule.

Find according to the given rates, the value of each given quantity, then taking the sum of these quantities, and the sum of their values, say, if that sum of quantities give that sum of values what will the quantity (proposed) give?

Application.

1. Suppose 15 bushels of wheat, at 5s. the bushel, and 12 bushels of rye, at 3s. 6d. ℔ bushel, were mixed together, what is the mean rate or price it may be sold at a bushel, without loss or gain.

	s.	d.		l.	s.
15 Bushels, at	5	0	come to	3	15
12 ditto, —	3	6	—	2	2

3)

Consequently 27 bush. their sum come to 5 17 what is that for 1 bushel?

27 { 9) 1 19

Answ. 4s. 4d.

The reason of this operation is manifest in itself.

To prove the truth of the WORK.

Find the value of all the mixture at the mean rate;

Likewise the value of each particular quantity proposed to be mixed at its given rates, and collect these particular values into one sum, if the said sum is equal to the value of all the mixture before found, the work is right, as

	s.	d.		s.	d.	l.	s.
27 bushels at	4	4	15 bush. at	5	0—	3	15
		3	12————	3	6—	2	2
	13	0	Total £.			5	17
		9					
Come to £. 5	17	0					

2. A tobacconist mixeth 36℔. of tobacco, worth 1s. 6d. a pound, with 12℔. of another sort, at 2s. a pound; and 12℔. of a third sort, at 1s. 10d. ꝑ ℔. How may he sell the mixture ꝑ ℔.? *Answ.* 1s. 8d.

3. A vintner mixeth $31\frac{1}{2}$ gallons of Malaga sack, worth 7s. 6d. the gallon, with 18 gallons of Canary, at 6s. 9d. the gallon; $13\frac{1}{2}$ gallons of sherry, at 5s. the gallon; and 27 gallons of white wine, at 4s. 3d. the gallon. 'Tis required to find what one gallon of this mixture is worth?

Answ. 6s. ꝑ gallon.

4. A druggist has 200℔. of ginger at 6d. the ℔. 300℔. at 7d. the ℔. and 400℔. at 8d. the ℔. Now if he mixes them together, what will 1℔. cost? *Answ.* $7\frac{2}{9}$d.

5. 100 casks of butter, *qt.* 200Cwt. 3qrs. 14℔. at 18s. 8d. ꝑ C. are mixed with 150 casks, *qt.* 306C. 1qr. 7℔. at 23s. 4d. ꝑ C. I demand what 1C. of said butter stands him?

Answ. 23s. $10\frac{1}{3}\frac{10}{63}$d.

6. There are melted and mixed together two sorts of silver; one sort is worth 5s. and the other 4s. an ounce; and there were 4 ounces of the first, and 8 ounces of the latter: what is the value of one ounce of this mixture?

Answ. 4s. 4d.

7. A goldsmith melts 8℔. $5\frac{1}{2}$ oz. of gold bullion of 14 * caracts fine, with 12℔ $8\frac{1}{2}$oz. of 18 caracts fine. I demand how many caracts fine this mixture is?

Answ. $16\frac{51}{127}$ caracts.

8. A refiner has 10℔. of gold of 20 caracts fine, and melts it with 16℔. of 18 caracts fine. The question is, how much alloy must be put to it to make it 22 caracts fine?

---

* An ounce of pure gold being reduced into 24 equal parts, these parts are called caracts: but gold is often mixed with some baser metal, which in the mixture is called the alloy: and according to the proportion of pure gold which is in every ounce, so the mixture is said to be so many caracts fine: Thus, if only 22 caracts of pure gold, and two of alloy, it is 22 caracts fine; if 20 caracts of pure gold and 4 of alloy it is 20 caracts fine; if there is no alloy, it is 24 caracts fine, or pure gold.

Answ. 'Tis not fine enough by 3$\frac{3}{13}$ caracts, so no alloy must be put to it but more gold.

SECT. II.

ALLIGATION ALTERNATE.

ALLIGATION Alternate is, when we have the several ingredients to be mixed, and the mean rate of the mixture given, to find such quantities of the simples or ingredients, as being mixed together, shall bear that common rate.

Rule.

The rates being all of, or reduced to one denomination, 2. Set the rates of the simples in a column under one another, and the mean rate on the left-hand of these. 3. Connect or link together the several simple rates, so that every one less than the mean be linked to one or more greater than it, and every one greater with one or more less. 4. Take the difference between the mean rate and that of the several simples, and write it over against all the simples with which that one (whose difference it is) is linked, then the sums of these differences standing against every simple rate, are such quantities of the several simples against which they stand as answer the question.

I. When the simple rates do not exceed three, there can be but one way of linking them; because the mixture or mean rate must be between the highest and lowest rate of the simples; else, 'tis plain the mixture would not bear that rate, but would be of a greater or lesser rate, as the simples were either all of a greater or lesser: So then of two simples one will be greater and one less than the mean rate, and of three one greater and two less, or two greater and one less, which cases can admit but one way of linking.

Examples.

1. How much wheat at 6s. the bushel, and barley at 3s. 8d. ⅌ bushel, will make a mixture that may stand in 4s. 4d. the bushel?

$$\text{Mean price } 52\left\{\begin{array}{l}\overset{\text{d.}}{72} \\ \\ 44\end{array}\right)\left.\begin{array}{r}8 \\ \\ 20\end{array}\right\}$$

Answ. 8 of wheat, at 6s.

20 of barley, at 3s. 8d.

The

The Proof.

Questions in this Rule are proved by Alligation Medial, as follows.

s. l. s.
8 at 6——2 8
20 at 3 8—3 13 4
— 4)————
If 28 ——— 6 1 4——— 1
————
£.0 4 4 Proof.

2. How much tobacco of 7½d. the ℔. and 9¾d. the ℔. must be mixt so that it will stand in 8½d. the ℔?
Answ. 1¼℔. of 7½d. and 1℔. of 9¾d.

3. What quantity of sugar at 11d. the ℔. and 7¼d. the ℔. would make a mixture so that it would stand in 10¼d. the ℔.? *Answ.* 3℔. at 11d. and ¾℔. at 7¼d.

4. How many oz. of silver, of 11 oz. fine, and 8 oz. fine, must be melted together to make the mass or mixture 9 oz. fine? *Answ.* 1oz. of 11oz. fine, and 2 oz. of 8 oz. fine.

II. When one rate is joined to two others, the sum of the differences of the said two and the mean rate, will be the quantity sought, at that rate to which the two are linked.

5. A merchant hath sugar of 5d. 10d. and 12d. ℘ ℔. How much of each sort must he take that he may sell a pound for 8d.

$$8\left\{\begin{array}{l}5\\10\\12\end{array}\right.\left|\begin{array}{l}2.4\\3\\3\end{array}\right|\begin{array}{l}\text{℔.}\\6 \text{ at } 5\\3-10\\3-12\end{array}\left.\right\}$$ *Answ.*

6 How much rye, at 4s. ℘ bushel, barley at 3s. ℘ bushel, and oats at 2s. ℘ bushel, will make a mixture worth 2s. 6d. ℘ bushel?
Answ. 6 bushels of rye, 6 bushels of barley, and 24 bushels of oats.

7. A grocer would mix three sorts of sugar together, viz. one sort at 10d. ⅌ ℔. another at 7d. and another at 6d. How much of each sort must he take, that the whole mixture may be sold for 8d. ⅌ ℔.

℔. d.
Answ. 3 at 10 ⅌ ℔.
2 at 7
2 at 6

8. A vintner has brandy, at 3, 5, and 6s. the gallon, and has a mind to mix a quantity of them together, so that it may stand him in 5s. 6d. the gallon. I demand how many gallons he must take of each sort?

Answ. ½ at 3s. ½ at 5s. and 3 at 6s.

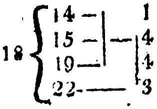
III. If the number of simple rates exceed 3 there may be several ways of linking them, and every way brings different answers; but all giving such numbers as will answer the thing required.

Examples.

9. A merchant would mix wines, at 14, 15, 19 and 22s. ⅌ dozen, so as the mixture may be worth 18s. What quantity of each may be taken?

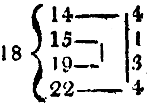
This sum may be linked 7 different ways as follow, viz.

*i.



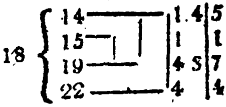
or

ii.



or

iii.



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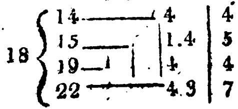
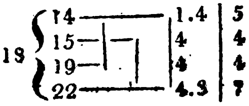
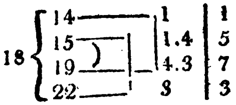


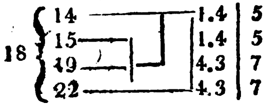
v.



vi.



vii.



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* Besides the different answers produced by this different manner of linking the terms or simple prices, questions in Alligation Alternate (being of that kind Algebraists term unlimited problems) have an infinite variety of other answers: for any other numbers in proportion to those found by this Rule (as above) will answer the questions as well as those.

10. A wine merchant has four sorts of wine, viz. of 20*d.* 16*d.* 12*d.* and 7d. the quart; how much of each sort must he take to sell a quart for 14*d.*?

Answ. 6 quarts of 7d. 2 quarts of 12d. 2 quarts of 16d. and 7 quarts of 20d. or otherwise.

11. A goldsmith has gold of 17, 18, 22 and 24 caracts fine; how much must he take of each to make it 21 caracts fine? *Answ.* 3 caracts of 17, 1 caract of 18, 3 caracts of 22 and 4 of 24 fine.

12. A vintner would make a mixture of Malaga worth 7s. 6d. ⅌ gallon, with Canary at 6s. 9d. ⅌ gallon; sherry at 5s. ⅌ gallon, and white wine at 4s. 3d. ⅌ gallon: what quantity of each must he take, that the mixture may be sold for 6s. ⅌ gallon?

Answ. 12 of Malaga; 18 of sherry; 21 Canary; 9 white wine, or otherwise.

SECT. III.

ALLIGATION PARTIAL.

THE particular rates of the ingredients proposed to be mixed, the mean rate of the whole mixture, and any one of the quantities to be mixed being given, to find how much of every one of the other ingredients is requisite to compose the mixture.

13. How much wheat at 5s. the bushel must be mixed with 12 bushels of rye, at 3s. 6d. ⅌ bushel, that the whole mixture may bear 4s. 4d. per bushel?

SOLUTION.

First, I find a quantity of wheat, which being mixed will bear the price proposed viz. 10 bushels of wheat and 8 of rye: but the given quantity of rye being 12 bushels, I must find a quantity of wheat so proportioned to 10 bushels as 12 to 8 viz. 8 : 12 : : 10 : 15, whence appears the reason of the

bu.

$$52\left\{\begin{array}{ll}60 & 10 \text{ of wheat.}\\ 42 & 8 \text{ of rye.}\end{array}\right.$$

8——12——10

10

8)120

Answ. 15 bushels.

or 8—10—12

Rule.

Set down all the particulars and find their differences, Then say, As the difference standing against the price of which the quantity is given, is to the said given quantity, so is each other difference to the quantity required.

14. How much alloy must be put to bullion of 10$\frac{4}{5}$oz. fine, to bring it to 7$\frac{1}{2}$ oz. fine? *Answ.* 5$\frac{2}{7}\frac{2}{5}$oz.

15. How much water must be mixed with 63 gallons of brandy, at 5s 5d. the gallon, to reduce it to 4s. 6d. ℔ gallon? *Answ.* 12$\frac{5}{6}$ gallons.

16. How much brass of 14d. ℔ lb. and pewter of 10$\frac{1}{2}$d. the lb must I melt with 50lb of copper worth 16d. the lb. so that the whole may stand me in 1s. the lb?
Answ. 200lb. at 10$\frac{1}{2}$d. and 50 at 14d.

17. How much gold of 21 and 23 caracts fine, must be mixt with 30*oz.* of 20 caracts fine, to bring it to 22 caracts fine? *Answ.* 30 of 21 and 90 of 23.

18. With 60 gallons of brandy at 6s. ℔ gallon, I mix brandy of 5s. 4d. ℔ gallon, and some water; then I find it stood me in 3s. 6d. ℔ gallon: I demand how much brandy, and how much water I took?
Answ. 60 at 5s. 4*d.* and 74$\frac{2}{7}$ of water.

19. How much Malaga of 7s. 5d. the gallon, and sherry of 5s. 2d. the gallon, and white wine at 4s. 2d. the gallon, must be mixed with 20 gallons of Canary at 6s. 8d. the gallon, so that one gallon of the mixture may stand in 6s. the gallon? *Answ.* 44 gallons at 7s. 5d. 16 gallons at 5s. 2d. and 34 gallons at 4s. 2d.

20. How much alloy, and how much gold of 21 and 23 caracts fine, must be put to 30*oz.* of 20 caracts fine, to bring it to 18 caracts fine? *Answ.* 16$\frac{2}{3}$*oz.* alloy, 30*oz.* of 21, and 30*oz.* of 23 caracts fine.

How is the above answer proved to be true?

SECT. IV.

ALLIGATION TOTAL.

THE particular rates of all the ingredients proposed to be mixed; and the sum of all their quantities, with the mean rate of that sum being given; to find the particular quantities of the mixture.

Rule.

Rule.

Set down all the particular rates with the mean rate, as before, and find their differences; and add together all the differences into one sum; then say, As the sum of all the differences: is to the sum of all the quantities given: So is every particular difference: to its particular quantity.

21. Let it be required to mix wheat at 5s. the bushel, with rye at 3s. 6d. the bushel, so as that the whole quantity may be 27 bushels, to be sold for 4s. 4d. a bushel: What quantity of each must be taken to make up the mixture?

$$52\left\{\begin{array}{c|r} 60 & 10 \\ 42 & 8 \end{array}\right.$$

18——27 < 10   *Answ.* 15 wheat
              8   —— 12 barley

PROOF.

s.	l.	s.
15 at 5 comes to	3	15
12 at 3 6——	2	2
27——	3)5	17——1
	9)1	19
£. 0	4	4 ⅌ bushel.

22. A goldsmith hath 2 sorts of silver bullion, the one of 10 oz. and the other of 5 oz fine, and has a mind to mix a ℔. of it, so that it shall be 8 oz. fine; I demand how much of each he must take? *Answ.* $4\frac{4}{5}$ of 5 oz. fine, and $7\frac{1}{5}$ of 10 oz. fine.

23. A grocer has sugar of 12d. the ℔. and of $6\frac{1}{2}$d. the ℔. and has a mind to mix a Cwt. of it, so that he may sell it at 8d. the ℔. I demand how much of each sort he must take? *Answ.* $162\frac{10}{11}$℔. of $6\frac{1}{2}$d. and $61\frac{1}{11}$℔. at 12d.

24. A refiner has silver of $11\frac{1}{2}$oz. and of 7 oz. fine, and has a mind to make a piece of work requiring 35℔. of $9\frac{3}{4}$oz. fine: How much must he take of each?

Answ. $21\frac{7}{13}$℔. of $11\frac{1}{2}$ oz. fine, and $13\frac{11}{13}$℔. of 7 oz. fine.

25. Brandy of 3s. 6d. and 5s. 9d. the gallon is to be mixed, so that a *hhd.* of 63 gallons may be sold for 12l. 12s. I demand how many gallons must be taken of each?

Answ. 14 gallons of 5s. 9d. and 49 gallons of 3s. 6d.

26. A vintner has 3 sorts of wine, viz. of 24d. 22d. and 18d. the gallon: now he has a mind to mix a cask of

60 gallons, so that he may sell it at 20*d.* the gallon: how much must he take of each? *Answ.* 12 at 24*d.* 12 at 22*d.* and 36 at 18*d.*

27. A goldsmith has 3 sorts of silver, viz. of 11, 8 and 5*oz.* fine, and has a mind to make a piece of work that shall weigh 10℔. of 9*oz.* fine: how much of each must he take? *Answ.* 5$\frac{5}{9}$lb. of 11*oz.* fine, 2$\frac{2}{9}$lb. of 8*oz.* fine, and 2$\frac{2}{9}$lb. of 5*oz.* fine.

28. A cask of 58 gallons is filled with liquor of 7, 8, and 10*d.* the gallon, and then it stands in 9$\frac{1}{4}$*d.* the gallon; I would know how many gallons of each sort was taken? *Answ.* 40.$\frac{6}{10}$ gallons of 10*d.* 8$\frac{7}{10}$ gallons of 8*d.* and 8$\frac{7}{10}$ of 7*d.*

BOOK IV.

EXTRACTIONS, PROGRESSIONS, &c.

CHAP. I.

OF THE SQUARE ROOT.

IF a Number be multiplied into itself any number of Times, the product is called a Power of that Number, and the Number multiplied in respect of the Product is called its Root, particularly

If a Number be multiplied into itself the Product is a Square Number, viz. the Square (or second power) of the number multiplied, which number is likewise the Square Root of the Product.

As 4×4=16, So 16 is the Square of 4, and 4 the Square Root of 16.

Having the Root given to find the Square thereof, is only to find the product of the given number multiplied by itself, and thus we construct.

A TABLE

Of the Squares of the single Figures.

Roots	1	2	3	4	5	6	7	8	9
Squares	1	4	9	16	25	36	49	64	81

This Table being committed to memory, we are to shew how to extract the square root of any number.

QUES-

QUESTIONS.

Quest. What is a square number?

A. That which is produced from the multiplication of any number into itself, which number is called the root with respect to its square.

Q. Repeat the squares of the single figures.

A. The square of 1 is 1, of 2 is 4, &c.

Q. How must I extract the square root?

A. By the following rule.

First to prepare the square, this do,
Point off the figures two by two:
Beneath the last the square next less
Put; and its root i' th' quotient place:
From the last period take the square,
Then the next lower period there
To the remainder must be brought;
Be this a dividend: The quote
Doubled must the divisor be
To all but units place; then see
How oft the greater holds the less,
That figure must the quote express,
And the divisor units too,
Then as in plain *Division* do.
Thus every period one by one
We manage and the work is done.

Q. How is the work proved?

A. By multiplying the root into itself, and adding the remainder, if any.

Examples

1. What is the square root of 256? *Answ.* 16.
2. ———————————— 576?
3. Find the square root of 234256?
4. What is the square root of 451584?
5. ———————————— 23097636?
6. ———————————— 151321?
7. ———————————— 14122564?
8. ———————————— 2985984?
9. ———————————— 15437041?
10. ———————————— 2990667969?
11. ———————————— 57219960721?

§ 2. *To extract the Square Root of Fractions.*

A Fractional power may be considered either as an immediate power, *i. e.* the immediate product of the multiplication of some fraction into itself, as $\frac{4}{9}=\frac{2}{3}\times\frac{2}{3}$; or as being only equivalent to some immediate power, as $\frac{8}{18}=\frac{4}{9}$.

Find the square root of $\frac{27}{75}$? *Answ.* $\frac{3}{5}$.

Find the square root of $\frac{36}{100}=\frac{6}{10}$ or $\frac{3}{5}$, &c.

§ 3. *To extract the Square Root of Surd Numbers.*

Such numbers as have not a perfect root, or are not perfect squares, cubes, &c. are called surd numbers.

From which definition it is manifest the square root of such numbers cannot be found exactly; but by approximation we may come as near the truth as we please, for which this is the

Rule.

Find the root of the given number, as if it was a perfect square, and when that is done there will be a remainder, to which prefix two cyphers (as the next lower period) and so to every succeeding remainder prefix two cyphers; and proceed at every step till one more than half the proposed number of decimal places be obtained (for all the figures arising when the cyphers are prefixed are decimals) and then the rest may be found by plain *Division.*

The USE *of the* SQUARE-ROOT.

Case I.

To find a mean proportion between any two given numbers.

Rule.

Multiply the two given numbers together, and extract the square root of the product, which roots shall be a mean proportional sought.

Examples.

1. What is the mean proportional between 4 and 9? *Answ.* 6.
2. What is the mean proportional between 16 and 36? *Answ.* 24.

Case II.

To find the side of a square equal in area to any given superficies.

Rule.

Extract the square root of the given superficies, which root will be the side of the square sought.

Examples.

Examples.

3. If the area of a given circle is 4276.5, I demand the side of a square whose superficial content shall be equal thereto? *Answ.* 65.395.

4. Suppose I have an elliptical or irregular fish-pond, containing in surface 9 acres, 2 roods, 15 perches, and would have a square one of the same content; I desire to know how many yards each side must be?

Answ. 274.2535 yards.

5. If the content of a given circle be 160, what is the side of a square equal thereto? *Answ.* 12.649.

Case III.

Having the area of a circle, to find the diameter.

Rule.

As 355 : 452 : : so is the area to the square of the diameter.

Examples.

6. Required the diameter of a circle that will comprehend within its circumference, the quantity of an acre of land?

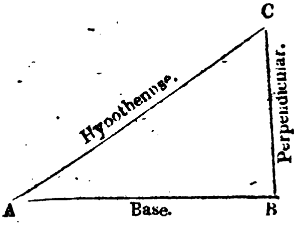
Answ. 99.91 yards.

7. In the midst of a meadow well stored with grass,
I took just two acres to tether my ass;
How long must the cord be, that feeding all round,
He mayn't graze less or more than these two acres of (ground?

Answ. 70.6475 yards.

Case IV.

Any two sides of a right-angled triangle, A, B, C being given, to find the remaining side.



1. The base and perpendicular being given to find the hypothenuse.

Rule.

Square each side, add the squares together, and the square root of this sum gives the hypothenuse required.

2. If the hypothenuse and one side be given, to find the other side.

Rule

Rule.

From the square of the hypothenuse, subtract the square of the given side, the square root of the remainder gives the side required.

8. A line 27 yards long, will exactly reach from the top of a fort, on the opposite bank of a river, known to be 23 yards broad: the height of the wall is required?

Answ. 14.1421 yards.

9. Suppose a light-house built on the top of a rock, the distance between the place of observation and that part of the rock level with the eye, and directly under the building, is given 310 fathoms; the distance from the top of the rock to the place of observation is 423 fathoms; and from the top of the building 425: the height of the edifice is required? *Answ.* 287.8 fathom height of the rock. 2.93156 ditto height of the light-house.

10. Two ships set sail from the same port, one of them sail'd due east 50 leagues, the other due north 84: How far are they asunder? *Answ.* 97.75 leagues.

11. The height of an elm, growing in the middle of a circular Island 30 feet in diameter, plumbs 53 feet, and a line, stretched from the top of the tree straight to the hither hedge of the water 112 feet; what then is the breadth of the moat, supposing the land on the other side the water to be level? *Answ.* 83$\frac{2}{3}$ feet.

12. Required the length of a shoar, that being to strut 11 feet from the upright of a building, will support a jamb 23 feet 10 inches from the ground?

Answ. 26 feet, 9 inches.

13. A castle wall there was, whose height was found
To be an hundred feet from th' top to th' ground;
Against the wall a ladder stood upright,
Of the same length the castle was in height.
A waggish youth did the ladder slide,
(The bottom of it) ten feet from the side:
Now I would know how far the top did fall,
By pulling out the ladder from the wall?

Answ. 6 inches nearly.

Case V.

Any number of men being given, to form them into square battle, or to find the number of ranks and files.

Rule.

Extract the square root of the number of men given, will give the number of men either in rank or file.

Examples.

Example.

14. A general disposing his army into a square battle, finds he has 23716 men: required the number in rank and file? *Answ.* 154 men.

THE EFFECTS OF LIGHT AND HEAT.

The effects or degrees of light, heat and attraction, are reciprocally proportional to the square of their distance from the centre whence they are propagated.

1. Suppose that in a room, where two men, A and B are sitting, there is a fire, from which A is 3 feet, and B is 6 feet distant: it is required to find how much hotter it is at A's seat than at B's? *Answ.* A's is 4 times as hot as B's.

2. Supposing the earth to be 81 millions of miles distant from the sun: I would know at what distance from him another body must be placed so as to receive light and heat double to that of the earth? *Answ.* 57275649 miles.

3. The distance between the earth and sun is accounted 81 millions of miles, the distance between Jupiter and the sun 424 millions of miles, the degree of light and heat received by Jupiter, compared with that of the earth is required? *Answ.* The sun's influence on the earth, to that on the planet Jupiter, is as $27\frac{4}{10}$ to 1.

4. Mercury the nearest of all the planets to the sun, is about 32 millions of miles from him; Saturn is distant about 777 millions of miles; what proportion is there between the solar influences on these two bodies?

Answ. The solar influence on Mercury to that of Saturn, is as $589\frac{1}{2}$ to 1 nearly.

5. Suppose with Dr. Keil, the distance of the sun to be from us 115 of his diameters: how much hotter is it then at the surface of the sun, than under our equator?

Answ. 13225 degrees hotter.

The less porous a Body is, the greater is its Density.

6. The compactness or density of the moon is to that of the earth, as $132\frac{1}{2}$ is to 100: what proportion then is there between the quantity of matter in the earth and that in the moon, since the earth's diameter is 7970 miles, and that of the moon 2170? *Answ.* The earth contains 40,117 times more matter than the moon.

VELOCITIES *acquired by heavy* BODIES *falling.*

The velocity acquired by heavy bodies falling near the surface of the earth, is $16\frac{1}{2}$ feet in the first second, and as

$16\frac{1}{12}$ feet are to the square of one second, or 1, so is the given distance to the square of the seconds required, or, on the contrary, to determine what space a heavy body has passed in any time given is,

By multiplying $16\frac{1}{12}$ the descent of a heavy body in one second of time, by as many of the odd numbers beginning from unity, as there are seconds in the given time, viz. by 1 for the first, 3 for the second, 5 for the third, 7 for the fourth, &c. the sum total will give the space it hath passed.

7. Suppose a stone let fall into an abyss should be stopped at the end of the eleventh second after its delivery, what space would it have gone through?

Answ. 1946.083 feet.

8. A ball descending by the force of gravity, from the top of a tower, was observed to fall half the way in the last second of time: required the tower's height, and the whole time of descent?

Answ. $\begin{cases} 187\frac{21}{25} \text{ feet, tower's height.} \\ 3\frac{2}{5} \text{ seconds, time of descent.} \end{cases}$

9. What is the difference between the depth of two wells, into each of which should a stone be dropped at the same instant, one will meet with the bottom at six seconds, the other at ten? *Answ.* 1029.3 feet.

10. If a stone be $19\frac{1}{2}$ seconds in descending from the top of a precipice to the bottom, what is the height of the same? *Answ.* 1019 fathoms, 1 foot, $8\frac{1}{4}$ inches.

11. In what time would a musquet ball, dropped from top of a steeple 400 feet high, be at the bottom?

Answ. 5 seconds nearly.

12. If a hole could be bored through the centre of the earth, in what time would a heavy body let fall from its surface, arrive at its centre? *Answ.* 18 minutes, 55 seconds, 33 thirds.

VIBRATIONS OF PENDULUMS.

It hath been found by experiment, that a pendulum 39.2 inches long, in our latitude, vibrates 60 times in 1 minute; and that the length of the pendulums are to one another reciprocally, as the square of the number of their vibrations made in the same space of time.

1. What difference is there between the length of a pendulum that vibrates half a second, or 120 times in a minute, and another that swings double seconds, or 30 time in a minute? *Answ.* 147 inches.

2. What difference will there be in the number of vibrations made by a pendulum of 6 inches long, and another of 12 inches long, in an hour's time?

Answ. 2695.08 the difference.

3. What difference is there in the length of two pendulums, the one swinging 30 times, the other 100 times in an hour? *Answ.* Difference 42806.4 feet.

4. Give the length of a pendulum that will swing once in a third, once in a minute, once in an hour, once in a day? *Answ.* 10108 inches the length of that which vibrates thirds.

$2\frac{5}{22}$ miles, the length of a pendulum which vibrates once in a minute.

$8018\frac{2}{11}$ miles, the length of a pendulum which vibrates once in an hour.

$4618188\frac{1933}{1760}$ miles, the length of a pendulum which vibrates once in a day.

CHAP. II.

THE CUBE ROOT.

A Number being multiplied into itself, and the Product again multiplied by the same, produceth a Cube Number, and the number multiplied is the Cube Root of the Product, as 4×4×4 produceth 64, which 64 is the Cube of 4, and 4 the Cube Root of 64.

Or if a Square number be multiplied by its Root, the product is the Cube of that Root, as 16×4=64 the Cube of 4.

From either of which definitions of a Cube, it will be easy to find the Cube of every single figure, according to the following

TABLE.

Roots	1	2	3	4	5	6	7	8	9
Cubes	1	8	27	64	125	216	343	512	729

Application.

Application.

Let it be required to extract the cube root of 157464?

157'464'(54
125

7500)32464 Resolvend.

30000
2400
64

32464 Subtrahend

(1) The given number being pointed over the first and fourth figures, (2) the last period is 157, and the next cube, is 125, whose root 5 I put in the quotient, and the remainder, after 125 is taken from 157, is 32; to which bringing down the next period 464, I get the resolvend 32464. (3) the square of 5 is 25, which being multiplied by 300 makes 7500 for a divisor. (4) Seeking how often this divisor will go in the resolvend, I find four times, which I put for the next quotient figure. (5) 4×7500=30000; Next the square of 4 (=16) ×5, the other figure of the quotient makes 80, which multiplied by 30 produces 2400, and the cube of 4 makes 64, which three numbers being added make the subtrahend 32464=the resolvend: So then the given number is a perfect cube whose root is 54

Again, let it be required to find the cube root of 164566592?

164'566'592'(548
125

7500)39566 1st Resolv.

30000
2400
64

32464 Subtrahend

874800)7102502 2nd Resol.

6998400
103680
512

7102592

The said number being pointed into periods of three figures, and proceeding as before, I get the resolvend 39566 and its divisor 7500 which will be contained therein 5 times; but then there is not a surplus in the resolvend equal to the other two numbers to be brought in, for if I take the quotient figure 5, and form a subtrahend therewith, ꝑ th[illegible] Rule, it will result 41475, which is greater than the resolvend,

solvend; and therefore cannot be subtracted from it, as the Rule directs, so I must take a lesser number as 4, and then I find the subtrahend 32464 (as before) which I subtract from the resolvend and there remains 7102, to which I bring down the next period, and get a new resolvend 7102592, with which I proceed as with the first, viz. I square 54 which makes 2916, this × 300 produces 874800 for a divisor to said resolvend in which it is found 8 times; and 8 × 874800=6998400; the square of 8, viz. 64 × 54 × 30=103680, and the cube of 8=512; these 3 numbers added make the subtrahend 7102592=the resolvend, and every period is brought down. So 548 is the cube root of the given number.

QUESTIONS.

Quest. What is a cube number?

Answ. A square number multiplied by its root.

Q. What are the cubes of the single figures?

A. The cube of 1 is 1; the cube of 2, 8; of 3, 27, &c.

Q. How must I extract the cube root?

A. First let the numbers pointed be
In periods each of places three;
Beneath the last the cube next less
Put; and its root i' th' quotient place;
The cube then from the period take;
Remainder with next period make
A resolvend: Then we must see
This resolvend divided be
By just 300 times the square
O' th' figures which in quotient are.
Next quotient figure such must be
As to allow for numbers three;
First for the product of the said
Figure, by the divisor made;
That of its square being multiplied
By all the quotient beside,
And then by 30 is the second;
And let its cube the third be reckon'd:
Their sum must be the subtrahend,
Not greater than the resolvend.
Then from the greater take the least;
To the remainder bring the next
Period; and the same way descend,
From point to point unto the end.

Which done, if ought remain there shall
Add treble cyphers for a decimal.

Q. How is the work proved?

A. Multiply the root or quotient into itself, and then the product by the same root, adding the remainder, (if any) to the last product: so shall we get the given number if the work is right.

Examples.

3. What is the cube root of 614125?
4. ——————————— 46656?
5. ——————————— 146363183?
6. ——————————— 41421736?
7. ——————————— 673373097125?
8. ——————————— 705919947284?

To extract the Cube Root of a Fraction.

Rule.

Bring the proposed fraction to its least terms, and extract the cube root of the numerator for the numerator, and the cube root of the denominator for the denominator of the root.

Application.

Let it be required to extract the cube root of $\frac{64}{216}$?

$$\begin{array}{c|c|c|c} 4) & 2) & & \\ \frac{64}{216} & \frac{16}{54} & \frac{8}{27} & \frac{2}{3} \end{array} \qquad \textit{Answ. } \tfrac{2}{3}.$$

To extract the Cube Root of Surd Numbers.

Extract the root of the given surd number by the Rule before delivered, but when the work is done there will be a remainder; to the remainder prefix three cyphers, and repeat the process, and so to every succeeding remainder, until a root be got sufficiently near, (for it cannot be found exactly,) and all the figures arising after the prefixing of cyphers are a decimal.

Example.

Example.

Let it be required to extract the Cube root of 9302348?

9'302'348'(210.312, *&c.*
8

1st Divisor ——
1200)1302—first Resolvend

1200=1200×1
60=Sq. 1×2×30
1=Cube of 1

1261 Subtrahend

2nd Divisor ———
13230000)41348000—second Resolvend

39690000=Divisor×3
56700=Sq. 3×210×30
27=Cube of 3

39746727—second Subtrahend

3rd Divisor————
1326782700)1601273000—third Resolvend

1326782700=Divisor×1
63090=Sq. 1×2103×30
1=Cube of 1

1326845791—third Subtrahend

4th Divisor ————
132690888300)274427209000—fourth Resolvend.

265381776600—fourth Divisor×2
2523720—Sq. 2×21031×30, *&c.*
8

265384300328—fourth Subtrahend

9042908672 Remainder.

1st Divisor is——1200=Sq. 2×300
2nd —— 13230000=Sq. 210×300
3rd —— 1326782700=Sq. 2103×300
4th ——— =Sq. 21031×300

THE USE OF THE CUBE ROOT.

Case I.

To find the side of a cube that shall be equal in solidity to any given solid, as a globe, cylinder, prism, cone, *&c.*

Rule.

Extract the cube root of the solid content of the given body, which root will be the side of the cube required.

Examples.

1. There is a stone of a cubic form, which contains 21952 solid feet; what is the superficial content of one of its sides? *Answ.* 28 feet.

Case II.

Having the dimension of any solid body, to find the dimensions of another similar solid, that shall be any number of times greater or less than the solid given.

Rule.

Multiply the cube of each side by the difference between the solid given and that required, if greater (or divide by the difference if less) than the solid given; then extract the cube root of each product or quotient, which will give the dimensions of the solid required.

Examples.

2. Suppose the length of a ship's keel to be 125 feet, the breadth of the midship beam 25 feet, and the depth of the hold 15 feet; I demand the dimensions of another ship of the same form, that will carry three times the burthen?

Answ.
Length of the keel, 180.28 feet.
Breadth of the beam, 36.05 ——
Depth of the hold, 21.63 ——

3. Again, I demand the dimensions of another ship of the same form, that shall be only half the burthen of that whose dimensions are given as above?

Answ.
Length of the keel, 99.21 feet.
Breadth of the beam, 19.84 ——
Depth of the hold, 11.9——

Case

Case III.

Having the dimension and capacity of a solid, to find the dimensions of a similar solid of a different capacity.

Rule.

Like solids are in triplicate proportion to their homologous sides, therefore it will be as the cube of a dimension : is to its given weight : : so is the cube : of any like dimension to the weight sought.

Examples.

4. If a ship of 300 tons burthen be 75 feet long in the keel, I demand the burthen of another ship, whose keel is 100 feet long? *Answ.* 711.111 tons.

5. Suppose a ball of 4 inches diameter weighs 18lb. I demand the diameter of another that weighs 114lb.? *Answ.* 7.4 inches.

Case IV.

To find two mean proportionals between two given numbers.

Rule.

Divide the greater extreme by the less, and the cube root of the quotient, multiplied by the less extreme, gives the lesser mean; multiply the said cube root by the lesser mean, and the product will be the greater mean proportional.

Examples.

6. What are the two mean proportionals between 7 and 189? *Answ.* 21 and 63.

7. Find two mean proportionals between 4 and 256? *Answ.* 16 and 64.

CHAP. III.

ARITHMETICAL PROGRESSION.

ANY rank of numbers encreasing one above another by a common Excess, as 1, 2, 3, 4, 5, &c. which exceed each other by 1; or 2, 4 6, 8, 10, &c. 1, 3, 5, 7, 9, &c. whose common Excess is 2; or contrariwise decreasing by a common Difference, as 5, 4, 3, 2, 1; 10, 8, 6, 4, 2, are said to be in Arithmetical Progression.

The numbers which form an Arithmetical Progression are called *Terms of the Progression*, and the number whereby the latter term exceeds or is deficient of the former is called the *Common Difference*.

As 2, 4, 6, 8, 10, these numbers are the terms, and 2 the common difference.

From the infinity of number, it is easy to conceive that an increasing Arithmetical progression may be infinitely continued; but a decreasing progression cannot be continued further than till the last term becomes less than the common difference.

Numbers in Arithmetical progression have sundry peculiar properties, of which are the following.

Proposition 1.

In any increasing series, if the first term be added to the product of the common difference multiplied by the number of terms less 1, the sum will be the last term: And in a decreasing series if the said product be subtracted from the first term, the remainder will be the last term.

Proposition 2.

If three numbers are in Arithmetical progression, the sum of the two extremes will be double the mean.

Example.

If 2, 3, 4, $2 + 4 = 6$ and $3 + 3 = 6$
4, 6, 8, $4 + 8 = 12$ and $6 + 6 = 12$
4, 3, 2, $4 + 2 = 6$, &c.

Proposition 3.

If four numbers be in Arithmetical progression, the sum of the means is equal to the sum of the extremes.

Example. $\left\{\begin{matrix}2, & 4, & 6, & 8\\ 8, & 6, & 4, & 2\end{matrix}\right\} 2+8=10$ and $4+6=10$

Problem I.

Having two or more numbers in Arithmetical progression, none less than the common difference, to continue the progression upwards and downwards.

Subtract the less from the greater, and thereby find the common difference, which add to the greater and subtract from the less, and so will two extreme terms be found, of which extremes the less being diminished and the greater encreased by the same common difference, we get two other extreme terms, and thus we may continue the progression upwards as far as we please, and downwards till we find a number less than the common difference, which is the first term of an encreasing progression.

Example. Let 8 and 10 be given, and let it be required to continue the progression both ways. By deducting 8 from 10 we find the common difference 2, whence we get this progression;

0, 2, 4, 6, [8, 10,] 13, 14, 16, 18.

Proposition 4.

In any Arithmetical progression whose number of terms is odd, the mean or middle term being doubled, is equal to the sum of any two terms equally distant therefrom: and if the number of terms be even, the sum of the means or two middle terms is equal to the sum of any two terms equally distant therefrom.

Example. In 1, 3, 5, 7, 9, 11, 13. 7 + 7 = 14 and 5 + 9; 3 + 11; 1 + 13 each = 14.

Again, 1, 3, 5, 7, 9, 11, 13, 15, 7 + 9 = 16 and 5 + 11; 3 + 13; 1 + 15 each = 16.

Cor. Hence if any two numbers be added, and their sum halved, that half is an Arithmetical mean between the said two numbers.

In an Arithmetical progression these five things are more especially to be noted; (1) the first term; (2) the last term; (3) the common difference; (4) the number of terms; (5) the sum of the series; any three of which being given the rest may be found, which admits of 21 problems, but the following 3 seeming principally useful in Arithmetic, for brevity sake we confine ourselves thereto.

Problem II.

Having the first term, the common difference and number of terms, to find the last term.

Rule.

Multiply the number of terms less 1 by the common difference, and to that product add the first term, the sum is the last term required.

Example.

Let 1 be the first term, 2 the common difference, and 11 the last term; 2 × 10(=20) + 1 = 21 last term.

Cor. If the first term be = common difference, then the first term multiplied by the number of terms will produce the last.

Problem

Problem III.

Having the first term, the last term, and the number of terms, to find the sum of the series,

Rule.

Add the first and last terms together, and multiply half the sum by the number of terms, or the whole sum by half the number of terms, and the product is the sum of the series.

Example.

Let the first term be 1; the last 21, and number of terms 11; then
1+21=22 whose 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21,
half is 11 × 11 =
121 sum of the series, or 1 + 19 = 20 × 5 half the number of terms) 100 the sum of the series of 10 terms.

Problem IV.

Having given the first term, common difference and number of terms, to find the sum.

Rule.

First find the last term by *Prob.* 2. and then the sum by *Prob.* 3.

Examples.

1. How many strokes doth a regular clock strike in a natural day, or 24 hours? *Answ.* 156.

For at one o'clock it strikes 1, at two, 2, &c. so it is required to find the sum of an Arithmetical progression 1, 2, 3, &c. up to 12, where we have 1 the first term, 1 common difference; the number of terms 12, whence the last term is found to be 12. So then 1 + 12 = 13 + 6 half the number of terms produceth 78 the number of strokes in the first 12 hours, which being doubled gives 156 in 24 hours.

2. A man buys 17 yards of kersey: for the first yard he gave 2 shillings; for the last 10s. the price of each yard encreasing in an Arithmetical progression; how much did the whole amount to? *Answ* 5l. 2s.

3. How many strokes doth the clocks of Venice (which go on to be 24 o'clock) strike in the compass of a natural day? *Answ.* 300.

4. The length of my garden is 94 feet; now if eggs be laid along the pavement 1 foot asunder, and be fetched up singly to a basket, removed one foot from the last: How much ground must he traverse that does it?

Answ. 1 mile, 5 furlongs, 21 poles, 3½ feet English.

5. A merchant hires a clerk by covenant 14 years, to give him 5l. the first year, and raise his salary 40s. a year during the term: The question is, to discover how much he paid him one year with another on an average?

Answ. 18l.

6. Supposing a press gang having a warrant to press for 30 days, press the first day 300 men, and every succeeding day 10 more than the former; how many men will they raise in the 30 days? *Answ.* 1335.

By these problems likewise may the questions relating to annuities in arrear be more readily solved: page 226. For the several yearly or half yearly interests form an Arithmetical Progression, of which the last interest due may be taken as the first term and common difference, and the number of years or half years, &c. less 1, the number of terms, because there is no interest due upon the last payment, as being only now due.

Example.

If an annuity of 70l. be forborn 5 years, what will be due at the end of that term for principal and interest thereof, interest being computed for every annuity from the time it became due, at 5 ℔ cent. ℔ annum simple interest?

OPERATION.

	l.	s.	
100—5—70	3	10	first term,
5		4	
3|50	14	0	last term,
20			
	17	10	sum of first and last.
10|00		2	half number of terms.
£,3 10 a year's int.	35	0	amount of interest.
70×5=	350	0	amount of the annuity.
Answ.	385	0	

Thus let all the questions in page 227 be solved, and likewise the following.

If 70l. annuity, payable by quarterly payments, were unpaid 5 years, what will it amount to in that time, simple interest being computed at 5 ⅌ cent.? *Answ.* 391l. 11s. 3d.

CHAP. IV.

GEOMETRICAL PROGRESSION.

WHEN a Rank or Series of Numbers do either encrease by one common Multiplier, or decrease by one common Divisor, they are said to be in Geometrical Progression, or Geometrical Proportion continued.

As { 2, 4, 8, 16, 32, 64, here 2 is a com. multiplier,
64, 32, 16, 8, 4, 2, —— 2 is a com. divisor,
2, 6, 18, 54, 162, —— 3 common multiplier,
162, 54, 18, 6, 2, —— 3 common divisor,

The common Multiplier or Divisor is commonly termed the Ratio of the Progression.

Proposition 1.

Any three numbers in geometrical progression will form an analogy, by making the consequent of the former ratio the antecedent of the latter.

As the numbers 2, 4, 8, will form this analogy 2:4::4:8.

Cor. 1. If three numbers be in geometrical progression, the product of the extremes multiplied into each other is equal to the square of the mean.

Cor. 2. If the product of two numbers be equal to the square of a third, these numbers are in geometrical progression.

Problem I,

Having two numbers to find a mean proportional between them.

Multiply the two numbers into each other, and extract the square root of the product.

Find a mean proportional between 4 and 9? *Answ.* 6.

What is the mean proportional between 4 and 64? *Answ.* 16.

Proposition 2.

Any four numbers in geometrical progression will form an analogy or proportion.

As 2, 4, 8, 16, 2 : 4 :: 8 : 16, which is manifest.

Cor. Therefore, if four numbers be in geometrical progression, the product of the means will be equal to the product of the extremes.

Proposition 3.

In any geometrical progression the product of the two extremes, is equal to the product of any two terms equally distant from the two extremes.

3, 6, 12, 24, 48, 96,
3, 6, 12, 24, 48, 96, 192,
As $3\times96=288$ and $6\times48=288$, &c.

If over a geometrical progression beginning with unity, we place 1 over the second term and so proceed orderly according to the natural progression of numbers, viz.

0 1 2 3 4 5 6 7 8 Indices.
1, 2, 4, 8, 16, 32, 64, 128, 256, Powers,

The numbers 1, 2, 3, &c. will express what power of the ratio the term is over which it stands, and are therefore called indices or exponents of the power.

If the first term given be the ratio, then every succeeding term is the same power of the ratio as the order of its place, or the index of the power will denote both the power and order of the place, for 1 must be placed over the first term, 2 over the second, &c.

1 2 3 4 5 6 7 Indices.
2, 4, 8, 16, 32, 64, 128, Powers.

Proposition 4.

In any geometrical progression beginning with unity, (if the indices be supposed placed over the terms of the progression) there will be this coherence or relation between the powers and their indices, viz. The sum of the indices of any two powers, or terms of the progression will be the index of the product of the said two terms, and if the index of any term be doubled, its double will be the index of the square of said term in the said progression.

0 1 2 3 4 5 6 7 8
1, 2, 4, 8, 16, 32 64, 128, 256, Powers.

As $3+5=8$ and $8\times32=256$, Likewise $4+4=8$ and $16\times16=256$, &c.

Proposition 5.

In a geometrical progression not proceeding from unity, if any term be squared, and the square be divided by the first or least term, the quotient gives a term of the same, progression doubly distant from the first.

0 1 2 3 4 5 6 7 8
3, 6, 12, 24, 48, 96, 192, 384, 768.

$$\frac{48\times48}{3}=768$$

0 1 2 3 4 5 6 7 8
1, 2, 4, 8, 16, 32, 64, 128, 256.

Proposition 6.

In any geometrical progression not proceeding from unity, if any two terms be multiplied together, and the product divided by 4 the first or least term, the quotient will be equal to that term denoted by the sum of the exponents of the other two.

As in the last progression $\frac{\overset{3}{24}\times\overset{5}{96}}{3}=\overset{8}{768}$

Proposition 7.

In any geometrical progression, as any one of the antecedents is to its consequent, so is the sum of all the antecedents to the sum of all the consequents.

2, 4 8, 16, 32, 64, &c.

2 : 4 : : 2+4+8+16+32(62) 4+8+16+32+64(124)

Problem II.

To continue a geometrical progression upwards or downwards.

1. Upwards; divide any consequent given by its antecedent, and the quotient will be the ratio, whereby multiply the consequent and the product will be the next term, which being again multiplied by the ratio, will produce a new term, and so on, and it is continued downwards by dividing each greater term by the ratio.

Problem III.

To find any assigned term of a geometrical progression proceeding from unity without producing all the terms.

Continue the progression from unity to the sixth term whose index is 5, square this sixth term, and it produces that whose index is 10, which being likewise squared, its square will be the term whose index is 20; and from those terms we may easily find all others whose indexes are decades or even tens, viz. 20+10=30; 20+20=40, and

$40+10=50$, &c. and from them any other whatever. For the units are either greater or not greater than 5; if not greater, multiply the term last found by the term whose index denotes the distance of the assigned term from that last found; and if greater, multiply first by the term under 5, and then by the surplus of the units or index of the assigned term above 5.

Application.

Let it be required to find the 43rd term of a geometrical progression, beginning with unity whose ratio is 2?

0	1	2	3	4	5	6	7
1,	2,	4,	8,	16,	32,	64,	128,

The progression being continued to the 5th place 32, whose index is 5, $32\times 32=1024$, which will be doubly distant from unity, viz. the 11th term of the progression, whose index is 10. Again, 1024×1024 produces 1048576 the 21st, which being again multiplied by itself, will produce that term whose index is 40, viz. the 41st; but it is proposed to find the 43rd, viz. that whose index is 42, and $40+2=42$; wherefore multiply the last found number which is 1099511627776 by 4, the number whose index is 2, and so we get the 43rd term required, 4398046511104.

Problem IV.

To find any assigned term of a geometrical progression not beginning with unity, without producing all the terms.

Proceed directly as in the last problem, only observe to divide every product by the first term.

Application.

Let it be required to find the 26th term of a geometrical progression whose first term is 2 and ratio 3?

0	1	2	3	4	5
2,	6,	18,	54,	162,	486,

I continue the progression to the sixth place, which being squared, and the product divided by the first term 2, the result is 118098 whose index is 10; this being again squared, and the product divided by 2, produces 6,973,568,802, whose index is 20, and this last being multiplied by 486 and divided by 2, produces 1,694,577,218,886, whose index is 25, which is the 26th term.

Problem V.

To find the sum of any geometrical progression.

If the last or greatest term be not given, let it be found by Prob. 3 or 4, then subtract the least from the greatest; divide the remainder by the ratio of the progression less 1, and to the quotient add the greatest or last term.

Application.

Let it be required to find the sum of the following progression, 1, 3, 9, 27, 81, 243, 729?

```
   1               From 729 the greatest.
   3               Take   1 the least.
   9                     ———
  27       Ratio 3—1=2)728
  81                     ———
 243                      364 Quotient.
 729                      729
————                     ————
1093                     1093
```

QUESTIONS.

1. A man bought a horse and was to give a farthing for the first nail, 2 for the second, 4 for the third, &c. in geometrical progression. The number of nails was to be 7 in each shoe, viz. 28 nails in all: What must be paid for the horse? *Answ.* 279620l. 5s. 3¾d.

```
      0 1 2 3  4  5
1st——1, 2, 4, 8, 16, 32, (ꝑ Prob. 3;) the last or
28th term will be found to be 134217728
                  Subtract            1
                           ——————————
Then ratio—1=1 which       134217727
divides not, therefore add 134217728 the gr.
                           ——————————
                          4)268435455
                           ——————————
                         12)67108863¾
                           ——————————
                        2|0)559240|5 3
                           ——————————
                       £· 279620 5 3¾
```

2. A merchant sold 15 yards of sattin; the first for 1*s.* the second for 2*s.* the third for 4*s.* the fourth for 8s. I demand the price of the 15 yards? *Answ.* 1638*l.* 7*s.*

3. A draper sold 20 yards of superfine cloth; the first yard for 3*d.* the second for 9*d.* the third for 27*d.* &c. in triple proportion geometrical. I demand the price of the cloth? *Answ.* 21792402*l.* 10*s.*

4. A goldsmith sold 1℔. of gold at a farthing for the first ounce, a penny for the second, 4d. for the third, &c. in quadruple proportion geometrical. I demand what he sold the whole for, also how much he gained by the sale thereof, supposing he gave for it 4l. ℘ ounce?

Answ. He sold it for 5825l. 8s. 5¼d.
And gain'd 5777l. 8s. 5¼d.

5. A cunning servant agreed with a master (unskilled in numbers) to serve him 11 years, without any other reward for his service but the produce of 1 wheat-corn for the first year; and that product to be sow'd the second year, and so on from year to year until the end of the time, allowing the increase to be but in a tenfold proportion; that 7680 wheat corns make a pint, and is sold at 3s. ℘ bushel? *Answ.* 33908l. 8s. 4½d.

6. A thresher work'd 20 days at a farmer's, and received for the first day's work 4 barley-corns, for the second 12 barley-corns, for the third 36 barley-corns, and so on in triple proportion geometrical. I demand what the 20 day's labour came to, supposing the pint to contain 7680 corns, and the whole quantity to be sold at 2s. 6d. ℘ bushel? *Answ.* 1773l. 7s. 6d. rejecting remainders.

7. A merchant sold 30 yards of fine velvet trimmed with gold very curiously, at 2 pins for the first yard, 6 pins for the second, 18 pins for the third, &c. in triple proportion geometrical. I demand how much the velvet produced when the pins were afterwards sold at an hundred for a farthing; also, whether the said merchant gain'd or lost by the sale thereof, and how much, supposing the said velvet to have been bought at 50l. ℘ yard?

Answ. The velvet produced 2144699292l. 13s. 0½d.
The merchant gain'd 2144697792l. 13s. 0½d.

CHAP. V.

COMPOUND INTEREST.

WHAT *Compound interest* is, is already signified, which see p. 212.

From which it follows, that if any sum, as 100 pound, be lent out, suppose at 5 ℘ cent. and that the interest be not paid at the year's end, there will arise a new principal of 105l. on which interest must be paid the second year, and if it runs on a third year, then the principal for the third year will be 105l. together with a year's interest of 105l. i. e. 110l. 5s. &c.

This being well considered, will point out a method for finding the amount of any sum for any number of years, at compound interest. As for

Example.

1. What will 500l. amount to in 3 years, at 8 ⅌ cent. interest upon interest?

1. If 100 : 108 : : 500 : 540 amt. of 1st year.
100 : 108 : : 540 : 583 4s.——2d. year,
l. *s.* *l.* *s.* d.
100 : 108 : : 583 4 : 629 17 $1\frac{11}{25}$ 3d year requir.

Or,

1 : 1.08 : : 500 : 540
1 : 1.08 : : 540 : 583 2
: : 583.2 : 629.856.

Which amounts 540,583.2, 629.856, being produced from 500 by the continual multiplication thereof by 1.08 the principal and several amounts are in a geometrical progression, (viz. 500, 540,583.2, 629.856) whose ratio is the amount of 1l. for a year, viz. here 1.08, and the number of years are continually indexes of the terms. Likewise.

If the amount of 1l. for any number of years be multiplied by any given principal, the product will be the amount of that principal for the same time.

From which consideration we draw the following

Rule.

Find the amount of 1l. for the given time (which is to find a term in the geometrical progression from 1. whose index is the number of years given) and multiply that amount of 1l. by the principal given.

Thus the foregoing example may be done as follows:

0 1 2 3
1, 1.08 1.1664, 1.259712
5|00

£.629.8560|00

629 17 1¼

Note. It will be sufficient to keep 5 or 6 decimal places complete, how many terms soever may be required.

2. How much will 320l. amount to at 5 ⅌ cent. in 10 years, at compound interest?

0 1 2 3
1, 1.05, 1.1025, 1.157625
52011

1.157625 × 1.1025 =1.276280=5th term.
826721

1.628897=10th term.
320

521.27040

£.521 4 $11\frac{1}{4}$

At 5 ⅌ cent. the continual multiplier is 1.05=$1\frac{5}{100}$ =$1\frac{1}{20}$, wherefore 1×1.05=1.05 second term, and 1.05 multiplied by 1.05 or $1\frac{1}{20}$ = third term. But this is done when $\frac{1}{20}$ of 1.05 is added to itself, and so continually adding to every new term $\frac{1}{20}$ of itself produces the next succeeding term as follows, viz.

1
$\frac{1}{20}$ 0.05
——— Year.
$\frac{1}{20}$ 1.05 ——— 1
.0525

1.1025 —— 2
$\frac{1}{20}$ .055715

1.157625 — 3
$\frac{1}{20}$ .057281

1.215506 — 4
$\frac{1}{20}$ .060775

1.276281 — 5

But 1.06 (the amount of 1l. at 6 ⅌ cent.) = 1.05 + .01 = $1\frac{1}{20}$ + $\frac{1}{100}$ or 1 + $\frac{1}{20}$ + $\frac{1}{5}$ of $\frac{1}{20}$. Therefore
1.06 ——

1.06——1
$\frac{1}{2}|_{10}$ .053
+⅕ 0.106

1.126——2
$\frac{1}{2}|_{10}$ .05618
⅕ .011236

1.191016——3, &c.

Or,

1.06
1.06

636
106

1.1236
1.06

67416
11236

1.191016

By which methods we may construct Tables of the amount of 1l. at 5 and 6, (or any other rate) ⅌ cent. which being done, the amount of any sum for any time at compound interest is found by multiplying the tabular number by the principal given.

QUESTIONS.

1. What sum will 450l. amount to in 3 years at 5 ⅌ cent. ⅌ annum? *Answ.* 520l. 18s. 7½d.

2. What will 256l. 10s. amount to in 7 years, at 6 ⅌ cent ⅌ annum? compound interest.
Answ. 385l. 13s. 7½

3. What will 136l. 15s. 6d. be augmented to, being forborne 20 years, at 6 ⅌ cent ⅌ annum?
Answ. 438l. 13s. 1½d.

4. What sum will 500l. amount to in 4 years, at 4½ ⅌ cent. ⅌ annum. compound interest?
Answ. 596l. 5s. 2½d.

SECT. II.

Of Annuities *or* Pensions *in arrear, computed at* Compound Interest.

To find the amount of an annuity or pension in arrear compound interest.

Rule.

Find the amount of the given yearly sum at compound interest, for the given years less 1, which will be the last term of a geometrical progression, of which the given sum is the first. Then find the sum of that progression, and it is the amount of the annuity required.

Otherwise.

Otherwise,

Find the amount of 1l. at compound interest for the given years less 1. Then find the sum of that progression, whose first term is 1, and last term the said amount, and multiply the said sum by the given annuity.

Example.

Suppose an annuity of 320l. be 10 years in arrear, it is required to find what is now due, compound interest being allowed on every payment, at 5 ⅌ cent. ⅌ annum?

First Method.	By 2d Rule.
521.247 last Term.	1.628897
320.	1
.05)201.247040	.05)0.628897
4024.9408	12.57794
521.247	1.628897
4546.1878	14.206837
£.4546 3 8¾	320
	28413674
	42620511
	£.4546.18784

QUESTIONS.

2. An annuity of 20l. ⅌ annum is forborne 7 years, what is then due at 6 ⅌ cent. compound interest?
Answ. 167l. 17s. 6½d.

3. If 30l. ⅌ annum, yearly rent be forborne 9 years; what will it amount to at 6 ⅌ cent. ⅌ annum compound interest? *Answ.* 344l. 14s. 9¼d.

4. Suppose a person who had an annuity of 20l. suffered it to be in arrear for 15 years, what had he then to receive; compound interest being computed at 6 ⅌ cent. ⅌ annum? *Answ.* 465l. 10s. 4½d.

SECT. III.

Of Rebate *at* Compound Interest.

Rule.

Find the amount of 1l. for the given time at the given rate, and divide the given sum to be rebated thereby, the quotient will be the sum to be paid down.

Examples.

Examples.

What ready money ought to be paid down for a debt of 629l. 17s. $1\frac{1}{2}$d. due 3 years hence, discount at 8 ℘ cent. ℘ annum, compound interest.

1.259712) 629.856000(500l. *Answ.*
6298560

2. Suppose 521l. 4s. $11\frac{1}{4}$d. were to fall due 10 years hence, how much ought to be paid now in full satisfaction for it, discount being allowed at 5 ℘ cent. ℘ annum, compound interest? *Answ.* 320l.

3. A legacy of 520l. 18s. $7\frac{1}{2}$d. is left to be paid in 4 year's time; but the Executor is willing to pay it at the expiration of 1 year, upon being allowed discount at compound interest at 5 ℘ cent. which being agreed to, what must he pay? *Answ.* 450l.

SECT. IV.

Of the present Worth of Annuities; *and of* Leases *in Reversion.*

Rule.

Find the present worth of the first and last year's annuity, which are the greatest and least terms of a geometrical progression: Then find the sum of that progression.

1. What is 30l yearly rent to continue 7 years, worth in ready money, allowing 6 ℘ cent. compound interest, to the purchaser?

$\frac{30}{1.06}=28.3019$ worth of the first year's ann. and comp. term.

$\frac{30}{1.50363}=19.9517$ the last and least term.

.06)1 3502

139.17
28.3019

167.4719

Answ. £.167 9 5

2. There is an annuity of 20l. ℘ annum, to continue 7 years to be sold for ready money: What is it worth

compound interest being allowed the purchaser at 5 ⅌ cent.? *Answ.* 115l. 14s. 6½d.

3. An annual rent of 365l. paid yearly, and to continue 12 years, is to be sold for ready money; what is it worth at 5 ⅌ cent. compound interest?

Answ. 3235l. 1s. 9d.

Now to find the value of an annuity or lease in reversion, this is the

Rule.

Find the present worth of the annuity as commencing immediately, and then find what ready money ought to be paid for that sum, rebate at compound interest being allowed for the term of years till the commencement of the annuity or lease.

Suppose it were required to compute the present worth of 75l. yearly rent, which is not to commence until 10 years hence, and then to continue 7 years after that time, at 6 ⅌ cent. compound interest.

1. An annuity of 75l. ⅌ annum, to continue 7 years, may be found at 6 ⅌ cent. compound interest to be worth 418l. 13s. 6¾d. 2. Then we are to find how much ready money ought to be paid for this sum as due 10 years hence, which will be found 233l. 15s. 9d. the answer required.

An annuity of 24l. ℔ annum, to begin 7 years hence, and to continue 21 years; what is it worth, allowing the purchaser 6 ℔ cent. compound interest?

Answ. 269l. 10s. 8d.

But as the finding the compound interest of any sum is troublesome, for a large term of years; the following Tables will make the work of questions relating to compound interest very easy.

TABLE I.

Shewing the Amount of 1l. for 31 Years, at 5 and 6 per Cent. Compound Interest.

Years.	5	6
1	1.050000	1.060600
2	1.102500	1.123600
3	1.157625	1.191016
4	1.215506	1.262477
5	1.276281	1.338225
6	1.340096	1.418519
7	1.407100	1.503630
8	1 477455	1.593848
9	1.551328	1.689479
10	1.628895	1.790848
11	1.710339	1.898298
12	1.795856	2.012196
13	1.885649	2.132928
14	1.979932	2.260904
15	2.078928	2.396558
16	2.182874	2.540352
17	2.292018	2.692773
18	2.406619	2.854339
19	2.526950	3.025599
20	2.653298	3.207135
21	2.785963	3.399564
22	2.925261	3.603537
23	3.071524	3.819750
24	3.225100	4.048935
25	3.386355	4.291871
26	3.555673	4.549383
27	3.733456	4.822346
28	3.920129	5.111687
29	4.116135	5.418388
30	4.321942	5.743491
31	5.438039	6.088101

TABLE II.

Shewing the present Worth Rebate of 1l. for 31 Years, at 5 and 6 per Cent. Compound Interest.

5	6
.952381	.943396
.907030	.889996
.863838	.839619
.822703	.792093
.783526	.747258
.746215	.704960
.710681	.665057
.676839	.627412
.644609	.591898
.613913	.558394
.584679	.526787
.556837	.496969
.530321	.468839
.505068	.442301
.481017	.417265
.458111	.393647
.436296	371364
.415520	350343
.395734	.330513
.376889	.311804
.358942	.294155
.341849	.277505
.325571	.261797
.310067	.246978
.295302	.232999
.281240	.219810
.267848	.207368
.255093	.195630
.242946	.184556
.231377	.174116
.220356	.164255

	TABLE III. Shewing the Amount of 1l. Annuity, forborne for 31 Years, or under, at 5 and 6 per Cent. Compound Interest.		TABLE IV. Shewing the present Worth of 1l. Annuity, to continue for 31 Years, at 5 and 6 per Cent Compound Interest.	
Years.	5	6	5	6
1	1.000000	1.000000	0.952381	0.943396
2	2.050000	2.060000	1.859410	1:833392
3	3.152500	3.183600	2.723241	2.673012
4	4.310125	4.374616	3.545950	3.465105
5	5.525631	5.637093	4.329477	4.212363
6	6.801913	6.975318	5.075692	4.917324
7	8.142008	8.393837	5.786373	5.582381
8	9.549108	9.897467	6.403212	6.209792
9	11.026564	11.491316	7 107821	6.801691
10	12.577892	13.180794	7.721734	7.360086
	14.206787	14.971643	8.306414	7.886673
12	15.917126	16.869940	8.863251	8.383843
13	17.712982	18.182137	9.393572	8.852682
14	19.598631	21.015065	9 898640	9.294983
15	21.578563	23.275969	10.379658	9.712248
16	23.657491	25.672527	10.837769	10.105894
17	25.840366	28.212879	11.274065	10.477258
18	28.132384	30.905651	11.689586	10.827602
19	30.539009	33.759992	12.085320	11.158115
20	33.065954	36.785590	12.462209	11.469920
21	35.719251	39.992727	12.821152	11.764075
22	38.505214	43.392291	15.163002	12.041580
23	41.430475	46.995826	13.488573	12.303377
24	44.501999	50.815575	13.798641	12.550356
25	47.727099	54.864510	14 093944	12.783354
26	51.113453	59.156381	14.375184	13.003164
27	54.669126	63.705763	14.943033	13.210531
28	58.402583	68.528112	14.898127	13.406162
29	62 322712	73.639799	15.141073	13.590721
30	66.438847	79.058184	15.372450	13.764829
31	70.760790	84.801677	15.592810	13.929084

CONSTRUCTION AND USE OF THESE TABLES.

The first Table may be constructed as shewn p. 308,309. The second may be constructed from the first, thus, Let 1 be divided by 4.538039 the last number of the first, and the quotient .220359 is the last number of the second. But the terms of the second Table are a decreasing geometrical progression, whose ratio is 1.05,1.06 ; so contrarywise the progression beginning at the last term and continued to the first will be an increasing progression, and therefore the last term being found as above, the rest may be found from it.

31——

31———220359
Add $\frac{1}{2}l_{0}$ 110179

30———231377
$\frac{1}{2}l_{0}$ 115688

242946, &c.

Table III. may be constructed from Table I. thus,
To 1. the amt. of annuity of 1l. at the end of 1 year.

Add 1.05 the amount of 1l. at compound interest.
To 2.05 the amount of an annuity of 1l. at 2 years end.
Add 1.1025 the second term of Table I.

3.1525 the third term of Table III.
Add 1 157625 the third of Table I.

4.310125 the fourth term of Table III. and so proceeding, add to each new term in Table III. the same term in Table I. For it is manifest that by this process we get the sums of this Progression, 1.1.05, 11025, 1.157625. But these sums are still the amounts of an annuity of 1l. It is manifest that Table IV. may be constructed in like manner from Table II.

The Use.

The use of these Tables is very easy, being only to multiply any given sum, by the tabular number, in the same row with the given number of years.

Example.

Suppose it be required to find the amount of 136l. 15s. 6d. in 20 years, at 6 ℘ cent. ℘ annum, compound interest?

£. 136 15 6=136 7525 which multiply by the tabular number under 6 ℘ cent and in the same row with 20, *viz.* 3.207135, and the product will be *l.* 438.5834. *i. e.* 438l. 11s. 8d. and so of any other.

Let the sundry Examples of this Chapter be done by the Tables.

CHAP. VI.

OF LOGARITHMS.

LOGARITHMS are Numbers so contrived and adapted to other Numbers that the Sums and Differences of the former correspond to, and shew the Products and Quotes of the latter, and also their Powers and Roots.

The Logarithms of these numbers in a decuple progression from 1, (to which progression the Logarithms now in use are applied) are called Characteristicks, because they denote how many places the corresponding natural number consists of, which is easily apprehended.

Num.	*Logarithms.*
1	0.0000000
10	1.0000000
100	2.0000000
1000	3.0000000
10000	4 0000000
100000	5.0000000

For the Logarithm of 1 is 0. for 1 is not distant from itself; of 10,1.0000000 wherefore the Logarithm of every number between 1 and 10 must be a decimal fraction: Likewise since the Logarithm of 10 is .1, and of 100, .2, the Logarithm of every number between 10 and 100 must be greater than 1, and less than 2, *i. e.* 1 and a decimal, and between 100 and 1000 the Log. will be 2 and some decimal; so on the contrary, if a Logarithm be a decimal fraction the natural number must be between 1 and 10, if it consists of 1 and a decimal, it is between 10 and 100, if of 2 and a decimal between 100 and 1000, &c. That is, if the characteristick of a Logarithm be 0, the natural number is a single figure; if the characteristick be 1, the natural number consists of 2 figures, if 2 of 3, if 3 of 4, &c. 2. The Logarithms of all numbers in a decuple proportion differ only in their characteristicks, as if the Logarithm of 6.748 be 0.8291751, then, the Logarithm of its decuples will be as under:

Numbers.	*Logarithms.*
6.748	0 8291751
67.48	1.8291751
67.48	2.8291751
67.48	3.8291751
67480	4.8291751

For 6.748×10 produces 67.48 wherefore the Log. of 67.48, *viz.* 0.8291751+1 the Log. of 10=1.8291751, the Log. of 67.48, and so of the rest.

A geometrical progression may in fractions be continued downward below unity, infinitely, in the same proportion as it ascends in whole numbers above it, *viz.*

$\frac{1}{1000}$, $\frac{1}{100}$, $\frac{1}{10}$, 1, 100, 1000, *&c.*

Or Decimally,

.001, .01, .1, 1, 10, 100, 1000, *&c.*

And since the distance of $\frac{1}{10}$ from unity is equal to the distance of 10; of $\frac{1}{100}=$ o that of 100 the Log. of $\frac{1}{10}$ will be equal to that of 10: of $\frac{1}{100}$ equal to that of 100, *&c.* But then since the Logarithm of unity is 0, the Logarithms of fractions are negative or descending below 0, for they go on the contrary way to whole numbers, and are there to be marked with the sign —, as,

$$\begin{array}{ccccccc} -3 & -2 & -1 & 0 & 1 & 2 & 3 \\ \frac{1}{1000}, & \frac{1}{100}, & \frac{1}{10}, & 1 & .10, & 100, & 1000 \end{array}$$

And as fractions in their multiplication and division have contrary effects to whole numbers, so have their Logarithms, *viz.* a fraction multiplying a whole number diminishes the whole number and the contrary. So a negative Logarithm must be subtracted from a positive Logarithm when addition is implied, and the contrary; but negative Logarithms are to be added or subtracted amongst themselves, as Addition or Subtraction is implied, as will be easily apprehended from an example or two of the foregoing progression, *viz.* $\frac{1}{10}\times 10=1-1+1$. *i. e.* 1 subtracted from $1=0$ the Log. of 1. $\frac{1}{100}\times 1000=10$ and $-2+3$, *i. e.* 2 subtracted from $3=1$. Again, if the fraction $\frac{1}{10}$ divide 10 the quotient will be 100. And so Subtraction being implied, add their Logarithms $+1=2$ the Logarithm of 100.

PROBLEMS, *shewing more particularly the use of Tables of Logarithms.*

To find the Logarithm of any number in general.

It is either found by inspection, being placed to the right-hand of the number, or if otherwise placed, suitable directions are prefixed or annexed to the tables for the use of them.

Problem II.

To find the Logarithm of an integral number exceeding the limits of the Table of Logarithms: for example exceeding 10,000.

Rule.

Take as many figures to the left hand of the given numbers as there are in the Table, (*viz.* 4 of them if the Table goes only to 10,000, or 5 if to 100,000) and in the place of the figures not taken, annex 0's: Again, to the number expressed by the figures taken, add 1, and annex the same number of 0's: Then take the difference of these two numbers; also the difference between the given number and the first of these, and make this proportion.

As the difference of the first two is to the difference of their Logarithms: So is the difference of the last: to the difference of their Logarithms, which added to the Logarithm of the number less than the given number, gives the Logarithm of the number proposed.

Application.

To find the Logarithm of 123459 from a table carried only to 10,000.

The two numbers less and greater than 123459 taken according to the rule, are 123400 and 123500 whose Logarithms are 5.0913152 and 5.0916670 for the Logarithm of 1234 is 3.0913152, to which add 2 the Logarithm of 100, (because 123400=1234 × 10) the sum 50913152 is the Log. of 123400; also the Logarithm of 1235 is 3.0916670, and so that of 123500 is 5.0946670, and the proportion is

From 123500 5.0916670 123459
Take 123400 5.0913152 123400

As 100 is .0003518 so is 59 to .00020756. &c. which added to 5.0913152 the Logarithm of 1234000 the sum is 50915276, &c. the Logarithm of 123459 nearly.

Problem III.

To find a number corresponding to any Log. which being the result of an operation with Logarithms, found in the table, is not itself found exactly in the table.

1. If the characteristick, and first 4 of 5 decimal figures are found in the table, that's near enough for common use; and the number against that Logarithm in the table, which is nearest the resulting Logarithm, may be taken as the number sought. But if greater exactness is desired, or the characteristick is beyond the limits of the table.

2. Take the two Logarithms, in the table, whose decimal figures are next less, and greater, and also their corresponding numbers, and make this proportion:

As the difference of the greater and lesser Logarithms is to the difference of their corresponding numbers.

So is the difference of the given and next lesser Logarithms to the diff. of their corresponding numbers.

Which difference added to the number corresponding to that lesser Logarithm, makes the number corresponding to the given Logarithm nearly.

Application.

Let the given Logarithm be 4669347; the next lesser and greater are .3010300 the Log. of 2, and .4771213 the Log. of 3: so the proportion is thus formed.

From .4771213 3 .4669347
Take .3010300 2 .3010300

.176 913 : 1 : : .1659047, : .94215, which added to 2, makes 2.94215. &c. number sought.

Again if it be required to find the number answering to the Logarithm 5.0915121 from a table not extending beyond 10,000, I seek for the highest Logarithms in the table, *viz.* those which have 3 for their characteristick, and the decimal figures next less and greater, than those of the given Logarithm, and find them to be 3.0913152 the Log. of 1234 and 3.0916670; from which we form the following analogy.

From 5.0916670 Log. of 123500 given L. 5.0915121
Take 5.0913152 Log. of 123400 5.0913152

.0003518 : 100 : : 0.0001969 : 56
which added to the lesser makes 123456.

Now it will be proper to shew the use of Logarithms in calculation, and then conclude.

1. *Of their Use in* MULTIPLICATION.

It is manifest that the sum of the Logarithms of the factors is the Log. of the product, wherefore

To multiply one number by another, add their Logarithms together, and in the table find the natural number corresponding to their sum: that number is their product.

Examples.

Multiply 144 Log.—2.1583625 } Add
By 12 Log.—1.0791812 }

1728 Log—3.2375537

2. Mult. 1385 by 185 4. Mult. .1385 by .0185
3. —— 138.5 by 18.5 5. —— 7589 by 6757

2. *In* DIVISION.

To divide one number by another, subtract the Logarithm of the divisor from the Log. of the dividend, and the remainder is the Log. of the quotient.

Examples.

$$\left.\begin{array}{l}\text{Divide } 1728\text{—Log. } 3.2375437\\ \text{By } \quad 12\text{—Log. } 1.0791812\end{array}\right\}\text{Subtract.}$$

Quotient 144—Log. 2.1583625

2. Divide 256225 by 185
3. ———.0256225 by 1.385
4. ——— 256.225 by 138.5
5. ——— 78956 by 278

3. *In the* Rule of Three *or* Proportion.

Add the Logarithms of the second and third numbers, and from their sum subtract the Logarithm of the first.

Example.

If 13℔. of tea cost 7l. 12s. what will $66\frac{1}{2}$℔. cost?

13℔.————Log. 1.1139433
l.
: 7.6————Log. 0.8808136
: : 66.5℔.————Log. 1.8228216
2.7636352
℔.
Answ. 138.8769——Log. 1.5896919

Or this may be done something easier yet, if instead of the Log. of the first term be taken its complement Arithmetical, or the difference of that Logarithm and the number 10.0000000, which is done by setting down the difference between each figure of the Logarithm, and the figure 9; for then, if the Arithmetical complement be added with the other two Logarithms, and if unity, which is the last, be taken away or erased from the sum, the remaining figures will be the Logarithm of the fourth term sought. As,

$$\left.\begin{array}{lr}13\text{℔. Co. Ar. —} & 8.8860567\\ :\text{£. }7.6\text{ — — —} & 0.8808136\\ ::66.5\text{℔. — — —} & 1.8228216\end{array}\right\}\text{Add}$$

1|1·5896919 as before.

4 *In* Involution *or raising Powers.*

Multiply the Logarithm of the root by the index of the power, i. e. by 2 for the square: 3 for the cube, &c.

What is the square and cube of 12?

Log. of 12—1.0791112 and 1.0791812
Mult. by 2 3
Log. of 144—2.1583624 3.2375436=Log. 1728

2. What is the square of 24? *Answ.* 576.
3. What is the cube of 9? —— 729.

5. *In* Evolution *or Extracting Roots.*

Divide the Logarithm of the power by its index, i. e. the Log. of a square by 2, of a cube by 3, &c. and the quotient is the Logarithm of the root.

Examples.

Find the square root of 144?

144 Log. 2)2 1583626

1.0791812 Log. of 12.

2. Find the cube root of 1728? *Ans.* 12; of 729, *Ans.* 9.
3. The square root of 160? *Answ.* 12.6491.
4. ———————— 225? —— .4743.
5. The cube root of 123456? —— 49.7982.

6. *In* Compound Interest.

To find the amount of any sum for any time at compound interest.

Multiply the Logarithm of the ratio (*i. e.* the amount of 1l. for 1 year) by the number of years, and to the product add the Logarithm of the principal; the sum will be the Logarithm of the amount.

Example.

What will 20l. amount to, forborne 7 years, at 6 ⅌ cent. ⅌ annum, compound interest?

Log. of 1.06, the ratio is — 0.0253059
Multiply by the time 7

0.1771418
Add Log. of 20 the princ.—1.3010299

The sum is — — — 1.4781712
Which is the Logarithm of 30 7, or 30l. 14s. the amount sought.

Rule.

From the Logarithm of the sum to be discounted, subtract the Logarithm of the rate multiplied by the time; the remainder is the Log. of the present worth.

Example.

Example.

What present money will pay a debt of 20l. due 7 years hence, at 5 ⅌ cent. ⅌ annum compound interest?

From Log. of 20l. — — — — 1.3010300
Subt. pro. of the Log. ratio + time — 0,1483251

The remainder is — — 1.1527049
Which is the Logarithm of 14.219 or 14l. 4s. 4¾d.

And thus may the other questions before delivered in *Compound Interest* be resolved.

A COLLECTION OF QUESTIONS

To exercise the Learner in the sundry Rules.

A Person dying left his widow 1780l. and 1250l. to each of his four children, 30 guineas a piece to 15 of his poor relations, and 150l. in charities; he had been 25½ years in trade, and at an average has cleared 126l. a year. What had he to begin with? *Answ.* 4228l. 17s. 6d.

2. The globe of the earth, under the line, is 360 degrees in circumference, each degree 69½ miles; and this body being turned on its own axis, in 23 hours, 56 min. At what rate an hour are the inhabitants of Bencoolen situate under the line, carried about from west to east by this rotation?
Answ. $1045\frac{145}{359}$ miles.

3. A fellow was saying, that when he told over his basket of chesnuts 2 by 2, 3 by 3, 4 by 4, 5 by 5, or 6 by 6, there still came an odd one; but when he told them 7 by 7 they came even: how many had he? *Answ* 721.

4. What number multiplied by 57 will produce just what 134 multiplied by 71 will do? *Answ.* $166\frac{52}{57}$.

5. There are two numbers, whose product is 1610, the greater is given 46; what is the sum of their squares, and what the cube of their difference? *Answ.* Sum of sq. 3341. The cube of their difference 1331.

6. A, B, and C trade in company: and at making up accounts, it appears that A and B together gained 13l. 10s. B and C together 12l. 10s. and A and C together gained 11l. 16s. 6d. what did they severally gain?
Answ. A 6l. 7s. 3d. B 7l. 2s. 9d. and C 5l. 9s. 3d.

7. Some others advance in trade, as follows, *viz.* W, X and Y raised 350l. 10s. W, X and Z 344l. 10s. X, Y and Z made up a stock of 400l. and W, Y and Z contribute 378l. 4s. In the conclusion, they parted with their joint property for 450 guineas. What did they gain or lose by their adventure? *Answ.* If the guineas are computed according to the English currency at 21s. they lost 18l. 11s. 4d. but at 22s. 9d. they gained 20l. 16s. 2d.

8. There is a mast or pole, $\frac{1}{9}$ of its length stands in the ground, 12 feet of it in the water, and $\frac{5}{6}$ of its length in the air, or above water: I demand the whole length?

Answ. 216 feet.

9. A and B traded together, and gained 100l. A put in 640l. B put in so much tha the must receive 60l. of the gain; I demand how much B put in? *Answ.* 960l.

10. What quantity of water must I add to a pipe of mountain wine value 33l. to reduce the first cost to 4s. 6d. ℘ gallon? *Answ.* $20\frac{2}{3}$ gallons.

11. What difference is there between the interest of 500l. at 5 ℘ cent. for 12 years, and the discount of the same sum, at the same rate and for the same time?

Answ. 112l. 10s. advantage to the interest.

12. If 12 apples are worth 21 pears, and 3 pears cost $\frac{1}{2}$d. what is the price of 4 score and 4 apples? *Answ.* 2s. $0\frac{1}{2}$d

13. Three merchants, A, B and C have gained 234l. which they are to divide so that when the profit of A is multiplied by 2, that of B by 3, and C's by 4; their products are to be equal; I demand how much each of them is to receive? *Answ.* A 108l. B 72l. C 54l.

14. There are 2 pieces of linen, the one is 9 yards shorter than the other, and cost 3l. $1\frac{1}{5}$s. the other piece at the same price cost 3l. 12s. I demand how many yards were in both pieces, and the price of 1 yard?

Answ. 111 yards, at $1\frac{1}{5}$s. ℘ yard.

15. A piece of sattin cost a certain sum, and being sold for 3l. 10s. there is lost $\frac{2}{5}$ in a shilling. I demand the first cost? *Answ.* 5l. 16s. 8d.

16. With 13 gallons of Canary at 6s. 8d. the gallon, I mingled 20 gallons of white wine, at 5s. a gallon; and to these add 10 gallons of cyder, at 3s. ℘ gallon: at what rate must I sell a quart of this mixture so as to clear 10 ℘ cent.? *Answ.* $16\frac{27}{43}$d.

17. It is a rule in some parishes, to assess the inhabitants in proportion to $\frac{8}{15}$ of their rents: What is the year'y

rent of that house, which pays 8l. 10s. to the king under this limitation, at 5s. ℔ l.? *Answ.* 42l. 10s.

18. If by selling hops at 3l. 10s. ℔ Cwt. the planter clears 30 ℔ cent. what was his gain per cent. when the same goods sold for 4l. 5s.? *Answ.* 57l. 17s. 1$\frac{5}{7}$d.

19. If by remitting to Holland, at 31s. 9d. Flemish ℔ l. sterling, 5 ℔ cent. is gained; how goes the exchange when by remittance I gain 10 ℔ cent.?

Answ. 33s. 3$\frac{1}{7}$d.

20. If when port wine is 17 guineas the hogshead, a company of 45 people will spend 20l. therein in a certain time; what is wine a pipe when 13 persons more will spend 63l, in twice the time, drinking with equal moderation? *Answ.* 43l. 12s. 6d. nearly, the guineas being taken as English, which is 47l. 5s. 2$\frac{1}{2}$d. Irish.

21. A is dispatched on a commission from London to Edinburgh, distant by computation, say 350 miles, and his rout is settled at 22 miles a day: 4 days after B is sent after him with fresh orders, and is to travel 32 miles a day; whereabout on the road will B overtake A?

Answ. 68$\frac{4}{10}$ miles on this side Edinburgh.

22. If 6℔. of pepper be worth 13℔. of ginger, and 19℔. of this be worth 4$\frac{3}{4}$℔. of cloves, and 10℔. of cloves be equivalent to 63℔. of sugar, at 5d. ℔ ℔. what is the value of 1Cwt. of pepper? *Answ.* 7l. 19s. 3d.

23. If 30 men can perform a piece of work in 11 days; how many will accomplish another, 4 times as big in one-fifth of the time? *Answ.* 600.

24. A may-pole 50 feet, 11 inches long, at a certain time of the day, will cast a shadow 98 feet, 6 inches long, I would hereby find the breadth of a river, that running 20 feet 6 inches from the foot of a steeple, 300 feet 8 inches high, the extremity of the shadow of the steeple reaches 30 feet 9 inches beyond the stream? *Answ.* 530 feet. 5 inches nearly.

25. In 81034 rundlets of brandy, each 18 gallons; how many gross of bottles, each $\frac{5}{9}$ of a quart?

Answ. 45581 gross 7 dozen and 6.

26. Bought hose in London for 4s. 3d. ℔ pair, and sold them in Dublin for 6s. a pair: now taking the charges at an average to be 8d. a pair, and the exchange is known to be 8$\frac{1}{3}$ ℔ cent. disadvantage; what do I gain ℔ cent. by this article of trade? *Answ.* 13$\frac{211}{253}$.

27. If the scavengers' rate at $1\frac{1}{2}$*d.* ⅌ l. comes to 6s. $7\frac{1}{2}$d. where they ordinarily assess $\frac{4}{5}$ of the rent: what will the king's tax for that house be, at 4s. in the pound, rated at the full rent? *Answ.* 13l. 5s.

28. A tradesman excreased his estate annually a third, abating 100l. which he usually spent in his family, and at the end of $3\frac{1}{4}$ years he found that his neat estate amounted to 3154l. 11s. 8d. What had he at outsetting?

Answ. 1411l. 12s. $9\frac{3}{4}$d.

29. A can do a piece of work in 10 days, B alone in 13; set them both about it together, in what time will it be finished? *Answ.* in $5\frac{15}{23}$ days.

30. A cistern is supplied with water by one pipe of such bigness, that if the cock A at the end of the pipe be set open, the cistern will be filled in $\frac{1}{2}$ hour; but at the bottom of the cistern are two other cocks B and C, whose capacities are such, that by the cock B set open alone (all the rest being stopt) the cistern supposed to be full will be emptied in $1\frac{3}{7}$ hour; also by the cock C alone it will be emptied in $2\frac{1}{3}$ hours: Now because more water will be infused by the cock A, than can be expelled by the cocks B and C in one and the same time; the question is, in what time the cistern will be filled, if all the said three cocks be set open at once? *Answ.* $1\frac{9}{61}$ hour.

31. A governor of a certain garrison being desirous to know how much money the port or passage of the garrison did amount to, in certain months, made choice of a loyal servant, giving him orders to receive of every coachman passing with a coach 4d. of every horseman 2d. and of every footman $\frac{1}{2}$d. Now at the year's end the servant making his account to the governor gives him 94l. 15s. 10d. and lets him know that as often as 5 passed with coaches 9 passed on horseback, and as often as 6 passed on horseback 10 passed on foot. The question is, how many coaches, horsemen and footmen passed? *Answ.* 2500 coaches, 4500 horsemen and 7500 footmen.

32. Twenty knights, 30 merchants, 24 lawyers and 24 citizens spent at a dinner 64 pounds, which sum was divided among them in such a manner that 4 knights paid as much as 5 merchants, 10 merchants as much as 16 lawyers, and 8 lawyers as much as 12 citizens; the question is, to know the sum of money paid by all the knights; also by the merchants, lawyers and citizens? *Answ.* the 20 knights paid 20l. the 30 merchants 24l. the 24 lawyers 12l. and the 24 citizens 8l.

33. There is an island which is 36 miles in compass: Now, if at the same time, and from the same place, two footmen A and B, set forward to travel round about said island, and follow one another in such a manner, that A travels every day 9 miles, and B 7 miles; the question is, to find in what space of time they will meet again; and also how many miles and how many times round the island each footman will then have travelled?

Answ. They will meet at the end of 18 days from their first parting; and then A will have travelled 162 miles (or $4\frac{1}{2}$ times the compass of the island) and B will have travelled 126 miles (or $3\frac{1}{2}$ times the compass of the island.)

34. A man dies and leaves a legacy of 900l. to be disposed of among his relations, viz. A, B, C and D, which legacy is to be disposed of in this order: B is to have twice as much as A, and C thrice as much as B, and D is to have as much and $\frac{1}{2}$ as C: what must each person have?

Answ. A's share is 50l. B 100l. C 300l. and D 450l.

35. Five merchants. viz. A, C, B, D and E have gained 2025l. which they divide in such sort that $\frac{1}{2}$ of the share of A is equal severally to $\frac{1}{4}$ of the share of B. $\frac{1}{5}$ of C, $\frac{1}{6}$ of D, $\frac{1}{8}$ of E: the question is, what was the share of every merchant? *Answ.* A 162l. B 324l. C 405l. D 486l. E 648l.

36. Two merchants A and B are in company, the sum of their stocks is 300l. the money of A continuing 9 months, the money of B 11 months; they gain 200l. which they divide equally; the question is, how much each man put in? *Answ.* A 165l. B 135l.

37. If 3481 soldiers are to be placed in a square battle, how many are to be set in rank, or in file? *Answ.* 59.

38. If 100l. being put forth for interest at a certain rate, will at the end of 2 years be augmented to $112\frac{36}{100}$l. (compound interest, or interest upon interest computed) what principal and interest will be due at the first year's end? *Answ.* 106l.

39. If 100l. being put forth for interest at a certain rate, will at the end of 3 years be augmented to *l.* 115.7625 (compound interest being computed) what principal and interest will be due at the first year's end? *Answ.* 105l.

40. The continual multiplication of the nine digits will give the number of changes that may be rung on 9 bells (as well as of any other combinations) how many are there? *Answ.* 362880.

41. There are 2 numbers; 75 is the less, to which the greater is in proportion as 8 to 5, what is the sum; and the product of their sum and difference; the difference and product of their squares; and the sum of the squares of their two quotes, the greater divided by the less, and the less again by the greater? *Answ.* Sum 195, sum and diff. 240, diff. sq. 8775, sum sq. of the two quotes $2\frac{1521}{1600}$.

42. In a series of proportional numbers the first is 5, the third is 8, the product of the second and third is 78.4, what is the difference of the second and fourth? *Answ.* 5.88.

43. If by sending pewter to Turkey, and parting with it at 25$\frac{2}{3}$*d.* ⅌ pound, the merchant clears cent ⅌ cent. what does he clear in Holland where he disposes of the cwt. for 8l.? *Answ.* 2l 0s. 2$\frac{2}{3}$d.

44. An accomptant told a gentleman who had constantly eight persons at his table, that he would gladly make a ninth, and was willing to give 200 guineas for his board, so long as he could place the said company at dinner differently from any one day before; this being accepted, what did his entertainment cost him a year?

Answ. 50d. and about $\frac{2}{3}$

45. It is proposed by an elderly person in trade, desirous of a little respite, to admit a sober and industrious young fellow to a share in the business, and to encourage him, offers that if his circumstances will allow him to advance 100l. his pay shall be 40l. a year; if he shall be able to put 200 into the stock he shall have 55l. and if 300 he shall receive 70l. annually: In this proposal what was allowed for his attendance simply? *Answ.* 25l. a year.

46. A merry young fellow in a small time got the better of $\frac{1}{5}$ of his fortune, by advice of his friends he then gave 2200l. for an exempt's place in the guards, his profusion continued till he had no more than 880 guineas left, which he found by computation, was just $\frac{3}{20}$ part of his money after the commission was bought; pray what was his fortune at first? *Answ.* 10450l.

47. A has kerseys at 4l. 5s, a piece ready money, in barter they are charged by him at 5l. 6s. each, and $\frac{1}{\text{[illegible]}}$ of that required down. B has flax at 3d. ⅌ pound, how ought he to rate it in truck, not to be hurt by the extortion of A? *Answ.* 5d. nearly.

48. Put out 384l. to interest, and in 8$\frac{1}{4}$ years there were 542l. 8s. found to be due, what rate of interest could then be implied? *Answ.* 5 ⅌ cent. ⅌ annum.

49. M of Amsterdam orders N of London to remit O of Paris, at 34 pence sterling ℘ crown, and to draw on P. of Antwerp for the value, at $33\frac{1}{2}$s. Flemish ℘ pound sterling, but as soon as N received the commission the exchange was on Paris at $54\frac{1}{2}$ ℘ crown, pray at what rate of exchange ought N to draw on P to execute his orders and be no loser? *Answ* 33s. $2\frac{34}{109}$d.

50. Suppose the sea allowance for the common men to be 5℔. of beef and 3℔. of biscuit a day, for a mess of four people, and that the price of the first barrelled to the king be $2\frac{1}{4}$d. a ℔. and of the second $1\frac{1}{2}$ such as was a ship's company that their flesh cost the government 12l. 12s. ℘ day; pray what did it pay for the bread ℘ week? *Answ.* 35l. 5s. $7\frac{1}{15}$d.

51. Three persons enter joint trade together, to which A contributed 210l. B 312l. they clear 140l. whereof 37l. 10s. belongs of right to C, that person's stock and the several gains of the other two are required?

Answ. C's stock 190l. 19s. 6d. A gained 41l. 4s. $8\frac{1}{2}$d.

52. A bond was made on the 7th of August. 1713, at 6 ℘ cent. ℘ annum, for the sum of 1114l. 10s. on the 11th of May, 1718: 140l. was paid off, and a fresh bond entered into for the remainder at $5\frac{1}{4}$ ℘ cent. ℘ annum, at the time the interest of this last was 21l. 16s. 8d. there was paid off 87l. 11s. 9d. the old bond being then taken up, a new one was then given for the residue, which being paid off on the 11th of September, 1724, the bond owner took no more than 1409l. 16s. 8d. in full payment; at what rate did he take interest ℘ cent ℘ annum upon the last renewal of the bond? *Answ.* 2l. 9s. $6\frac{1}{2}$d.

53. A, B and C will trench a field in 12 days, B, C and D in 14, C, D and A will do it in 15, and D, A, B in 18, in what time will it be done by all of them together; and by each of them singly?

Answ. Together in 10.83 days: by A in 47.848, B in 38.969, C in 27.194, D in 111.176 days.

54. A, B and C company, A put in share of the stock for 5 months, and laid claim to $\frac{1}{6}$ of the profit, B put in his for 8 months, C advanced 400l. for 7 months, and required on the balance $\frac{2}{3}$ of the gain, the stock of the other two adventurers is sought? *Answ.* A 168l. B 70l.

55. If $120\frac{1}{3}$l. is to be distributed among 3 persons, A, B, C, in such sort, that as often as A takes 5, B shall take

4; and as often as B takes 3, C shall take 2; what will be the share of each of them?

Answ. A $51\frac{4}{7}$l. B $41\frac{9}{35}$l. C $27\frac{53}{105}$l.

56. Divide 1000 crowns: give A 129 more than B, and B 718 fewer than C?

Answ. A 360, B 231 and C 409.

57. Part 250l. give A 37 more than B, and let C have 28 fewer? *Answ.* A $117\frac{1}{3}$l. B $80\frac{1}{3}$l. and C $52\frac{1}{3}$l.

58. A father divided his fortune among his sons; he gave A 7 as often as B 4; to C he gave as often 2 as B 5: the dividend of C came to $2166\frac{3}{8}$l. what was the value of the whole legacy? *Answ.* 17060l. 4s. $0\frac{3}{4}$d.

59. A stationer sold quills at 11s. ℔ thousand, by which he cleared $\frac{3}{8}$ of the money; but growing scarce raised them to 13s. 6d. ℔ thousand; what might he clear ℔ cent. by the latter price? *Answ.* 96l. 7s. $3\frac{3}{11}$d.

60. A person was possessed of $\frac{3}{5}$ share of a copper mine, and sold $\frac{3}{4}$ of his interest therein for 1710l. what was the value of the whole property at the same rate?

Answ. 3800l.

61. What sum of money at $3\frac{1}{2}$ ℔ cent. ℔ annum, will clear 38l. 10s. in a year and quarter's time?

Answ. 880l.

62. X, Y and Z working together, can complete a staircase in 12 days; Z alone can do it in 24 days; X in 34: In what time then can Y get it done himself?

Answ. $81\frac{3}{5}$ days.

63. Two merchants enter into partnership, and each of them put in 12 pieces of cloth; but those of A cost 48l. more than those of B; the cloth being sold they find they have gained $273\frac{3}{5}$l. of which B has for his share $122\frac{2}{5}$l. The question is to know at how much a piece each of their cloths was valued? *Answ.* A's at 21l. and B's at 17l:

64. If when a *hhd* of wine is sold for 10l. there is lost 6 ℔ cent. how much is the gain or loss ℔ cent. when 5 hhds. are sold for 45l. 10s.? *Answ.* $14\frac{23}{56}$ ℔ cent. loss.

65. A person dying left his wife with child, and by his will ordered that if she went with a son, $\frac{2}{3}$ of the estate should belong to him, and the remainder to his mother; and if she had a daughter, he appointed the mother $\frac{2}{3}$ and the daughter $\frac{1}{3}$: but it happened she was delivered of both son and a daughter; by which she lost in equity 2000l. more than if it had been only a girl. What would have been her dowry if she had only a son? *Answ.* 1750l.

66. In distress at sea they threw out 17 hhds. of sugar, worth 34l. ⅌ hhd. the worth of which came up to but the $\frac{4}{7}$ of the indigo they threw overboard; besides which they threw out 18 iron guns worth 18l. 10s. a piece, the value of all these amounted to $\frac{3}{7}$ of $\frac{9}{13}$ of that of the ship and lading: what value came into port?

Answ. 4337l. 15s. $6\frac{2}{3}$d.

67. A gay young fellow, had 18200l. left him by an old uncle, to whose memory he expended 3 ⅌ cent. of his whole fortune, in a sumptuous funeral and monument; 9 ⅌ cent of the remainder he made a present of to his cousins, forgotten by the old man; with $\frac{2}{7}$ of the remainder he bought a fine seat, and with $\frac{1}{8}$ of the residue a stud of horses; he squandered 550l. in gaming; and after having lived at the rate of 2000l. a year for 19 months, and ruined his health, he died. What was there left for his sister who was heir at law? *Answ.* 6324l. 0s. 11d.

68. Three persons purchase a West India sloop, toward the payment whereof A advanced $\frac{3}{8}$, B $\frac{3}{7}$, and C 140l. how much paid A and B, and what part of the vessel had C?

Answ. A paid $267\frac{3}{11}$l. B $305\frac{5}{11}$l. and C's part $\frac{11}{56}$.

69. A and B clear by an adventure to sea 50 guineas; their stocks were as 13 to 10, and they gained 45 ⅌ cent. I demand their respective stocks? *Answ.* A's 71l. 8s. $8\frac{64}{69}$d. B's 54l. 19s. $0\frac{28}{69}$d.

70. A and B join stocks, and purchase brandies, A's stock was 19l. 19s. 8d. more than that of B. Now by selling out their commodity at 55s. ⅌ anker, A cleared 74l. 11s. and B 52l. 10s. The quantity of brandy dealt for is required, and the gain upon the anker?

Answ. 88 ankers, and 1l. 8s. $10\frac{1}{2}$d. ⅌ anker gained.

71. In an article of trade, A gains 14s. 6d. and his adventure was 35s. more than B's, whose share of the profit is but 8s. 6d. what are their respective stocks?

Answ. A's 4l. 4s. 7d B's 2l. 9s. 7d.

72. Suppose I would exchange 527l. 17s. 6d. for dollars at 4s. 6d. ⅌ piece, ducats at 5s. 8d. ⅌ piece, and crowns at 6s. 1d. ⅌; and would have 2 dollars for 1 ducat, and 3 dollars for 2 crowns: how many of each sort must I have? *Answ.* 927 dol. $463\frac{1}{2}$ duc. 618 cro.

73. A lets B have a hhd. of sugar of 18Cwt. worth 31s. for 42s. the Cwt. $\frac{1}{3}$ of which he is to pay in cash; B hath paper cost 14s. ⅌ ream, which he gives A for the rest of his sugar, at 15s. 6d. Which gained most by the bargain? *Answ.* A by 7l. 9s. 3d.

74. A and B in partnership equally divide the gain: A's money, which was 84l. 12s. 6d. lay for 19 months, and B's for no more than 7: the adventure of the latter is sought? *Answ.* 229l. 13s. 11$\frac{1}{7}$d.

75. In 117 times 406 pieces, worth 3s. 8$\frac{2}{3}$d. a piece, how many reas at 20 for 3d.? *Answ.* 14145040.

76. Lent 100 guineas, at 4 ℘ cent. which by the 18th November 1756, was raised to as many moydores, bating half a crown. Pray what day did the bond bear date?

Answ. October 7, 1749, or rather allowing for the alteration of style, September 26, 1749.

77. A for a nine month's adventure received 20l. B for one of 7 months received 23 guineas and 1s. 9d over; and C for lying out of his contribution 5 months, had a title to 32l. the total of their adventures, multiplied into their respective times is 640: what were their particular stocks? *Answ.* A's 18l. 3s. 6d. B's 30l. 13s. 6d. and C's 52l. 6s. 10$\frac{3}{4}$d.

78. A had 15 pipes of Malaga wine, which he parted with to B at 4$\frac{1}{3}$ ℘ cent. profit, who sold them to C for 38l. 11s. 6d advantage; C made them over to D for 500l. 16s. 8d. and cleared thereby 6$\frac{1}{2}$ ℘ cent. what did this wine cost A ℘ gallon? *Answ.* 4s. 4$\frac{1}{2}$d.

79. I have imported 80 jars of Lucca oil, each containing 1190 solid inches? what came the freight to at 4s. 6d. ℘ Cwt. tare 1 in 10; counting 7$\frac{1}{2}$ lb. of oil to the wine gallon of 231 cubic inches? *Answ.* 6l. 15s. 5d.

80. A, B and C company, and put in together 3860l. A's money was in 3 months, B's money was in 5 months, and C's was in 7 months; they gained 234l. which was so divided that $\frac{1}{2}$ of A's gain was equal to $\frac{1}{3}$ of B's, and $\frac{1}{3}$ of B's to $\frac{1}{4}$ of C's: what did each gain and put in?

Answ.	A gained 52l.	A put in 1400l.
	B —— 78	B —— 1260
	C —— 104	C —— 1200

81. There are 7 chests of drawers, in each of which there are 18 drawers, and in each of these are 6 divisions; in each of which is 16l. 6s. 8d. how much money is there in the whole? *Answ.* 12348l.

82. If 3 dozen pair of gloves be equal in value to 2 pieces of ribbon: 3 pieces of ribbon to 7 dozen of points; 6 dozen of points to 2 yards of Flanders lace; and 3 yards of Flanders lace to 81 shillings; how many dozen of gloves may be bought for 28 shillings? *Answ.* 2 dozen.

83. A with intention to clear 30 guineas, on a bargain with B, rates hops at 16d. the pound, that stood him in 10d. B apprized of that, set down malt which cost him 10s. 10d. ⅌ barrel at an adequate price: how much malt did they contract for? *Answ.* 420 bushels.

84. A and B, venturing equal sums of money, clear by joint trade 154l. By agreement A was to have 8 ⅌ cent. because he spent time in the execution of the project, and B was only to have 5: the question is, what was allotted A for his trouble? *Answ.* 35 l. 10s. 9$\frac{1}{4}$d.

85. A, in order to put off to B 720 ells of damaged Holland, worth 5s. an ell, at 6s. 8d. proposes, in case he has half the value in money, to give B a discount of 10 ⅌ cent. The rest A is to take out in saffron, which B apprized of the whole management, rates in justice 30s. ⅌ ℔. Pray what was it really worth in ready money; and what quantity of saffron was he to deliver on the change? *Answ.* 72℔. worth 20s. ⅌ ℔.

86 Laid out in a lot of muslin 480l. 11s. upon examination of which, 2 parts in 7 proved damaged; so that I could make but 5s. 6d. a yard of the same, and by so doing I lost 49l. 6s. by it. At what rate ⅌ ell am I to part with the damaged muslin to make up the said loss?

Answ. 12s. 3$\frac{33}{160}$d.

87. A young hare starts 5 roods before a greyhound, and is not perceived by him, till she has been up 34 seconds; she scuds away at the rate of 12 miles an hour, and the dog in view makes after her at the rate of 20: How long will the course hold, and what ground will he run, beginning with the outsetting of the dog?

Answ. 58$\frac{1}{32}$ seconds, 1702$\frac{1}{4}$ feet run.

88. A leaves Exeter at 10 o'clock in the morning for London, and goes at the rate of 2 miles an hour without intermission. B sets out of London for Exeter, at 6 the same evening, and rides 3 miles an hour constantly: the question is, whereabouts on the road will they meet if the distance of the two cities be 130 miles?

Answ. 61$\frac{3}{5}$ miles from Exeter.

89. A reservoir of water hath 2 cocks to supply it: by the first it may be filled in 44 minutes; by the second in just an hour; and it hath a discharging cock, by which it may, when full, be emptied in half an hour: now suppose these three cocks, by accident, should all of

them be left open, and the water should chance to come in: what time would this cistern be in filling?

Answ. $2\frac{3}{4}$ hours.

90. A set out from London for Lincoln, at the very same time that B set out from Lincoln to London, distant 100 miles: at 8 hours end they meet on the road, and it then appeared that A had ridden $2\frac{1}{2}$ miles an hour more than B. At what rate an hour did each of them travel?

Answ. A $7\frac{1}{2}$ miles, B 5.

91. Bought 7 tons of wine at 17l. ℘ hogshead, which I sell again at 1s. ℘ pint, what is the whole gain and how much ℘ cent.? *Answ.* the whole gain 229l. 12s. and 48l. 4s. $8\frac{56}{119}$ ℘ cent.

92. There are 100 stones which lie 1 yard one from the other, and there is one employed to gather up the stones 1 by 1, and bring them to a basket which is standing a yard from the first stone: the question is, how many miles he will go, before the last stone is brought into the basket? *Answ.* 4 miles, 4 furl. 2 perch. 6 yds. Irish measure.

93. A minor of 12 years of age was left an estate of 150l. ℘ annum: his guardian was allowed by his father's will 40l. ℘ annum for his board, education, and other contingent charges, and was to put out the surplus to interest for his benefit, at 5 ℘ cent. simple interest: Now supposing no loss of principal or interest, what sum had his guardian to pay him when he was of age.

Answ. 1188l.

94. A person said he had 20 children, and that it happened there was a year and a half between each of their ages, his eldest was born when he was 24 years old, and the age of the youngest is now 21: what was the father's age? *Answ.* $73\frac{1}{2}$ years.

95. One at a country fair had a mind to a string of 20 fine horses, but not caring to take them at 20 guineas the head, the jockey consented that he should, if he thought good, pay but a single farthing for the first, doubling it only to the 19th, and he would give the 20th into the bargain: this being accepted, what what were they sold for a piece? *Answ.* 27l. 6s. $1\frac{47}{80}$d. each.

96. What ought a man to give down in ready money for the reversion of 1000l. a year, to continue 20 years, on a lease, which cannot commence till 5 years are at an

end, allowing the purchaser compound interest at 6 ⅌ cent.? *Answ.* 8570l. 19s. 10d.

97. A minor of 14 had an annuity left him of 70l. a year, the proceeds of which, by will, was to be put out both principal and interest yearly, as it fell due at 5 ⅌ cent. until he should arrive at 21 years of age. The utmost improvement being thus made upon this part of his fortune: what had he then to receive?

Answ. 569l. 18s. 9½d.

98. Value the lease of a house in tolerable repair, the rent 54l. 17s. a year, the ground-rent 7 guineas; 3 years of it only to come; the rent payable every 6 months, discount ⅌ compound interest on this kind of purchase, at 10 ⅌ cent.? *Answ.* 120l. 10s. 11½d. the guineas being taken as English currency; but at 1l. 2s. 9d. ⅌ the *Answ.* is 118l. 19s. 10d.

99. A lease for 7 years is agreed for, at 250l. fine on the old rent 44l. a year; but the purchaser being desirous to reduce the rent to 20l. a year and pay a proper fine, computing as before, after the rate of 10l. a year: to what must the fine be advanced?

Answ. 336l. 16s. 10d.

100. Suppose I would add 5 years to a running lease of 15 years yet to come, the improved rent being 186l. 7s. 6d. ⅌ annum: what ought I to pay down for the favour. Discount being allowed at 4 ⅌ cent. ⅌ annum compound interest? *Answ.* 460l. 14s. 1¼d.

101. A person dying left a piece of land, let on a 20 year's lease for 31l. 10s. tax free; the profits of this he bequeathed to the poor of the parish where he was born, for the first 4 years after his decease; the proceeds of the next 6 years he left to the poor of the parish where he lived; and the residue or last 10 years he gave to his niece; now this young woman having money, and being willing to come into the immediate possession of her uncle's land, comes to a compromise with the parishes, on a discount of 10 ⅌ cent. What did it cost her?

Answ. 193l. 11s.

AN

APPENDIX OF ALGEBRA.

ALGEBRA (or the great Art) is a Method of managing arithmetical and geometrical Computations by letters, by means whereof any Question may be clearly solved, and curious Theorems deduced for solving all Questions wherein Numbers or lines are concerned, which would be in vain to attempt either by Arithmetic or Geometry. I have here subjoined the principles of Algebra, with sundry examples to exercise the young Algebraist.—Having made every thing short and plain, that the Rules may not be burthensome to Youth.

The Characters or Signs which are used in the following Appendix are in the Preceding Book, except the following; x^2 signify that x is squared x^3 = the Cube of x, and x any Power of x, &c.

$\sqrt{}$ = any Square Root. $\sqrt[3]{}$ = the Cube Root, and $\sqrt[x]{}$ = any Root at Pleasure.

Quantities that are not known are represented by x, y, z, and v, those that are known by a, b, c, d, &c.

ADDITION may be comprehended in one Case, provided the following Directions be well understood.

1. When Quantities are of the same Kind, whether they have co-efficients or not, add them together, and their Sum will be the Sum required; but if unlike, subtract the co-efficients (if any) and set down the Difference with the Sign of the greater, + or — as the Question may require.

Example 1.

$$\begin{array}{lr} & x+y+z \\ & x+y+z \\ & x+y+z \\ \hline \text{Sum} & 3x+3y+3z \\ \hline \end{array}$$

Example 2.

$$\begin{array}{lr} & 3x-4y+2z \\ & 4x-y+7z \\ & 6x-9y+6z \\ \hline \text{Sum} & 13x-\ 14y\ 15z \\ \hline \end{array}$$

N. B. Where no Sign is, the Sign + is understood.

Example 3.

$$\begin{array}{lr} & x^2-4xy+5y^2 \\ & 6x^2+9xy+7y^2 \\ & -2x^2-3xy+8y^2 \\ \hline \text{Sum} & 5x^2+2xy+6y^2 \\ \hline \end{array}$$

$$\begin{array}{r} x^2+2xy+y^2 \\ x^2-2xy+y^2 \\ \hline 2x^2 \text{———} 2y^2 \\ \hline \end{array}$$

Subtraction may be performed by changing all the terms that are to be ſubtracted,— in +, or + into —, and then adding them together, and the Sum will be the Difference.

$$\begin{array}{lrr}
\text{Exam. From} & 6x^2-7y+4z & 4x^3-3y^2+4z \\
\text{Take} & 3x^2-9y+6z & 6x^3-9y^2-2z \\
\hline
& 3x^2+2y-2z & -2x^3+6y^2+6z
\end{array}$$

$$\begin{array}{lrr}
\text{Exam. 2.} & ax & ax-by \\
& bx & x-z \\
\hline
& ax-bx & ax-by-x+z
\end{array}$$

MULTIPLICATION is performed by one general Rule; obſerving that like Signs produce +, and unlike — in the Product.

N. B. If the quantities have co-efficients multiply them, if not, join the Letters like the Letters of a Word, obſerving to place the ſign + or — (as above) before them.

$$\begin{array}{lrrr}
\text{Exam. 1 Multiply} & x & -x & x^6 \\
& \text{by } x & y & y^8 \\
\hline
\text{Product} & x^2 & -xy & x^6y^8
\end{array}$$

$$\begin{array}{l}
x+y+z \\
x+y+z \\
\hline
x^2+xy+xz \\
\qquad xy+y^2+yz \\
\qquad\qquad xz+yz+z^2 \\
\hline
x^2+2xy+y^2+2xz+2yz+z^2
\end{array}$$

$$\begin{array}{l}
x^3+y^3+z^3 \\
x^3+y^3+z^3 \\
\hline
x^6+x^3y^3+x^3z^3 \\
\qquad xy^3+y^6+y^3z^3 \\
\qquad\qquad x^3z^3+y^3z^3+z^6 \\
\hline
x^6+2x^3y^3+y^6+2x^3z^3+2y^3z^3+z^6
\end{array}$$

N. B. The addition of Exponents is the ſame as Multiplication of Quantities.

$$\begin{array}{lrr}
\text{Multiply} & x^6+y^6 & x^n+v^n \\
\text{by} & x^6+y^6 & x^n-v^n \\
\hline
& x^{12}+x^6y^6 & x^{2n}+x^nv^n \\
& -x^6y^6-y^{12} & -x^nv^n-v^{2n} \\
\hline
& x^{12}-y^{12} & x^{2n}-v^{2n}
\end{array}$$

This laſt ſhews that the Rectangle or Product of the Sum and Difference of any two Quantities is equal to the Difference of their Squares, which is very uſeful.

Division—may be performed by one general Rule, being nothing but the Proof of Multiplication; and therefore if the Quantities in the Diviſor have like Signs to thoſe in the Dividend, the Quotient will be Affirmative; but if unlike, it will be Negative.

Exam. 1. $x)xy+x(y+1$

Exam. 2. $xy)xy^2(y$ Exam. 3. $x^2)x^5(x^3$

Exam. 4. $x+v)x^2+2xv+v2(x+v^3$

$$x^2+xv$$
$$xv+v^2$$
$$xv+v^2$$

$$x-v)x^4-v4(x^3+x^2v+xv^2+v^3$$
$$x^4-x^3v$$
$$.+xv^3-v^4$$
$$x^3v-x^2v^2$$
$$.+x^2v^2-v^4$$
$$x^2v^2-xv^3$$
$$.+xv^3-v^4$$
$$xv-v^4$$
$$.\quad .$$

$$x)-xv-xy(-v-y$$

$$x-y)x^2-2xy+y^2(x-y$$
$$x^2-xy$$
$$-xy+y^2$$
$$-xy+y^2$$

$$)x^5-v^5(x^4+x^3v+x^2v+xv^3+v^4$$
$$x^5-x^4v$$
$$+x^4v-v^5$$
$$x^4v-x^3v^2$$
$$.+x^3v^2-v^5$$
$$x^3v^2-x^2v^3$$
$$.\ +x^2y^3-v^5$$
$$x^2y3-xv^4$$
$$.\ +v^4-y^5$$
$$x\quad -v^5$$

Involution of Quantities is nothing but multiplying them continually together as the Power they are to be raiſed to— Thus $x\times x=x^2$ $\times x=x^3$ &c.

Sir Iſaac Newton has given the following rule for raiſing any Binominal to any Power, which is this.

Rule.

If the Index of the firſt Letter of any Term be multiplied into its own Co-efficient, and the Product be divided by the Number of Terms to that Place; the Quotient will be the Co-efficient of the next ſuceeeding Term forward.

Required to raiſe $x+v$ to the 6th Power regularly up.

$\surd\overline{x+v}|^{2}=x^{2}+2xv+v^{2}=$ the Square.

$\surd\overline{x+v}|^{3}x^{3}+3x^{2}v+3xv^{2}+v^{3}=$ the Cube.

$\surd\overline{x+v}|^{4}=x^{4}+4x^{3}v+6x^{2}v^{2}+4xv^{3}+v^{4}=$ 4th power.

$\surd\overline{x+v}|^{5}=x^{5}+5x^{4}v+10x^{3}v^{2}+10x^{2}v^{3}+5xv^{4}+v^{5}=$ 5th Power.

$\surd\overline{x+v}|^{6}=x^{6}+6x^{5}v+15x^{4}v^{2}+20x^{3}v^{3}+10x^{2}v+4xv^{5}+v^{6}=$ 6th Power. And ſo for as many Powers as you pleaſe.

$\surd\overline{x-v}|^{2}=x^{2}-2xv+v^{2}$

$\surd\overline{x-v}|^{3}=x^{3}-3x^{2}v+5xv^{2}-v^{3}$

$\surd\overline{x-v}|^{4}=x^{4}-4x^{3}v+6x^{2}v^{2}-4xv^{3}+v^{4}$

$\surd\overline{x-v}|^{5}=x^{5}-5x^{4}v+10x^{3}v^{2}-10x^{2}v^{3}+5xv^{4}-v^{5}$

$\surd\overline{x-v}|^{6}=x^{6}-6x^{5}v+15x^{4}v^{2}-20x^{3}v^{3}+15x^{2}v^{4}-6xv^{5}+v^{6}$.

Note, All even Terms end with + and thoſe that are odd with —

Evolution, or the Extraction of Roots being the reverſe of Involution, or raiſing of Powers, is performed by converſe Operations (viz.) by the Diviſion of Indices, as Involution was by their Multiplication—Thus the ſquare Root of x^{6}, $=\underline{x^{6}}=x^{3}$ and $x^{5}=x^{\frac{5}{2}}=\surd^{2}\overline{x^{5}}|$ and univerſally, $xn=xn=\surd^{2}\overline{xn}$——Required the Square Root of $x-a^{2}$ it will be $\surd\overline{x-a^{2}}|^{\frac{1}{2}}$.

If the Cube Root of any Quantity were to be extracted, put the Radical Sign over the Quantity, with the Index 3 above the Radical, thus: the Cube Root of ax. will be $\surd^{3}\overline{ax}|$ of $x^{12}=\frac{12}{3}=x^{4}$ and $xn=\underline{x^{n}}_{3}$, &c.

Required the Cube Root of $x^{3}-3x^{2}y+3xy^{2}-y^{3}$. ∴ $\surd^{3}\overline{x^{3}-3x^{2}y+3x^{2}y-y^{3}}=x-xy$.

See the Work

$$
\begin{array}{rl}
 & x^3-3x^2y+3xy^2-y^3(x-y \\
 & x^3 \\
\hline
3x^2) & -3x^2y+3xy^2-y^3 \\
\hline
 & -3x^2y- \\
 & \quad +3xy^2- \\
 & \qquad -y^3 \\
\hline
 & -4x^2y+3xy^2-y^3 \\
\hline
\end{array}
$$

N. B. As Surd quantities are not eaſily managed without the help of a Maſter, I thought it beſt to omit them.

Of Equations.

An Equation is when two equal quantities, differently expreſſed, are compared together, by means of the ſign $=$ placed between them.

Thus, $9-4=5$ is an equation, expreſſing the Equality of the quantities $9-4$ and 5.

Alſo $x=n+m$ is an equation ſhewing that x is equal to the Sum of the Quantities n and m.

Equations are the means whereby we come at ſuch concluſions as to anſwer the conditions of any problem that may be propoſed; and this is called reduction of equations.

Reduction of Equations,

Has ſundry Rules, according as the Problem is propoſed; therefore when a Problem is propoſed, having but one unknown Quantity, it is called a ſingle Equation, though before the Quantity can be cleared we muſt examine it whether it requires Addition, Subtraction, Multiplication, or Diviſion, &c. And then having cleared the unknown Quantity, and brought it to one ſide of the Equation, and all the known Quantities to the other, the Problem will be done by ſome of the above Rules.

Here follow a few Examples to illuſtrate the Rule. Note. Any Quantity may be tranſpoſed to either ſide of the Equation, by changing its Sign. Thus $x+6=14 \therefore x-14-6=8$——And if $x-9=3 \therefore x=12$. Let $5x-9=4x+8$. Then it is plain $5x$ muſt be $=4x+17$ and by taking $4x$ out of both ſides of the Equation we have $x=17$.

Reduction where two or more unknown Quantities are concerned; find the value of any of them, and then compare them together, remembering to get as many Values of each unknown Quantity as you have Equations, otherwiſe it will be in vain to attempt the Solution of any

Problem by any Learner —— Here follow a few Examples that will illuſtrate the whole.

Let	1	$5x+8y=124$
—	2	$3x-2y=20$
2×4	3	$12x-8y=80$
1×3	4	$17x=204$
4÷17	5	$x=\frac{204}{17}=12$
2×5	6	$15x-10y=100$
1×3	7	$15x+24y=372$
7−6	8	$34y=272$
8÷34	9	$y=8$

Note, When you multiply any Equation by an abſolute Number, you muſt daſh the Figure as in Step the 3d, 5th, 7th, &c. to ſhew that they are not Steps multiplied together.

Given. $\begin{cases} 5x-3v=90 \\ 2x+5v=160 \end{cases}$

Now in order to exterminate x, let the firſt Equation be multiplied by 2, and the ſecond by 5, to make the Coefficients of x alike, there will ariſe the two following Equations, $10x-6v=180$

$10x+25v=800$

the firſt of which ſubtracted from the ſecond, we have $31v=620$ which Difference being divided by 31 the Coefficient of v, gives $v=20$, then by tranſpoſing 6v in the firſt Equation, we have $10x=180+6v$ or $10x=180+120 \therefore x=30$.

Let	1	$x+y=13=a$
	2	$x+z=14=b$
	3	$y+z=15=d$
1+3+3	4	$2x+2y+2z=42=a+b+d$
4÷2	5	$x+y+z=21=\frac{a+b+d}{2}$
5−3	6	$x=\frac{a+b+d}{2}d=6$
5−2	7	$y=\frac{a+b+d}{2}b=7$
5−1	8	$x=\frac{a+b+d}{2}a=8$

REDUCTION of EQUATIONS.

If the Quantities of the Equation be fractional, make them on each ſide pure Fractions, then multiply them croſs-wiſe and the Product may be reduced as before.

Example. $\frac{x}{4}=\frac{a}{3}$ then $x\times3=a\times4$ ∴ $3x=4a$. hence $x=\frac{4a}{3}$

Exam. 2. $\frac{x}{4}=\frac{16}{x}$ ∴ $x^2=64$ by extracting the Square Root on both ſides of the Equation, we have $x=8$.

Exam. 3. $\frac{x-b}{a}=\frac{b-x}{n}$, which Equation being multiplied croſs-wiſe, we have, $nx-nb=ab-ax$, and by Tranſpoſition and Diviſion, we have $x=\frac{ab+nb}{n+a}$

When any Equation has a Radical Sign on one Side, the other Side muſt be raiſed to the ſame Dimenſion.

Exam. 1. Let $\sqrt{ax}=b$, then by ſquaring b, we take off the Radical, and we have $ax=b^2$, Now in order to find x we muſt divide both ſides of the Equation by a ∴ $x=\frac{b^2}{a}$, and ſo for any other.

When a Problem hath two Dimenſions, it cannot be anſwered by any of the Methods before laid down; and therefore we muſt have recourſe to ſome other Method; which is by completing the Square, and is performed by the following

Rule

Add the Square of half the Co-efficient of the unknown quantity to both ſides of the Equation, and the Square will be complete,

Given $x^2+2ax=b$ Required x.

Firſt, let a be ſquared, and added as above; we have $x^2+2ax+a^2=a^2+b$, Then by extracting the root on both Sides of the Equation we have $x+a=\sqrt{a^2+b}$ and by tranſpoſing a we have $x=\sqrt{a^2+b}-a$.

Of

Of PROBLEMS.

1. Problems are Queſtions to be ſolved.

2. The Solution of a Problem is, the Anſwer to a Queſtion, or the Determination of the quantity ſought.

3. The Problem has oftentimes various Anſwers, and therefore it is neceſſary to know when it is truly limited; which may be known by the following

Rule.

When the number of Quantities ſought, exceeds the Number of Equations given, the Queſtion admits of various anſwers.

Suppoſe $x+y=20$ now it is plain x may be any Number, whole or broken, being leſs than 20, and y the Remainder.

Rule.

When the Number of Quantities ſought are equal the number of Equations (not depending on each other) the Queſtion is truly limited.

Axioms for the more ready ſolving of Queſtions.

Ax. 1. If from the Sum of any two Quantities either Quantity be taken, the remainder is the other Quantity.

Ax. 2. The difference of any two Quantities being added to the leſs, the Sum is the greater.

Ax. 3. The Product of any two Quantities being divided by either Quantity, the Quotient is the other.

Ax. 4. The Quotient of any two Quantities being multiplied by the leſs, the Product is the greater.

Ax. 5. The Rectangle of the Sum, and Difference of any two Quantities, is equal to the difference of their Squares.

Ax. 6. The Difference of the Squares of the Sum, and difference of any two Quantities, is equal four times their Rectangle.

Ax. 7. The Sum of the Squares of the Sum, and difference of any two Quantities, is equal twice the Sum of their Squares.

$x+y=$ Sum.
$x-y=$ Difference.
$xy=$ Product.
$\frac{x}{y}=$ Quotient.
$x^2+y^2=$ Sum of the Squares.
$x^2-y^2=$ Difference of the Squares.

Note, The above being underſtood, any Problem may be eaſily taken in the Algebraic Method.

SOLUTION OF PROBLEMS.

Prob 1. The ſum of Two Numbers being 14, and their difference 4, required the Numbers, with a Theorem for all ſuch Queſtions?

Let $14=a$ $4=b$ and $x=$ leſs, than
$b+x$ will be the greater number per Ax. 2.
1 | $2x+b=a$ P.Q.
1 — ÷ | 2 | $x=\frac{a-b}{2}=5$
$b+x=9$
N. B. I ſhall follow the above Numbers for 10 Queſtions.

Prob. 2. The Sum of two Numbers $=2a$ and their Product $=p$.

Required the Number?
$x=\frac{1}{2}$ Difference of the Numbers $a+x=$ greater
$a-x=$ leſs
— | 1 | $a^2-x=p$
1 + | 2 | $a^2-p=xz$
2 √ | 3 | $x=\sqrt{a^2-p2}=2$ ∴ $a+x=9$, $a-x=5$

Prob. 3. The Sum of two Numbers 14 (a) and their Quotient 1, 8 (q) required the Numbers.

$x=$ leſs & $qx=$ greater Number
1 | $x+qx=a$ P.Q.
1 ÷ | 2 | $x=\frac{a}{q+1}=5$
$qx=9$

Prob. 4. The Sum of any two Numbers being 14, and the Sum of their Squares 106, required the Numbers?

14=

		$14 = 2a$, $x = \frac{1}{2}$ Difference, $106 = b$
		$a + x =$ greater and $a - x =$ leſs Number
	1	$2a^2 + 2x^2 = b$
1 — ÷	2	$x^2 = \frac{b - 2a^2}{2}$
		$x = \sqrt{\frac{b - 2a^2}{2}} = 2$
2 √	3	$a + x = 9$
		$a - x = 5$

Prob. 5. The Sum of two Numbers 14, and the Difference of their Squares 56, required the Numbers?

		$x = \frac{1}{2}$ Differ. of the Numbers, $14 = 2a$, $56 = b$
		$a + x =$ greater $a - x =$ leſs
	1	$4ax = b$
1 ÷ 4a	2	$x = \frac{b}{4a} = 2$
		$a + x = 9$; $a - x = 5$

Prob. 6. The Difference of two Numbers 4, and their Product 45, required the Numbers?

		$x = \frac{1}{2}$ Sum, $4 = 2a$, $45 = b$.
		$x + a =$ greater, $a - x =$ leſs
	1	$a^2 - x^2 = b$
1 + √	2	$x = \sqrt{a^2 - b} = 2$
		$x + a = 9$
		$x - a = 5$

Prob. 7. The Difference of two Numbers being 4, and their quotient 1, 8 required the Numbers?

		$x =$ leſs, $1, 8 = q$, $4 = b$
		$qx =$ greater
	1	$qx - x = b$
1 ÷	2	$x = \frac{b}{q - 1} = 5$
		$qx = 9$

Prob. 8. The Difference of two Numbers 4, and the Sum of their Squares 106, required the Numbers?

$x =$

		x=½ Sum 4=2a, 106=b
		x+a=greater x—a=leſs
	1	$2x^2+2a^3=b$
1 — ÷ √	2	$x=\frac{\sqrt{b-2a^2}}{2}=2$
		x+a=9; x—a=5

Prob. 9. The Sum of two Numbers 14, and the difference of their Squares 56, required the Numbers?

		14=2a, 56=b, x=½ Difference
		a+x=greater a—x=leſs
	1	4ax=b
1 ÷	2	$x=\frac{b}{4a}=2$
		a+x=9
		a—x=5

Prob. 10. The product of two Numbers being 45, and their Quotient 1, 8 required the Numbers?

		45=b, 1, 8=q.
		x=leſs, qx=greater
	1	qx =b
1 ÷ √	2	$x=\frac{\sqrt{b}}{q}=5$
		qx=9

Prob. 11. What two Numbers are thoſe whoſe Sum is 40, and Product only 4, with a Theorem for all ſuch Queſtions?

		Let 40=2a, 4=b, x=½ Diff. of the Numbers.
		then a+x=greater and a—x leſs Number.
	1	$a^2-x^2=b$
1 + —	2	$a^2-b=x^2$
2 √	3	$x=\sqrt{a^2-b}=19,899$
		a+x=39,899
		a—x=0,101

Prob. 12. Required two Numbers whoſe Product is 12, and the difference of their Squares 7?

$\sqrt{x}$ = greater. $\frac{12}{\sqrt{x}}$ = leſs

1 | $x - \frac{144}{x} = 7$

$1 \times x$ | 2 | $x^2 - 144 = 7x$

$2 - +$ | 3 | $x^2 - 7x = 144$

3 □ | 4 | $x^2 - 7x + 12{,}25 = 126{,}25$

$4 \sqrt{}$ | 5 | $x - 3{,}5 \quad 12{,}5$

$5 +$ | 6 | $x = 16$

$6 \sqrt{}$ | 7 | $\sqrt{x} = 4$

8 | $\frac{12}{\sqrt{x}} = 3$

Prob. 13. A Gentleman being aſked how many Guineas he had in his pocket? anſwered, if the Number I have be multiplied by 4, and the Square Root of the Product be added to the Number of Guineas I have in my pocket, that Sum will be equal 440, required the Number?

x^2 = the Guineas he then had.

1 | $x^2 + 2x = 440$ P. Q.

1 □ | 2 | $x^2 + 2x + 1 = 441$

$2 \sqrt{}$ | 3 | $x + 1 = 21$

-3 | 4 | $x - 20 . \therefore . x^2 \mid 400$

Prob. 14. What number is that which being divided into 4 or 5 equal parts, the Product of all the parts in both Caſes ſhall be exactly the ſame.

x = the Number.

1 | $\frac{x}{4} \times \frac{x}{4} \times \frac{x}{4} \times \frac{x}{4} = \frac{x}{5} \times \frac{x}{5} \times \frac{x}{5} \times \frac{x}{5} \times \frac{x}{5}$ P.Q

$1 \times$ | 2 | $\frac{x^5}{3125} \quad \frac{x^4}{256}$

$2 \div$ | 3 | $x = \frac{3125}{256} = 12\frac{53}{256}$

Prob. 15. A Merchant ſhips Goods for Jamaica, to the Amount of £.500; pays for Inſurance 5 per Cent. for prompt Payment 3 per Cent. how much money ought the Merchant to inſure, ſuppoſing the Goods to be loſt, that he ſhall be no loſer?

x=

		$x=$ the Sum to be inſured.
	1	$x-\frac{8x}{100}=500$
1 ×	2	$100x-8x=50000$
2 ÷	3	$x=\frac{50000}{92}=$ £.543-9-6$\frac{3}{4}$, $\frac{3}{23}$

Prob. 16. You that are skill'd in mathematic Arts,
Divide 100 into two ſuch Parts,
That when thoſe Parts each other hath divided,
Their Quotients make just 5, if right decided?

		$x=\frac{1}{2}$ Diff. of the Numbers, $100=2a$; $b=5$ $+x=$ greater and $a-x=$ less Number.
		$\frac{a+x}{a-x}+\frac{a-x}{a+x}=b$ P. 2. which reduced
Gives.	2	$x=\frac{\sqrt{ba^2-2a}}{b+2}=32{,}7326$ hence 82, 7326 and 17, 2674 are the Parts.

Prob. 17. x Shews the Years, and y the Months explain,
Hence by a Quadratic, you my age May gain.

	1	$x^2y+xy^2=8580=a$ } Reqd. x and y
	2	$x^2y^3+x^3y^2=1338480=b$
2 ÷ 1	3	$xy=\frac{b}{a}=156=n$
3 ÷	4	$y=\frac{n}{x}$
4 + 2	5	$y^2=\frac{n^2}{x^2}$
∴	6	$nx+\frac{n^2}{x}=a$, by Substitution.
6 × −	7	$nx^2-ax=n^2$
7 ÷	8	$x^2-\frac{ax}{n}=-n$ let $-2mx=-\frac{ax}{n}$
then	9	$x^2-2mx-m^2=m^2-n$
9 √	10	$x-m=\sqrt{m^2-n}$
10 +	11	$x=m+\sqrt{m^2-n}=52$ Years.
P 4	12	$y=\frac{n}{x}=8$ Months, answering to August.

FINIS.